Phase Shifter Operation of the Azimuthally Magnetized Coaxial Ferrite Waveguide

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(Received May 5, 2012)

The terms for operation of the coaxial waveguide, entirely filled with azimuthally magnetized latching ferrite, as a digital nonreciprocal phase shifter for the normal \(TE_{01}\) mode, are found. They are classified as physical, mathematical and functional ones. The physical prerequisites are drawn from the phase curves of the structure and specify the boundaries of the interval in which it produces differential phase shift for a given numerical equivalent of the modulus of off-diagonal ferrite permeability tensor element. The mathematical condition brings the parameters of configuration together with certain roots of its characteristic equation, derived in terms of complex Kummer and Tricomi confluent hypergeometric functions and with the related to them positive real \(L_2(c, \rho, n)\) numbers \([c = 3, \ 0 < \rho < 1, \ n = 1]\). The functional criteria determine the borders of the domain of phase shifter operation of the geometry. These are functions, defined for a fixed central conductor thickness which express in normalized form the impact of the guide radius on the phase shift at the cut-off frequencies and at the envelopes, denoting the termination of the phase curves for negative ferrite magnetization from the side of higher frequencies. The same are reckoned, employing iterative methods, consisting in a repeated numerical solution of the equation mentioned, followed by a computation of the guide radius and phase constant of the wave and are plotted graphically. The influence of the parameters of transmission line on the area referred to is analyzed.

PACS: 02.60.Lj, 02.90.+p, 41.20.-q, 41.20.Jh, 85.70.Ge

1. Introduction

It is thought that the circular waveguides, comprising coaxially positioned cylindrical or toroidal inserts of azimuthally magnetized ferrite that support normal \(TE_{01}\) mode, are natural microwave structures for digital nonreciprocal phase shifters \([1–35]\). This is due to their ability to afford differential phase shift when latching the magnetization between its two stable states and to their symmetry by reason of which the electromagnetic wave interacts with the entire body of the anisotropic medium. The devices in question could be employed in the design of electronically scanned antenna arrays \([36–40]\). Such is e.g. the Kashin and Safonov monopulse transmit-receive phased array with polarization of targets in the main beam \([40]\). As an antenna element for the \(X\) and \(C\) bands a composition has been proposed, constructed of a phase shifter of the type referred to and of axially symmetric transmitters which could be manufactured of the same material. The magnetizing conductor passes along the centre-line of the setup and has two shoulders, serving like current inputs. Symmetrizing wires are placed in their neighbourhood, providing independence of the propagation conditions of the wave inside the phase shifter of its polarization. Steps are taken to match the wires inhomogeneity. This mechanism enables a phasing of the elements of the array with the help of one and the same control program for whatever polarization. It is nonreciprocal and needs rephrasing in changing the transmit and receive regimes of the antenna.

In this investigation three kinds of criteria for phase shifter operation of a coaxial ferrite waveguide with azimuthal magnetization which sustains normal \(TE_{01}\) mode, are established. With that end in view, the phase curves of the transmission line, some roots of its characteristic equation, written by complex confluent hypergeometric functions and the bound up with them positive real \(L_2(c, \rho, n)\) numbers \([30]\) (denoted also as \(L(c, \rho, n)\) \([19, 28]\)), are harnessed. Besides, a numerical approach, widely exploiting iterative techniques is used, too. The domain in which phase shift is produced, is depicted graphically. The influence of structure parameters on it is examined.

2. Synopsis of boundary-value problem

An infinitely long, perfectly conducting coaxial waveguide of inner and outer conductor radii \(r_1\) and \(r_0\), respectively, filled entirely with latching ferrite, magnetized in azimuthal direction to remanence, is thershed out (see inset, Fig. 6), propagating normal \(TE_{01}\) (\(H_r\), \(E_\theta\), \(H_z\)) modes of phase constant \(\beta\). A cylindrical co-ordinate system \((r, \vartheta, z)\) is accepted. The anisotropic medium possesses a Polder permeability tensor \(\bar{\mu} = \mu_0 [\mu_{ij}]\), \(i, j = 1, 2, 3\), with nonzero components \(\mu_{11} = 1\) and \(\mu_{13} = -\mu_{31} = -\mu_{0} \alpha \gamma M_r / \omega\), \(\omega = 1 < \alpha < 1\) (\(\gamma\) — gyromagnetic ratio, \(M_r\) — ferrite remanent magnetization, \(\omega\) — angular frequency of the wave) and a scalar permittivity \(\varepsilon = \varepsilon_0 \varepsilon_r\) (\(\varepsilon_0\) and \(\mu_0\) — free space permittivity and permeability, respectively). The characteristic equation of configuration \([20, 25, 28]\):

