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Neutral Bosonic Condensates in Layered 2D Structures under Artificial Magnetic Field

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We show that the properties of the ideal Bose gas in three-dimensional optical lattice can be closely mimicked by finite two dimensional systems with only ten of layers. The match between critical properties strongly depends on the anisotropy of the hopping amplitudes in and between layers which we fully control. The theory we provided can be directly used in the experiments and results in less challenging requirements of the setups. We also present the phase diagram with its non-monotonic dependence of the ratio of tunneling to on-site repulsion when artificial magnetic field is applied to the system.

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1. Introduction

The merging of atomic and condensed matter physics since the experimental realization of the Bose–Einstein condensation (BEC) has opened exciting new perspectives for the creation of novel quantum states. Systems of dilute bosonic gases confined in optical lattices are ideal toolboxes for testing theoretical models and their solutions [1]. Surprisingly, the quantum phase transitions in systems under uniform magnetic field can be also analyzed considering rotating Bose–Einstein condensates trapped in a two-dimensional (2D) lattice potential. In frame of reference rotating about the z-axis with angular velocity Ω the kinetic term in Hamiltonian is equivalent to that of a particle of charge q experiencing a magnetic field B with $qB = 2m\Omega$, where m is the mass of the particle. This connection shows that the Coriolis force in the rotating frame plays the same role as the Lorentz force on a charged particle in an uniform magnetic field. The presence of angular velocity induces vortices in the system described by the rotation frustration parameter f ($\equiv ma^2\Omega/\pi\hbar$, with a being the lattice spacing). Frustration occurs in this system because two different area scales are in competition. One characteristic area is the unit cell a of considered lattice. The other $\pi\hbar/m\Omega$ is associated with the rotation of the lattice. However, this approach puts limit on maximum rotational velocity, thus large synthetic magnetic field SMF (e.g. required for quantum Hall physics) cannot be reached. To overcome those difficulties, imprinting of the quantum mechanical phase is used, which is based on superimposing of an external potential on a BEC [2]. The developing in trapping techniques provides presently one layer resolution of creation stacked structures. The method many groups are operating with involves another one-dimensional optical lattice that is used to split a magnetically trapped 3D BEC into a small array of 2D clouds. After that the sample is cleaned with the resonant depumping laser. From the experimental point of view our work provides results that can be immediately accessed.

2. Model

In optical lattices, two main energy scales are set by the hopping amplitude t (the kinetic energy of bosons tunneling between the lattice sites), and the on-site repulsive interaction U (resulting from repulsion of multiple boson occupying the same lattice site). For $t \gg U$, the superfluid order is well established in zero-temperature limit. However, for sufficiently large repulsive energy U, the quantum phase fluctuations lead to suppression of the long-range phase coherence resulting in SF (superfluid) to MI (Mott-insulator) transition. The synthetic magnetic field $\boldsymbol{B}$ (resulting either from rotation of the system, phase imprinting, or external electric field) introduces the Peierls phase factor $\exp\left(\frac{2\pi \mathrm{i}}{\Phi_0}\int_{\boldsymbol{r}_j}^{\boldsymbol{r}_i}\boldsymbol{A}\cdot \mathrm{d}\boldsymbol{l}\right)$, where $\boldsymbol{B} = \nabla \times \boldsymbol{A}(\boldsymbol{r})$, and $\Phi_0 = hc/e$ is the flux quantum, with $\boldsymbol{A}(\boldsymbol{r})$ being the vector potential (which can be realized experimentally, see Ref. [2]), and h, c and e – the Planck constant, speed of light and charge of electron, respectively. Thus, the system can be described by the following quantum Bose-Hubbard Hamiltonian [3, 4]:

$$\mathcal{H} = \frac{U}{2}\sum_{\boldsymbol{r}} n_{\boldsymbol{r}}(n_{\boldsymbol{r}}-1) - \sum_{\langle \boldsymbol{r},\boldsymbol{r}'\rangle} t_{\mathbf{rr}'}\exp\left(\frac{2\pi\mathrm{i}}{\Phi_0}\int_{\boldsymbol{r}_j}^{\boldsymbol{r}_i}\boldsymbol{A}\cdot\mathrm{d}\boldsymbol{l}\right)a^{\dagger}_{\boldsymbol{r}}a_{\boldsymbol{r}'} - \mu\sum_{\boldsymbol{r}} n_{\boldsymbol{r}}, \tag{1}$$

where $a^{\dagger}_{\boldsymbol{r}}$ and $a_{\boldsymbol{r}'}$ are for the bosonic creation and annihilation operators that obey canonical commutation relations $[a_{\boldsymbol{r}}, a^{\dagger}_{\boldsymbol{r}'}] = \delta_{\boldsymbol{rr}'}$, $n_{\boldsymbol{r}} = a^{\dagger}_{\boldsymbol{r}}a_{\boldsymbol{r}}$ is the boson number operator on the site $\boldsymbol{r}$. Here, $\langle \boldsymbol{r}, \boldsymbol{r}'\rangle$ denotes summation over the nearest-neighbor sites. Furthermore, $t_{\boldsymbol{rr}'}$ is the hopping matrix element with the dispersion $t_{\boldsymbol{k}} = 2t_{\parallel}\left(\cos k_x + \cos k_y + \frac{t_\perp}{t_\parallel}\cos k_z\right)$ where $t_\perp$ is the hopping between layers and $t_\parallel$ within the planes. Since, we are interested in investigating the influence of the lattice geometry on the system properties, we consider a stack an arbitrary number (L) of two-dimensional planes coupled with $t_\perp$. As a result, the values of k_x and k_y

are continuous ($k_{x,y} = -\pi, \dots, \pi$), while k_z is discrete ($k_z = \frac{2\pi}{L} l$, where $l = 0, \dots, L-1$). Also, we allow for z-axis anisotropy, which is a ratio of inter-plane to in-plane hopping $\eta = t_\perp / t_\parallel$.

