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Mechanisms of Gap Solitons Formation in Periodic Ferromagnetic Structures

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The features of envelope soliton formation in one-dimensional periodic ferromagnetic structure were considered. The model based on the coupled nonlinear Schrödinger equations was used for investigation. The parameter space showing the region in which solitons similar to the Bragg solitons with different features can form was calculated. The mechanisms of the formation of the solitons localized on the limited length of the structure were considered.

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1. Introduction

At present, the investigation of envelope solitons (localized wave packets) has become a topic of great interest. These types of solitons are formed from pulses propagating in different media with nonlinearity and dispersion [1]. A new type of solitons called the Bragg soliton or gap solitons are formed in nonlinear media whose properties vary periodically with length, in a definite direction [2]. The photonic crystals are an example of such media in optics. In this structure the refractive index is a periodic function of spatial coordinates [3]. The investigation of the Bragg solitons is of great interest not only from a fundamental point of view, but also has great potential for practical use in telecommunication systems in optical communication lines. The magnonic crystals, where spin waves propagate, are similar to the photonic crystals [4–6]. The magnonic crystals have a number of significant advantages compared to the photonic crystals — the ability to manage their properties by an external magnetic field; the use of planar technology, and creating crystals with magnonic band-gap at microwave frequencies (of the order of several millimeters). The nonlinear effects in ferromagnetic films appear at relatively low power levels [5]. The magnonic crystals, by analogy with the photonic crystals, demonstrate more interesting nonlinear phenomena in comparison with the effects observed in homogeneous ferromagnetic films. However, we can conclude that the nonlinear processes in such periodic structures, including those associated with the peculiarities of formation of solitons, are insufficiently investigated. There are very few specific studies in this field [7–10] that show the experimental and numerical simulation results based on one-dimensional nonlinear Schrödinger equation (NSE). The coefficients of dispersion and nonlinearity were calculated based on the assumption that only one magnetostatic wave (MSW) propagates in the ferromagnetic film. The dispersion of this wave depends on the parameters of the periodic structure.

The coupled-mode theory was used to investigate fiber-optic gratings [3]. The essence of this method lies in the fact that the nonlinear wave processes in such structures are mainly due to a superposition of incident and reflected waves. The system of coupled NSE was used to describe this structure. In this case, the use of one NSE is a simplified approach to describe the nonlinear dynamics of periodic structures. The aim of this study was to investigate the features of formation of the solitons that are similar to the Bragg solitons in the ferromagnetic one-dimensional periodic structure. The system of coupled nonlinear Schrödinger equations for the amplitude envelope of forward-propagating (FP) and backward-propagating (BP) waves was used for numerical simulation. Great attention was paid to the conditions of formation of solitons, such as the Bragg or gap solitons.

2. Theoretical model

A one-dimensional periodic ferromagnetic structure (magnonic crystal) was considered. The structure is infinite in the x and y directions (see inset in Fig. 1). A constant magnetic field is applied perpendicularly to the film plane. The value of this field was chosen in such a way that the forward volume MSW (FVMSW) propagates in the y direction. FVMSW is multimode, but in this work a single-mode approximation is considered. In this case only one basic mode is excited and all parameters of the model are calculated for this mode. Following [5], dispersion relation for the one-dimensional system consisting of alternating sections of the ferromagnetic medium, in which wave propagates with different velocities, can be written as

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$$\cos KL = \cos(k_1(\omega)a_1)\cos(k_2(\omega)a_2) - \frac{k_1^2(\omega)+k_2^2(\omega)}{2k_1^2(\omega)k_2^2(\omega)} \times \sin(k_1(\omega)a_1)\sin(k_2(\omega)a_2), \quad (1)$$

where K is wave-number for a wave propagating in the structure with the period $L = a_1 + a_2$, a_2 is the width of the groove, and functions $k_1(\omega)$ and $k_2(\omega)$ are the dispersion relations of FVMSW for film thicknesses d_1 and d_2, respectively. These functions were determined by the ratio (1). Numerical solution of Eq. (1) leads to appearance of band gaps on the dispersion characteristics of $KL = \pi$. From the last condition (the Bragg condition), it follows that $K_{\rm B} = \frac{\pi}{L} = \frac{2\pi}{\lambda_{\rm B}}$, where $K_{\rm B}$ and $\lambda_{\rm B}$ are the Bragg propagation constant and the wavelength, respectively.