\[\Phi(a, c; x_0) / \Psi(a, c, x_0) = \Phi(a, c, \rho x_0) / \Psi(a, c, \rho x_0) \] (1)

is represented in terms of the Kummer and Tricomi confluent hypergeometric functions \(\Phi(a, c, x)\) and \(\Psi(a, c, x)\).
respectively [41], in which $a = 1.5 - jk$, $c = 3$, $x_0 = j z_0$, $k = \alpha / (2\beta)$, $-\infty < k < +\infty$, $\beta_2 = (1 - \alpha^2 - \beta^2)^{1/2}$, $z_0 = 2\beta_2 r_0$, $z_0 > 0$, $\rho = \bar{r}_1 / r_0$, $0 < \rho < 1$ — central conductor to waveguide radius ratio. The introduction of barred (normalized) quantities: $\beta = \beta / (\beta_0 \sqrt{\gamma})$, $\beta_2 = \beta_2 / (\beta_0 \sqrt{\gamma})$ ($\beta_2 = \omega_0^2 \mu_0 \varepsilon_0 (1 - \alpha^2) - \beta^2)^{1/2}$ — radial wavenumber, $\bar{r}_1 = \beta_0 \bar{r}_1 \sqrt{\gamma}$, and $\bar{r}_0 = \beta_0 r_0 \sqrt{\gamma}$, $(\beta_0 = \omega_0 \varepsilon_0 \mu_0$ — free space phase constant) permits to obtain general results, holding for all admissible values of parameters and all frequency bands in which transmission may be realized. If $\chi_{k,n}^{(c)}(\rho) \ (n = 1, 2, 3, \ldots)$ designates the positive purely imaginary roots of Eq. (1), the same is valid in case of $\bar{\beta}_2 = \chi_{k,n}^{(c)}(2\bar{r}_0)$ that yields the eigenvalue spectrum of the fields explored. In what follows normal $TE_{01}$ mode $(n = 1)$ is examined solely.

3. Some features of the coaxial waveguide with azimuthally magnetized ferrite

3.1. Main points

The analysis of the phase curves of geometry under study [19], parts of which are reproduced in Figs. 1–3, discloses its nonreciprocal character which is manifested in the following [28]:

i) For all allowable numerical equivalents of the magnitude of off-diagonal ferrite permeability tensor element $[\alpha]$ there are values of the normalized guide radius $\bar{r}_0$ for which propagation may take place with two different phase constants $\beta_+$ and $\beta_-$, corresponding to positive and negative magnetization. Without exception it holds: $\beta_- > \beta_+$. Accordingly, the $\beta_-(\bar{r}_0)$ — characteristics (drawn by dashed lines) are situated above the $\beta_+(\bar{r}_0)$ — ones (plotted by solid curves), (cf. Figs. 1–3 and Fig. 1 in [19]). For every single pair of parameters

\[ \{ \bar{r}_0, [\alpha] \} \]

differential phase shift $\Delta \beta = \beta_- - \beta_+$ may be obtained. The quantity $\Delta \beta$ is always positive (see Fig. 1);

