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The Multiple Phase Slip Phenomena in the Narrow Superconducting Channels

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The current–voltage characteristic of the narrow superconducting channel is investigated by direct numerical integration of the time-dependent Ginzburg–Landau equations. We have demonstrated that the steps in the current–voltage characteristic correspond to a number of different bifurcation points of the time-dependent Ginzburg–Landau equations. We have analytically estimated the period and the averaged voltage of the oscillating solution for the relatively small currents. We have also found the range of currents where the system transforms to the chaos.

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1. Introduction

It is well known that the superconducting state transforms to a resistive state at some critical current j_c. In this state superconductivity and a static electric field coexist. Above a certain current j_2 the superconducting state is absolutely unstable and a system transforms to a normal state. Therefore, the resistive state exists in the current interval $j_c < j < j_2$. The order parameter (OP) of a superconductor in the resistive state vanishes in a set of sample points. In these points the phase of the OP exhibits 2π jump. This process appears periodically in time. These points are well known as the phase slip centers (PSCs). The PSC's phenomena were comprehensively investigated in the past. It has been shown that the current–voltage characteristic (CVC) of the resistive state of a superconductor possesses a stair like structure. Every step of it corresponds to the point where a new PSC penetrates to the wire [1, 2].

Recently, the properties of the resistive state of different types of superconductors in the j = const [3–6] or V = const [3, 7, 8] regime have been investigated on the basis of the time-dependent Ginzburg–Landau equations (TDGLEs) [9]. The authors of Ref. [6] have decomposed the two-dimensional parameter space of temperature and current into regions of stability of a normal, steady, and oscillatory state. It has been found out that different periodic and quasiperiodic in time solutions emerge with the change of voltage [7, 8]. Recent experimental observations [10] have revealed the space–time arrangement of the PSCs. Using the low temperature laser scanning microscopy technique it has been demonstrated that each voltage jump on the CVC corresponds to a generation of a new PSC [10]. The authors have observed the creation of one PSC at certain critical current. Further increase of the current leads to the spatial rearrangement of the PSCs — two PSCs appear symmetrically with respect to the center of the wire. Finally, the third PSC appears in the middle of the wire (see Fig. 2 of Ref. [10]).

Based on the previous works [7, 8, 11] we know the significance of the length of the wire on the properties of the system. Here we present the more detailed study of the narrow superconducting channel of length $L/\xi = 21.76$, where ξ is the coherence length. Firstly, we analytically estimate the period of the oscillatory solution in the vicinity of the critical current. Then we demonstrate that each step on the CVC correspond to a number of different bifurcation points of the TDGLEs and reveal the bifurcations' types.

2. Theory

For the complex order parameter $\psi = \rho\exp(\mathrm{i}\theta)$, where ρ and θ are the modulus and phase of the OP respectively, the TDGLEs in dimensionless units take the form

$$u\left(\frac{\partial\psi}{\partial t} + \mathrm{i}\phi\psi\right) = \frac{\partial^2\psi}{\partial x^2} + \psi - \psi|\psi|^2, \quad (1)$$

$$j = -\frac{\partial\phi}{\partial x} + \frac{1}{2\mathrm{i}}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right), \quad (2)$$

wherein the distance and time are measured in units of the coherence length ξ and phase relaxation time $\tau_\theta = 4\pi\lambda^2\sigma_{\mathrm{n}}/c^2$, respectively, where λ is the penetration depth, σ_{n} is the normal state conductivity, and c is the speed of light. The electrostatic potential ϕ is written in units of $\phi_0/2\pi c\tau_\theta$, where $\phi_0 = \pi\hbar c/e$ is the flux quantum, e is the electric charge, $\hbar$ is the reduced Planck constant. The current density j is defined in units of $\phi_0 c/8\pi^2\lambda^2\xi$. The only parameter left is $u = \tau_\rho/\tau_\theta$, where τ_ρ is the relaxation time of the amplitude of the OP.

Here we consider the superconducting wire of a length L with the following boundary conditions: $\rho(-L/2) = \rho(L/2) = 1$ and $\mathrm{d}\phi(-L/2)/\mathrm{d}x = \mathrm{d}\phi(L/2)/\mathrm{d}x = 0$. The

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absence of the electric field at the end of the wire determines the gradient of the phase: $\mathrm{d}\theta(-L/2)/\mathrm{d}x = \mathrm{d}\theta(L/2)/\mathrm{d}x = j$. In the present paper, to model a realistic situation we assume $u = 1/2$ [12].

The analysis of the steady state of the TDGLEs (1, 2) demonstrates that the equation for the current $j = k(1-k^2)$ has two roots when $j < j_c$. One of them is stable and the other is unstable. As soon as the current reaches its critical value $j = j_c$, they collide with each other. In addition, according to our calculations, a limit cycle appears in the system at this point. Besides, only one Lyapunov exponent crosses zero at this moment [13]. Therefore, we deal with a saddle-node homoclinic bifurcation at $j = j_c$ [14].