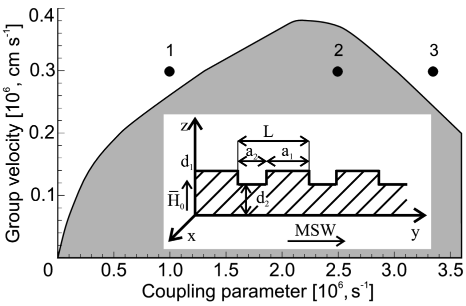

Fig. 1. The parameter space $(\chi, V_{\rm g})$, corresponding to envelope solitons formation ($\varphi_{0f} = 0.04$ and $\varphi_{0b} = 0$). Inset shows the scheme of the YIG film with the grating of grooves.

The Bragg condition provides the addition of a weak reflected wave on a phase along the entire length of the lattice. It leads to an effective reflection of the incident wave. To construct a nonlinear model of a periodic ferromagnetic structure, similar to optical systems [3], the coupled-wave approximation is used. The distribution of magnetostatic potential near the gap represents as the sum of FP and BP waves

$$\Psi(y,t) = \varphi_f(y,t)\exp(\mathrm{i}(\omega t - K_{\rm B})y) + \varphi_b(y,t)\exp(\mathrm{i}(\omega t + K_{\rm B})y), \quad (2)$$

where $\varphi_f(y,t)$ and $\varphi_b(y,t)$ are slowly varying complex envelopes of FP (incident) and BP (reflected) waves, respectively. Taking [3] into account for the approximation of weak nonlinearity, the nonlinear equations for the envelopes of direct and reflected waves can be represented as

$$\mathrm{i}\left(\frac{\partial\varphi_f}{\partial t} + V_{\rm g}\frac{\partial\varphi_f}{\partial y}\right) - \beta\frac{\partial^2\varphi_f}{\partial y^2} + \eta\varphi_f + \chi\varphi_b + \gamma\left(|\varphi_f|^2 + 2|\varphi_b|^2\right)\varphi_f = 0,$$

$$\mathrm{i}\left(\frac{\partial\varphi_b}{\partial t} - V_{\rm g}\frac{\partial\varphi_b}{\partial y}\right) - \beta\frac{\partial^2\varphi_b}{\partial y^2} + \eta\varphi_b + \chi\varphi_f + \gamma\left(|\varphi_b|^2 + 2|\varphi_f|^2\right)\varphi_b = 0, \quad (3)$$

where $V_{\rm g}$ is the group velocity; $\eta = \omega_0 - \omega_{\rm B}$ is frequency detuning (ω_0 — center frequency of the pulse, $\omega_{\rm B} = K_{\rm B}V_{\rm ph}$; $V_{\rm ph}$ is the MSW phase velocity in homogeneous structure); β is the coefficient of dispersion; χ is the coefficient of coupling; γ is the nonlinear coefficient. Equations (3) are similar to the system of two coupled nonlinear Schrödinger equations describing the propagation of direct and reflected waves in the Bragg optical lattices [2, 3]. However, in contrast to the model of NSE in optics, the coefficient of coupling in Eq. (3) is mostly determined by periodicity of the boundary conditions. In addition the electrodynamic parameters of the medium do not change. This approach is actual when features of nonlinear phenomena are considered in ferromagnetic films, in which periodic structure is created on the film surface. It should be noted that the system (3), without taking into account the dispersion ($\beta = 0$), as shown in [3], may have the soliton solutions, namely, the family of the Bragg solitons. This type of soliton represents a combination of two waves either moving together or remaining in place. If $\varphi_f(y,t) = \varphi_b(y,t)$, soliton does not move — a stationary gap soliton. In the case of excitation of magnetostatic waves with a carrier frequency near the band gap, the dispersion medium plays a more important role than the dispersion caused by the structure's periodicity. Moreover, the coupling parameter, the coefficient of dispersion and the nonlinearity depend significantly on the type of MSW excited in a ferromagnetic film. When FVMSW is excited in a periodic structure the coefficient $\beta = \frac{\partial^2\omega}{\partial k^2}$ and the group velocity $V_{\rm g} = \frac{\partial\omega}{\partial k}$ are calculated if the thickness of the lattice $d = d_0 = \frac{a_1d_1 + a_2d_2}{L}$ is the effective thickness. The nonlinear coefficient for FVMSW at $kd \ll 1$ is $\gamma = -\frac{1}{4}\left(1 + \frac{\omega_H^2}{\omega^2}\right)\omega_M(\frac{kd^2}{2})$. To calculate the coupling coefficient, we assume that the thickness of film in the direction of wave propagation in the periodic structure is described by the expression