ii) All $\beta_-(\bar{r}_0)$ — curves are finite and the $\beta_+(\bar{r}_0)$ — ones — infinite. The couple of $\beta_-(\bar{r}_0)$ and $\beta_+(\bar{r}_0)$ — characteristics for the same $[\alpha]$ originates in the cut-off frequency (bifurcation) point $(\bar{r}_{0cr}, \beta_{cr})$ at the horizontal axis of phase portrait, denoted by a circle (cf. Figs. 1–3 and Fig. 1 in [19]). A special $En_{\bar{r}_1}$ — envelope (depicted by a green dotted line) exists, restricting the $\beta_-(\bar{r}_0)$ — curves from the side of higher frequencies. The $\beta_+(\bar{r}_0)$ — characteristics are unlimited from above (see Figs. 1 and 3, and Fig. 1 in [19]);
iii) The $\beta_-(\bar{r}_0)$ – curves have a bulge (area of double-valuedness) below cut-off. The leftmost (inversion) point $(r_{0-}, \beta_{-})$ (shown by a star) denotes the lowest frequency for which propagation may take place for $\alpha_- < 0$ (cf. Fig. 2). For the same $\bar{r}_0$ from the interval $(r_{0-}, r_{0cr})$ the structure may guide a forward and a backward wave (whose characteristics are portrayed by red and blue dashed lines, respectively) with two different phase constants $\beta_{-1}$ and $\beta_{-2}$ ($\beta_{-1} > \beta_{-2}$), respectively on condition that $\alpha_- < 0$. No transmission is possible in this area provided $\alpha_+ > 0$ (see Fig. 2).

iv) A magnetically controlled cut-off is observed. If $\bar{r}_0 = r_{0ncr}$, the propagation stops when $\alpha_+ > 0$ and $\beta_{cr+} = 0$ ($k_{cr+} = 0$). In case $\alpha_- < 0$, however, for the same $\bar{r}_0$, $(r_{0-} = r_{0ncr} = r_{0cr})$ two waves of different phase constants may be excited. The first of them is in cut-off regime and for it $\beta_{cr+} = 0$ ($k_{cr+} = 0$, $\beta_{cr+} \equiv \beta_{-1}$). The second wave is in transmission state with constant $\beta_{cr-} \neq 0$ ($k_{cr-} \neq 0$, $\beta_{cr+} \equiv \beta_{-1}$) (cf. Fig. 2).

3.2. Some important formulae

i) The phase curves are calculated from [25, 28]:

$$\bar{r}_0 = \left( k \chi_{k,2}(\rho)/\alpha \right) \left[ \left( 1 + (\alpha/(2k))^2 \right) / (1 - \alpha^2) \right]^{1/2},$$

(2)

ii) The critical guide radius is determined by the expression [25, 28]:

$$\bar{r}_{0ncr} = \chi_{r,0}(\rho)/\left( 2 \sqrt{1 - \alpha^2} \right).$$

(3)

iii) The equation $\beta_{cn} = \beta_{cn}(\bar{r}_{0ncr})$ of the envelope, written in parametric form, is [19, 25, 28]:

$$\bar{r}_{0ncr} = L_2(c, \rho, n)/\left[ \alpha_{cn} / (1 - \alpha_{cn}^2) \right]^{1/2},$$

(4)

$$\beta_{cn} = \left( 1 - \alpha_{cn}^2 \right)^{-1/2},$$

(5)

where $L_2(c, \rho, n)$ are certain positive real numbers [30] and $\alpha_{cn} < 0$ is a parameter. Assuming $\rho = 0.1$ and 0.5, it is found out that: $\chi_{r,0}(\rho) = 7.8818832204$ and $12.7863135322$, and $L_2(c, \rho, n) = 7.65209$ and $30.628585 (c = 3, n = 1)$. The following subscripts are used: (i) “+” (−)” for the quantities, answering to the positive (negative) magnetization; (ii) “cr” (“cn”) for the ones, relating to cut-off (to transmission state for $\alpha_- < 0$, linked with the cut-off, cf. item (iv) in Sect. 3.1); (iii) “en” (“e+)” for those, describing the envelope (the point from the $\beta_1(\bar{r}_0)$ – curve for certain $|\alpha|$ of abscissa equal to that of the end point of the $\beta_1(\bar{r}_0)$ – one for the same $|\alpha|$ at the envelope); see Figs. 1, 2.

4. Physical prerequisites for phase shifter operation

Bearing in mind the above said, it might be concluded that for any $\rho$ and fixed $|\alpha|$ [28]:

i) At positive (negative) ferrite magnetization normal $TE_{01}$ mode could be sustained in the semi-closed interval $\Delta_+ = [\bar{r}_{0cr}, +\infty)$ (bilateral restricted interval $\Delta_- = [r_{0cr}, r_{0ncr}]$);

ii) For both signs of magnetization the wave may be guided in the interval $\Delta = \Delta_+ \cap \Delta_-$ (cf. Fig. 1) of overlapping (of intersection) of $\Delta_+ \cap \Delta_-$;

iii) Differential phase shift may be afforded in $\Delta$.