To find out the features of the saddle-node homoclinic bifurcation we rewrite the TDGLEs (1, 2) in the limit of an infinite channel. By introducing the gauge invariant scalar $\Phi = \phi + \partial\theta/\partial t$ and vector $Q = \partial\theta/\partial x$ potentials we obtain

$$u\partial\rho/\partial t = \partial^2\rho/\partial x^2 + \rho(1-\rho^2-Q^2),$$
$$u\rho^2\Phi = \partial(\rho^2 Q)/\partial x,$$
$$j = -\partial\Phi/\partial x + \partial Q/\partial t + \rho^2. \qquad (3)$$

These equations have the steady state solution $\rho = \sqrt{2/3}$, $\Phi = 0$, $Q = 1/\sqrt{3}$, $j = j_c$, which becomes unstable and has one Lyapunov exponent λ_0 which crosses zero at $j = j_c$. Expansion of Eq. (3) up to the second order in deviations from the steady state solution, rescaling of the time $t/u \to t$, and projection of this equation on the direction of the eigenvector corresponding to λ_0 yield

$$\dot{y} = \beta + a(0)y^2, \qquad (4)$$

where y is a representative phase variable, $\beta = -2^{3/2}u(j-j_c)/3(u+2)j_c$, $a(0) = -2^{3/2}u/(u+2)$. Integration of Eq. (4) over y between $\pm y_1$, where y_1 is of the order of 1 yields $T = 2\tan^{-1}(y_1\sqrt{a(0)/\beta})/\sqrt{a(0)\beta}$. Therefore, if $(j-j_c)/j_c \ll 1$, the period of oscillations is determined by the formula

$$T = \pi/\sqrt{a(0)\beta} = \frac{\pi\sqrt{3}(u+2)}{2^{3/2}u}[(j-j_c)/j_c]^{-1/2}. \qquad (5)$$

We have solved Eqs. (1) and (2) numerically by using the fourth order Runge–Kutta method. Since we investigate the wire of the finite length, the critical current in our system j_c is not equal to the critical current in the Ginzburg–Landau theory $j_{GL} = 2/3\sqrt{3}$ for an infinite wire. Our calculations show that the critical current $j_c/j_{GL} = 1.002$.

3. Results and discussion

The results of our calculations of the period of the oscillating solution together with the analytical estimation (Eq. (5)) are presented in Fig. 1. There is a very good agreement of numerical results with Eq. (5) over more than two orders of magnitude in $(j-j_c)/j_c$.

According to the Josephson relation [1] and Eq. (5) the voltage in the vicinity of the saddle-node homoclinic bifurcation $V \propto T^{-1} \propto [(j-j_c)/j_c]^{1/2}$. Inset (f) of Fig. 2 clearly demonstrates this behaviour for the region of currents $j_c < j < j_{c1}$. For this range of currents the PSC appears in the middle of the wire periodically in time (Fig. 2a) in agreement with the experimental results [10].

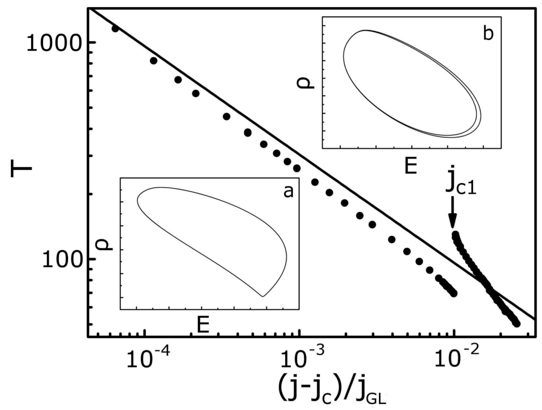

Fig. 1. The period of the solution as a function of the current. The solid line represents the result of Eq. (5). Arrow indicates the period-doubling bifurcation point j_{c1}. Insets represent projection of the limit cycle trajectory to the $(\rho(0), E(0))$ plane before (a) and after (b) the bifurcation point. Here $\rho(0)$ is the modulus of the OP, $E(0)$ is the electric field both in the center of the wire.

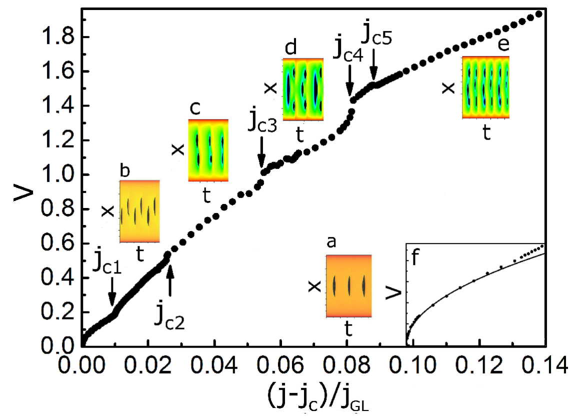

Fig. 2. The CVC of the channel. Insets (a)–(e) show the space–time PSC's arrangement in the different regions of the CVC. Inset (f) represents the CVC for $j < j_{c1}$ in comparison with analytical formula (see the text).