$$d = d_2 + \delta(y), \quad (4)$$

where

$$\delta(y) = \delta(y+L) = \begin{cases} \Delta d = d_1 - d_2, & 0 \le y \le a_1, \\ 0, & a_1 \le y \le L. \end{cases}$$

The function δy is expanded as a Fourier series, and by restricting the expansion coefficients with $n = 0, \pm 1$, Eq. (4) can be represented as

$$d = d_0\left[1 + \delta d\cos\left(\frac{2\pi y}{L}\right)\right], \quad (5)$$

where $\delta d = \frac{2\Delta d}{\pi d_0}\sin\frac{\pi a_1}{L}$. Taking (5) into account, the coupling parameter for a one-dimensional periodic lattice of first order for $kd_0 \ll 1$ can be written as

$$\chi = \frac{\pi V_{\rm g}}{\lambda}\delta d, \quad (6)$$

where λ is a wavelength of FVMSW at frequency ω.

3. Simulation results

The results relating to the formation of solitons in this system were obtained based on the numerical solution of the coupled system of NSE (3) using a split--step Fourier method [3]. The coefficients in (3) were calculated taking into account the relation (6) and were accepted as $\beta = -2 \times 10^4$ cm^2 s^{-1}, $\gamma = 3 \times 10^{10}$ s^{-1}, and $\eta = 1 \times 10^6$ s^{-1}. The pulse was specified only on FP wave as the initial conditions $\varphi_{0f} = \varphi_0 \exp(-y^2/y^2_{\mathrm{imp}})$ and $\varphi_{0b} = 0$, where y_{imp} is the pulse width and φ_0 is the dimensionless pulse amplitude during the initial moment of time, which rose above a soliton threshold [3]. We consider the features of the wave evolution at a fixed value V_{g} depending on the parameter χ. This parameter characterizes the geometrical parameters of the periodic structure and the relationship between FP and BP waves accordingly.

The extreme case (when $\chi = 0$) corresponds to a homogeneous film ($a_1 = 0$), and BP wave is not excited in the structure. When $\chi \neq 0$, a linear relationship leads to the exchange of power between the waves. In this case, BP wave is excited and the solitons are formed, such as the Bragg solitons. The parameter space (V_{g}, χ) corresponding to soliton formation with different features is shown in Fig. 1. The parameter space corresponding to solitons, which move with some velocity $V_{\mathrm{s}} < V_{\mathrm{g}}$, is shown in white. The parameter space corresponding to the solitons that remains localized on the limited length of structure is shown in grey in Fig. 1. Figure 2a demonstrates the dynamics of soliton formation for parameter values conformable to point 1 in Fig. 1. For small values χ, there is incomplete transfer of power of FP wave $P_f = \int_0^l |\varphi_f^2| \mathrm{d}y$ into BP wave $P_b = \int_0^l |\varphi_b^2| \mathrm{d}y$, where l is the length of a structure (Fig. 2a). FP wave dominates and solitons move in the positive direction of y-axis at a velocity $V_{\mathrm{s}} < V_{\mathrm{g}}$ (Fig. 2a).