iv) For the abscissa $r_0$ of the working point $(\bar{r}_0, \beta_+)$, lying at the curve $\beta_+(\bar{r}_0)$ ($\beta_+(\bar{r}_0)$) which conforms to $\alpha_+ > 0$ ($\alpha_- < 0$), it is true: $\bar{r}_0 \in \Delta_+(r_0 \in \Delta_-)$.

v) For the common abscissa $\bar{r}_0$ of the pair of working points $(\bar{r}_0, \beta_+)$ and $(\bar{r}_0, \beta_-)$, situated at the curves $\beta_+(\bar{r}_0)$ and $\beta_-(\bar{r}_0)$, respectively for the same $|\alpha|$, it is valid: $\bar{r}_0 \in \Delta$.

Thus, the physical criterion the geometry of any $\rho$ to behave as a phase shifter for certain $|\alpha|$, is

$$\bar{r}_{0ncr} < \bar{r}_0 < \bar{r}_{0nc} - \bar{r}_{0ncr}.$$  

(7)

(The above discussion is visualized in Fig. 3.)

5. Mathematical condition for phase shifter operation

The inequalities [28]:

$$\chi_{r,0}(\rho)/2 < \bar{r}_0 \sqrt{1 - \alpha^2} < L_2(c, \rho, n)/|\alpha|$$

(8)

express the mathematical criterion of the structure to work as a phase shifter. It is a direct corollary of relation (7), combined with Eqs. (4) and (5) in case $|\alpha| \equiv |\alpha_{cr}| \equiv |\alpha_{cn}|$ and specifies the sets of parameters $\{\rho, \bar{r}_0, |\alpha|\}$ for which $\Delta_\beta$ is produced. (Everywhere in Ref. [28] the symbol $L(c, \rho, n)$ stands for $L_2(c, \rho, n)$.)

6. Functional criteria for phase shifter operation

6.1. Left limiting function

Since $\beta_{cr+} = \beta_{cr-} = \beta_{cr} = 0$, for the couple of points $(\bar{r}_{0cr}, \beta_{cr-})$ and $(\bar{r}_{0cr}, \beta_{cr+})$, $(\bar{r}_{0cr}, \equiv \bar{r}_{0cr})$, it is fulfilled: $\Delta_\beta_{cr} = \beta_{cr}$(cf. Figs. 1, 2). The function $\Delta_\beta_{cr} = \Delta_\beta_{cr}(\rho, \bar{r}_0, n)$, presented in parametric form as [28]:

$$\beta_{cr} = \beta_{cr}(\alpha_-, k_{cr}),$$

(9)

$$\bar{r}_0 = \bar{r}_{0cr}(\rho, \alpha_-, k_{cr}),$$

(10)

determines the left limit (bounded with cut-off) of the domain in which phase shift is observed, if $\bar{r}_0 \equiv \bar{r}_{0cr}$, $\bar{r}_{0cr} = \bar{r}_{0cr}(\rho, \alpha_{cr}, n)$ (see Eq. (4)) and $\alpha_- = \alpha_{cr}$. Equations (9) and (10) are given explicitly by the ones (2) and (3) in which $\alpha$ and $k$ are substituted by $\alpha_-$ and $k_{cr}$.

6.2. Right limiting function

At certain $|\alpha| = |\alpha_{en}|$ for the pair of points $(\bar{r}_{0en}, \beta_{en-})$ and $(\bar{r}_{0en}, \beta_{en+})$, $(\bar{r}_{0en} \equiv \bar{r}_{0en})$ (see Fig. 1), it is true [28]:

$$\Delta_\beta_{en} = \beta_{en} - \beta_{en+}.$$  

(11)

The function $\Delta_\beta_{en} = \Delta_\beta_{en}(\rho, \bar{r}_0, n)$, written parametrically as [28]:

...
\[ \beta_{en^-} = \beta_{en^-}(\alpha^-), \]
\[ \beta_{en^+} = \beta_{en^+}(\alpha^+, k_{en^+}), \]
\[ \tilde{r}_0 = \tilde{r}_0(\rho, \alpha^+, k_{en^+}, n), \]
specifies the right limit (connected with the envelope) of the area in which \( \Delta \beta \) is available in case \( \tilde{r}_0 \equiv \tilde{r}_{0en^-}, \tilde{r}_{0en^+} = \tilde{r}_{0en^-}(\rho, \alpha_{en^-}, n) \). Equations (13) and (14) are equivalent to the ones (2) and (3) with \( \alpha^+ \) and \( k_{en^+} \), replacing \( \alpha \) and \( k \), and Eq. (12) represents the one (6), respectively in it \( \alpha^- \) stands for \( \alpha_{en^-} \) (\( n = 1 \) for \( TE_{01} \) mode).