Further increase of the current leads to the bifurcation of the periodic solution. In the insets to Fig. 1 we have plotted the phase trajectories of the periodical solutions below and above $j = j_{c1}$ in coordinates $\rho(x=0)$ and electric field $E(x=0) = -\mathrm{d}\phi/\mathrm{d}x$. A single-loop (period-1) limit cycle transforms to a double-loop (period-2) limit cycle at $j_{c1}/j_{GL} = 1.012$ as a result of the period-doubling bifurcation. This bifurcation causes an increase of the period as it is clearly seen from Fig. 1. It also results in the change of the CVC's slope. As it follows from Fig. 2b, the space–time arrangement of the PSCs is changed similarly to Ref. [7]: two adjacent PSCs are shifted in the opposite directions with respect to the center of the wire in agreement with the experimental observations [10]. Now the period includes two PSCs. It causes an increase of the period of the limit cycle by twice. As a result of period-doubling, a new frequency $\omega_2 = \omega_1/2$ appears in the spectrum of an electromagnetic radiation generated by the current. However, we have not observed the PSCs in the center of the wire, in contrast to results in Ref. [7]. It may be related to the fact that the authors of Ref. [7] have performed calculations with different parameter $u = 1$. Moreover, we consider the channel in the $j =$ const regime, while in Ref. [7] the TDGLEs have been studied at constant voltage.

The next bifurcation is the destruction of the limit cycle. At $j_{c2}/j_{GL} = 1.027$ the limit cycle loses stability. The space–time arrangement of the PSCs in this area is similar to the current's region $j_{c1} < j < j_{c2}$ (see Fig. 2c). For the range of currents $j_{c2} < j < j_{c5}$ we have found oscillating, but non-periodic solution.

To investigate the system's behavior in the vicinity of this bifurcation we calculate the Poincaré map. The projection of the phase trajectory to the plane $(\rho(-L/4), E(-L/4))$ never crosses the space near the center of the trajectory. Therefore we chose the Poincaré section as a plane which crosses the center of this trajectory $E(-L/4) = 0.04$ for $j/j_{GL} = 1.0278$. In Fig. 3a we plot the projection of the Poincaré map to $\rho(0)$ and $E(0)$ as a function of discrete time, when trajectory crosses the Poincaré section. Our calculations demonstrate that the wire has two possible states in the vicinity of this bifurcation. There are a number of long time intervals where the system stays near the laminar oscillation phase. Then, due to the instability, it moves far away from the limit cycle and enters the turbulent motion phase. After a while it comes back to the regular orbit. It is clearly seen from Fig. 3b that the period of the laminar phase is increasing with the decrease of the current. The solid line here represents the $\tau \propto (j - j_{c2})^{-1/2}$ law. Therefore, the chaotic behaviour of the solution of the TDGLEs is developing via the intermittence [15].

The bifurcation point $j_{c3}/j_{GL} = 1.058$ corresponds to the appearance of a new PSC in the center of the wire (Fig. 3d) without changing the non-periodic character of the oscillating solution. At this point the CVC exhibits the voltage jump. In the vicinity of this bifurcation we have observed so-called "periodic windows" — areas of the parameter values, where the limit cycles with a relatively large period become stable [15].

The fourth PSC appears in the wire at $j_{c4}/j_{GL} = 1.084$ (Fig. 2e). It considerably changes the slope of the CVC, but does not destroy the chaos. And finally, at $j_{c5}/j_{GL} = 1.092$ the limit cycle becomes stable. At this bifurcation point the voltage decreases in contrast to all previous cases.

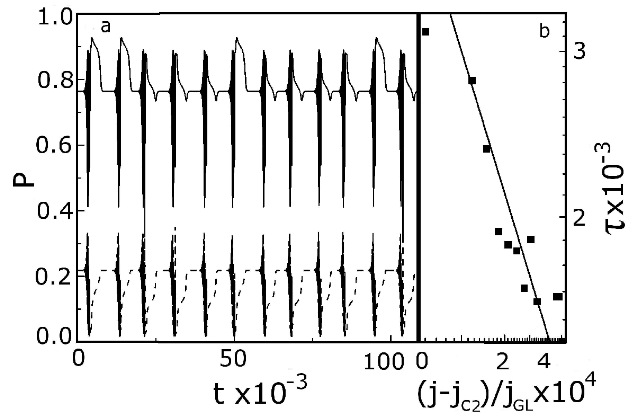

Fig. 3. (a) The projection of the Poincaré map on $\rho(0)$ (solid line) and $E(0)$ (dashed line) as a function of time. (b) The period of the laminar oscillation phase as a function of the current (squares). The solid line shows analytical prediction (see the text).

4. Conclusions

We have explored the CVC of the narrow superconducting channel. We have demonstrated that the steps of the CVC correspond to a number of different bifurcation points of the TDGLEs. The voltage appearance in the system corresponds to the saddle-node homoclinic bifurcation leading to the formation of the limit cycle with a diverging period when $j \to j_c$. The voltage $V \propto (j - j_c)^{1/2}$ in this region. We have also analytically estimated the period of oscillations in the vicinity of this bifurcation point. The second singularity corresponds to the period-doubling bifurcation. As a result of this bifurcation, a new frequency equals to the half of the frequency before the bifurcation appears in the spectrum. We have also proved that the chaos appears via the intermittence.

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