Figure 3 demonstrates the dynamics of soliton formation for parameter values conformable to point 2 in Fig. 1. A complete power swap between waves with period T is observed (Fig. 2b). Thus, on FP wave, a soliton is formed moving in the positive direction of y-axis (Fig. 3a). Figure 3b demonstrates that its power proceeds to the soliton on FP wave, traveling in the same direction. After the time cell $T/2$, the power of these solitons turns equal and solitons stop. Thus, the solitons change their propagation direction periodically in time and move in the direction of the major power wave. One can notice some "kinking" of the solitons, but it remains localized on a limited length of structure. The period T decreases and "zigzags" become smooth with increasing χ, and solitons can exist without moving ($V_{\mathrm{s}} = 0$). In the parameter space situated to the right of the gray area (point 3 in Fig. 1), solitons are not localized in space and travel with a certain velocity. This phenomenon can be explained as: when the period T becomes small in comparison with soliton period $T_{\mathrm{s}} \approx 1/\varphi_0$, it leads to the power swap periodicity getting disturbed and provokes soliton movement. On increasing V_{g} power swapping becomes less effective (period T decreases and swapping becomes incomplete), which leads to a pulling down of pulses with increase in V_{g}.

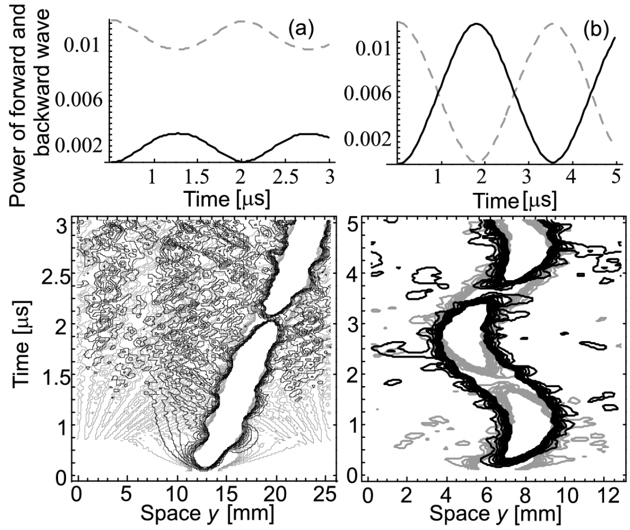

Fig. 2. The power swap of FP (grey dotted curve) and BP (black solid curve) waves eventually; lines of equal level of envelope amplitudes for φ_f (shown by grey color) and φ_b (shown by black color); (a) at $\chi = 1 \times 10^6$ s^{-1} (point 1 in Fig. 1) and (b) at $\chi = 2.5 \times 10^6$ s^{-1} (point 2 in Fig. 1) ($V_{\mathrm{g}} = 0.3 \times 10^6$ cm s^{-1}).

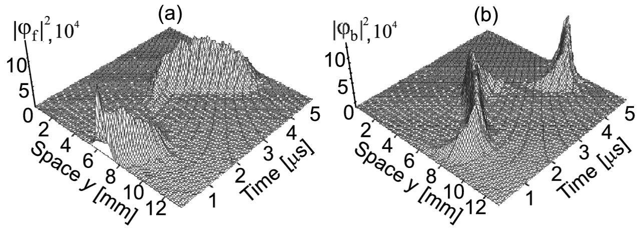

Fig. 3. The spatio-temporal evolution of the envelope amplitude φ_f (a) and φ_b (b) at $\chi = 2.5 \times 10^6$ s^{-1}, $V_{\mathrm{g}} = 0.3 \times 10^6$ cm s^{-1} (point 2 in Fig. 1).

4. Conclusion

In this paper, a model based on coupled nonlinear Schrödinger equations was used to calculate the parameter spaces corresponding to solitons, similar to the Bragg solitons, with different properties. The features of envelope soliton formation in one-dimensional periodic ferromagnetic structure were considered in FP and BP waves. In particular, the basic mechanisms for the formation of solitons, similar to the Bragg solitons and localized on the limited length of structure, were found to be the mutual capture of pulses on FP and BP waves. These pulses moved with the cumulative velocity (velocity, in turn, is defined by relative power of two waves). The power swapping between FP and BP waves is defined by value coupling between the waves. The features of wave evolution depending on coupling parameter and group velocity, and the areas of parameters corresponding to formation of pulses, similar to the Bragg solitons and localized on the limited length of structure, are investigated.

Acknowledgments

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