7. Iterative method for tracing the left limit of the domain of phase shifter operation

7.1. Description of the method

First, values of the ratio \( \rho^{ch} \) and of the off-diagonal tensor element \( |\alpha_{ch}| \) are singled out and the relevant one of the critical radius \( \tilde{r}_{0ch} \) is found from formula (4). Next, for an arbitrarily picked out negative numerical equivalent \( k_{ch} \) of the imaginary part \( k \) of the parameter \( a \) of the confluent functions, the positive purely imaginary roots \( \chi^{(ch)}_{k_{ch}n}(\rho^{ch}) \) of Eq. (1) in \( x_0 \) (in \( z_0 \)) are determined (\( n = 1, 2, 3, \ldots \)). Then, the numbers, answering to \( \alpha_{ch}^-, k_{ch}^- \) and \( \chi^{(ch)}_{k_{ch}^-n}(\rho^{ch}) \) are put in expressions (2), (3). After that \( k_{ch}^+ \) is changed and the scheme is hammered away, until an interval of values \( \Delta k_{ch} = [k_{ch}^- - \Delta k_{ch}^- < k_{ch}^+ < k_{ch}^+] \) is specified, such that \( \tilde{r}^{ch}_{0ch} \) lies in the pertinent one \( \Delta k_{ch} = [\tilde{r}^{ch}_{0ch} < \tilde{r}^{ch}_{0ch}]. \) Then \( k_{ch} \) is divided into \( k_{ch}^{ch} \) parts and that of them is considered for which the relevant interval for \( \tilde{r}^{ch}_{0ch} \) contains \( \chi^{(ch)}_{k_{ch}n}(\rho^{ch}) \). The procedure goes on, until it becomes true: \( |\tilde{r}^{ch}_{0ch} - \tilde{r}^{ch}_{0ch}| < \varepsilon^{ch} \varepsilon^{ch} \) — the prescribed accuracy. The value of \( \beta_{ch}^- \), corresponding to the last iteration is taken as that, searched for. Afterwards, [\( \alpha_{ch} \)] is altered and what has been described is repeated. Finally, a new \( \rho^{ch} \) is accepted and the calculations are done again. The chosen (computed) quantities are marked by a superscript “\( ch \)” ("comp")

7.2. Numerical example

Let \( \rho^{ch} = 0.1 \) and \( |\alpha_{ch}| = 0.1 \). Accordingly, it is found that: \( \tilde{r}^{ch}_{0ch} = 3.96009 53061 \). Let now \( k_{ch} = -0.1 \) be an arbitrary chosen negative value of the parameter \( k \). On doing it for the relevant computations, it is obtained:

\[ \chi^{(ch)}_{k_{ch}n}(\rho^{ch}) = 7.50329 93839, \tilde{r}^{ch}_{0ch} = 8.43120 56878 \]
and \( \beta_{ch}^+ = 0.88994 38185 \). Due to the fact that \( \tilde{r}^{ch}_{0ch} \neq \tilde{r}^{ch}_{0ch} \left( \tilde{r}^{ch}_{0ch} > \tilde{r}^{ch}_{0ch} \right) \), the process should continue. Taking \( k_{ch} = -0.2 \), yields: \( \chi^{(ch)}_{k_{ch}n}(\rho^{ch}) = 7.14087 84197, \tilde{r}^{ch}_{0ch} = 14.79546 11178 \) and \( \beta_{ch}^- = 0.96527 95998 \), respectively. In this case \( \tilde{r}^{ch}_{0ch} \) is even larger than in the previous one. Hence, the value of \( k_{ch} \) sought is smaller than the ones, already used. For \( k_{ch} = -0.0024 \)

\( k_{ch} = -0.0025 \) it is reckoned: \( \chi^{(ch)}_{k_{ch}n}(\rho^{ch}) = 7.87261 37226, \tilde{r}^{ch}_{0ch} = 3.96009 20961, \beta_{ch}^- = 0.04770 44730 \) and \( \chi^{(ch)}_{k_{ch}n}(\rho^{ch}) = 7.87222 7658 \).

\( \tilde{r}^{ch}_{0ch} = 3.96008 50996, \beta_{ch}^- = 0.04969 73015 \), respectively. Thus, an interval \( \Delta k_{ch} \) has been found, containing \( \tilde{r}^{ch}_{0ch} \), respectively ones for \( k_{ch} \) and \( \beta_{ch}^- \) have been determined, involving the values sought.

Let \( m_{ch} = 10 \) and the interval for \( k_{ch} \) be divided into 10 parts. Then \( \tilde{r}^{ch}_{0ch} \) lies between the values of \( \tilde{r}^{ch}_{0ch} \), answering to \( k_{ch} = -0.00245 \) and

\[ 0 \leq \tilde{r}^{ch}_{0ch} \leq 4.00339 \]
\( k_{ch} = -0.00246 \), these are admitted as left and right-hand sides of the subsequent interval to be treated. The relevant numerical equivalents of \( \tilde{r}_{0\text{comp}} \) and \( \beta_{\text{comp}} \) are accepted as first approximations to the quantities looked for (cf. Table I). In a similar manner, it is ascertained that the following intervals for \( k_{ch} \) to be thresholded out should be: \([-0.002455, -0.002454], [-0.002456, -0.002454]\), etc. Table I contains the results of computations at both the both limits of pertinent intervals for the first 6 iterations in case \( \rho = 0.1 \) and \( \rho = 0.5 \), assuming \( |\alpha| = 0.1 \). Data, corresponding to the arbitrary change of \( k_{ch} \), are not included. Table II yields the upshot of numerical investigation for the same \( \rho = 0.1 \) and \( |\alpha| = 0.1 \). Besides, its fifth and sixth columns represent the function \( \Delta \beta_{\text{comp}} = \Delta \beta_{\text{comp}}(\rho, r_0, n) \) (cf. Sect. 6.1) numerically for discrete values of parameters. Using the outcomes of numerical study, this function is plotted graphically in Figs. 4-7 by dashed green \( LE_{\text{in}} \) lines in the intervals \( r_0 = (0 \pm 80) \) and \( (0 \pm 25) \) for the aforesaid numerical equivalents of \( \rho \) and \( n \).

**Table I**

<table>
<thead>
<tr>
<th>( N )</th>
<th>( k_{-ch} )</th>
<th>( \chi_{k_{-ch}^{-1}}^{(c)}(\rho^{ch}) )</th>
<th>( r_{\text{comp}} )</th>
<th>( \beta_{\text{comp}} )</th>
<th>( N )</th>
<th>( k_{-ch} )</th>
<th>( \chi_{k_{-ch}^{-1}}^{(c)}(\rho^{ch}) )</th>
<th>( r_{\text{comp}} )</th>
<th>( \beta_{\text{comp}} )</th>
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<td>0.04878 42400</td>
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<td>7.87200 30907</td>
<td>3.96079 10009</td>
<td>0.04878 42400</td>
</tr>
</tbody>
</table>

**Table II**

| \( |\alpha| \) | \( \rho = 0.1 \) | \( \rho = 0.1 \) | \( \rho = 0.1 \) | \( \rho = 0.1 \) |
|-------|--------|------------------|------|-------|
| 0.1   | 3.96079 53400 | -0.00245 40573 | 7.87200 30354 | 0.04878 53316 |
| 0.3   | 4.31220 93321 | -0.02232 73862 | 7.79599 19919 | 0.14044 57845 |
| 0.5   | 4.55000 73990 | -0.06340 30242 | 7.64001 24956 | 0.21289 46115 |
| 0.7   | 5.51842 20491 | -0.12870 14588 | 7.39759 54837 | 0.24464 83154 |
| 0.9   | 9.04114 01165 | -0.22387 61940 | 7.05680 58873 | 0.19415 58038 |
| 0.1   | 2.10500 42645 | -0.0105 79595 | 7.87200 30354 | 0.04878 53316 |
| 0.3   | 7.67083 92930 | -0.00953 89116 | 7.67053 75130 | 0.06054 13195 |
| 0.5   | 1.31825 90358 | -0.02659 27654 | 7.12749 90059 | 0.09160 66057 |
| 0.7   | 8.05221 06218 | -0.05407 22650 | 6.26451 53139 | 0.10576 29670 |
| 0.9   | 14.66090 75022 | -0.08724 88707 | 6.25234 46762 | 0.08300 46218 |

7.3. Maximum value of the normalized differential phase shift

Analyzing the numerical results obtained, it might be concluded that \( \Delta \beta_{\text{comp}} \) is a smooth continuous convex function of \( |\alpha| \) (of \( r_0 = 10 \)) to, possessing a maximum which is attained in both cases in the vicinity of point \( |\alpha| = 0.7 \) (cf. Table II). Its exact location is found by means of an iterative procedure, consisting in a successive computation of \( \Delta \beta_{\text{comp}} \) for varying \( |\alpha| \) around the latter. Performing a number of iterations, the next is calculated for \( |\alpha| \) by \( \tilde{r}_{0\text{comp}}, k_{-ch}^{-1}, \chi_{k_{-ch}^{-1}}^{(c)}(\rho^{ch}) \) and \( \beta_{\text{comp}}^{-1} \) when \( \rho = 0.1 \) and \( 0.5 \), respectively: 0.709183, 5.58979 42553 16039, -0.13235 98055 22100, 7.38421 88422 72688, 0.24655 08703 95035, 0.707148, 9.04181 60087 88339, -0.05350 05006 62200, 12.64240 38742 22261, 0.10578 43737 58764.
Numerical values of the quantities $k_{e+}^{ch}$; $\chi_{k_{e+}^ch}^{(c)\comp}(\rho^{ch})$, $(c = 3)$; $\beta_{e-}^{\comp}$; $\beta_{e+}^{\comp}$ for normal $TE_{01}$ mode in case $|\alpha^{ch}| = 0.1$ and $m^{ch} = 10$ as a function of the number of iteration $N$, assuming $\rho^{ch} = 0.1$, $r_{0en-c}^{\comp} = 76.88630$ and $\rho^{ch} = 0.5$, $r_{0en-c}^{\comp} = 302.14331$.

| $N$ | $k_{e+}^{ch}$ | $|\chi_{k_{e+}^ch}^{(c)\comp}(\rho^{ch})|$ | $\beta_{e-}^{\comp}$ | $\beta_{e+}^{\comp}$ | $N$ | $k_{e+}^{ch}$ | $|\chi_{k_{e+}^ch}^{(c)\comp}(\rho^{ch})|$ | $\beta_{e-}^{\comp}$ | $\beta_{e+}^{\comp}$ |
|-----|---------------|---------------------------------|----------------------|----------------------|-----|---------------|---------------------------------|----------------------|----------------------|
| 4   | 0.7000        | 0.896094 6402646               | 0.99245 87874       | 0.99246 03860       | 5   | 0.70210       | 0.897608 899806               | 0.99246 03860       | 0.99246 03860       |
| 2   | 0.7002        | 0.897608 360806               | 0.99246 03860       | 0.99246 03860       | 5   | 0.70210       | 0.897608 992686               | 0.99246 03860       | 0.99246 03860       |
| 3   | 0.70022       | 0.897608 899806               | 0.99246 03860       | 0.99246 03860       | 6   | 0.70023       | 0.897627 021556               | 0.99246 03860       | 0.99246 03860       |

![Fig. 6. Domain of phase shifter operation of the azimuthally magnetized coaxial ferrite waveguide for normal $TE_{01}$ mode in case $\rho = 0.5$, for the intervals $\bar{r}_0 = (0 \div 80)$ and $\Delta \beta = (0 \div 30)$.](image)

The maximum of the $LEn_1$ – curve (the maximum value of $\Delta \beta$ which the relevant geometry might afford at all) is marked in Figs. 4–7 by the points (1).

8. Iterative method for tracing the right limit of the domain of phase shifter operation

8.1. Description of the method

As before, values of the parameters $\rho^{ch}$ and $|\alpha^{ch}|$ are selected. Afterwards, the corresponding numerical equivalents of the normalized guide radius $r_{0en-c}^{\comp}$ and phase constant $\beta_{en-c}^{\comp}$ are reckoned from formulae (5) and (6). The $L_2(3, \rho^{ch}, 1)$ numbers are taken from Table 3 in [19]. Then the iterative scheme, described in Sect. 7.1, is effectuated for positive values of parameter $k_{e+}^{ch}$ to get $\chi_{k_{e+}^ch}^{(c)\comp}(\rho^{ch})$, respectively $r_{0en-c}^{\comp}$ and $\beta_{e+}^{\comp}$ looked for. The computations go on until the inequality $|\rho^{\comp} - \bar{r}_{0en-c}^{\comp}| < \varepsilon^{ch}$ becomes true. The quantity $\Delta \beta_{en-c}^{\comp}$ is determined as a difference between $\beta_{en-c}^{\comp}$ and $\beta_{e+}^{\comp}$.

8.2. Numerical example

Let like previously $\rho^{ch} = 0.1$, $|\alpha^{ch}| = 0.1$ and $m^{ch} = 10$. Formulae (5) and (6) with $L_2(3, \rho^{ch}, 1) = 7.65009$ yield $r_{0en-c}^{\comp} = 76.88630$ and $\beta_{en-c}^{\comp} = 0.99439$. For $k_{e+}^{ch} = 0.7$ and $k_{e+}^{ch} = 0.71$ (two arbitrary picked out positive values of $k$) it is obtained: $\chi_{k_{e+}^ch}^{(c)\comp}(\rho^{ch}) = 10.89669 40203$, $r_{0en-c}^{\comp} = 76.85644 21512$. $\beta_{e+}^{\comp} =
9. Domain of phase shifter operation

The $LE_{n_1}$ and the $RE_{n_1}$ lines in Figs. 4-7 portray the limits of the domain of existence of $\Delta \beta$ (represented by blue color), linked with the cut-off frequencies and with the envelope of the $\beta_{en}$ ($\rho_0$) characteristics [19], respectively. As seen, for specific $\rho_0 \in [\rho_0/n, 2L_2(c, p, n)], \Delta \beta$ is produced for all $[\alpha] \in [\alpha_{cr}, \alpha_{cr}]$, where $\alpha_{cr} = \sqrt{1 - [\chi^{(c)}_{en}(\rho_0)/(2\rho_0)]^2}$ and its value varies from $\alpha_{cr}$ to $2L_2(c, p, n)$, which is separated by one where it does not exist. In the lower (upper) zone phase shift is obtained for $[\alpha] \in [0, \alpha_{cr}]$ ($[\alpha] \in [\alpha_{cr}, \alpha_{cr}]$), $\alpha_{cr} = 0.9\sqrt{1 \pm (1 - 4L_2(c, p, n)/\rho_0)^2}$.

No $\Delta \beta$ is available, if $[\alpha] \notin [\alpha_{cr}, \alpha_{cr}]$. The absolute minimum of $\Delta \beta$ in $LE_{n_1}$ line is defined by two intervals for $\rho_0$. Moreover, the growth of central conductor thickness lessens the maxima of $LE_{n_1}$ - line (the maximum value that $\Delta \beta$ may attain, decreases). Simultaneously, the $RE_{n_1}$ - curve moves faster to the side of higher frequencies and the domain studied expands.

10. Conclusion

The criteria under which the azimuthally magnetized coaxial ferrite waveguide operates as a phase shifter for normal $TE_{01}$ mode, named as a physical, a mathematical and a functional one, are obtained. The cut-off frequency points and the peculiar envelope lines in the phase picture of configuration are used to formulate the first of them. The second condition connects the parameters of geometry with definite roots of its characteristic equation, presented by complex confluent hypergeometric functions and with the linked with them $L_2(c, p, n)$ numbers. The third one yields the differential phase shift afforded by the transmission line at cut-off and at the envelopes as a function of its parameters. The same is evaluated by iterative techniques, using the roots of the equation referred to. Its graphical image determines the limits of the domain of phase shifter operation of the waveguide. The potentiality of the latter as an element of a special kind of electronically scanned antenna array is also revealed.

Acknowledgments

We express our gratitude to our mother Trifonka Romanova Popivohlova and to our late father Nikola Georgiev Popivohlova for their self-denial and for their tremendous efforts to support all our undertakings.

References