A boson hopping around a lattice cell of the area of A will gain an additional phase $2\pi f$ resulting from the synthetic magnetic field, where $f = ABe/2\pi\hbar$. As a result, the periodic potential leads to splitting of the Landau levels into integer number q of sub-bands. Of special interest are the values of the SMF which correspond to rational numbers of $f \equiv p/q = 1/2, 1/3, 1/4, \dots$ (p is an integer), since for those values the energy spectra and the density of states can be obtained exactly. Here, we present results for $f = 1/8$ and $f = 3/8$, which up to now have been analytically inaccessible (see Ref. [5]).

To proceed, we rely on the quantum rotors approach. The method is extensively described in Refs. [4, 6–8], so here we only summarize its main points. We use the functional integral representation of the model with bosonic operators becoming complex fields $a_{\boldsymbol{r}}(\tau)$ (where τ is imaginary Matsubara's time). The most important element of our method is a local gauge transformation to the new bosonic variables: $a_i(\tau) = b_i(\tau) \exp(i\Phi_i(\tau))$. This allows to cast the strongly correlated bosonic problem into a system of weakly interacting bosons, submerged into the bath of strongly fluctuating gauge potentials on the high energy scale set by U. As a result, the superfluid order parameter can be written as $\Psi_\mathrm{B} \equiv \langle a_i(\tau)\rangle = b_0 \Psi_\mathrm{B}$, where non-zero value of $\Psi_\mathrm{B} = \langle \exp(\mathrm{i}\Phi_i(\tau))\rangle$ results from phase ordering and b_0 is the amplitude of the bosonic field

$$b_0^2 = \left[4 + 2\left(1 - \frac{1}{L}\right)\frac{t_\perp}{t_\parallel}\right]\frac{t_\parallel}{U} + \frac{\mu}{U} + \frac{1}{2}. \tag{2}$$

The coefficient $4 + 2\left(1 - \frac{1}{L}\right)\frac{t_\perp}{t_\parallel}$ is an effective number of nearest neighbors averaged over all lattice sites. In the zero-temperature limit we arrive at the equation for the phase order parameter

$$1 - \psi_\mathrm{B}^2 = \frac{1}{2N}\sum_{\boldsymbol{k}} \frac{1}{\sqrt{\frac{J_{\boldsymbol{k}=0} - J_{\boldsymbol{k}}}{U} + v^2\left(\frac{\mu}{U}\right)}} \tag{3}$$

with

$$v\left(\frac{\mu}{U}\right) = \mathrm{frac}\left(\frac{\mu}{U}\right) - \frac{1}{2},$$

where $\mathrm{frac}(x) = x - [x]$ is the fractional part of the number and $[x]$ is the floor function which gives the greatest integer less than or equal to x and the phase stiffness $J_{\boldsymbol{k}} = b_0^2 t_{\boldsymbol{k}}$.

Since the number of layers L in the system is finite, the summation in Eq. (3) runs over discrete values of k_z and continuous values of k_x and k_y. However, because density of states of a single layer under SMF (ρ_f) is known, we explicitly derive the density of states of the whole stack of L coupled planes (for calculation details, see Ref. [9]) $\rho_f^L(\eta, \xi) = \frac{1}{L}\sum_{k_z} \rho_f(\xi - \eta\cos k_z)$. As a result, the critical line equation ($\psi_\mathrm{B} = 0$) including the effects of SMF and z-axis anisotropy reads

$$1 = \frac{1}{2}\int_{-\infty}^{+\infty} \mathrm{d}\xi \frac{\rho_f^L(\eta, \xi)}{\sqrt{2(\xi_0 - \xi) b_0^2 \frac{t_\parallel}{U} + v^2\left(\frac{\mu}{U}\right)}}, \tag{4}$$

with ξ_0 being the half-width of the band dispersion for selected value of $f = p/q$.

3. Results

Equation (4) allows us to calculate the zero--temperature phase diagram of the investigated Bose–Hubbard model from Eq. (1) as a dependence of critical interaction on the chemical potential, SMF, number of layers and z-axis anisotropy

$$\left(\frac{t_\parallel}{U}\right)_z = x_z = x_f^L\left(\frac{\mu}{U}, \eta\right). \tag{5}$$

The diagram is plotted in Fig. 1 for different number of layers L, synthetic magnetic field $f = 1/2$, in isotropic case ($\eta = 1$). In the weak coupling limit ($t_\parallel \gg U$), the kinetic energy dominates and the ground state is a delocalized superfluid, described by nonzero value of the superfluid order parameter $\Psi_\mathrm{B} \neq 0$.

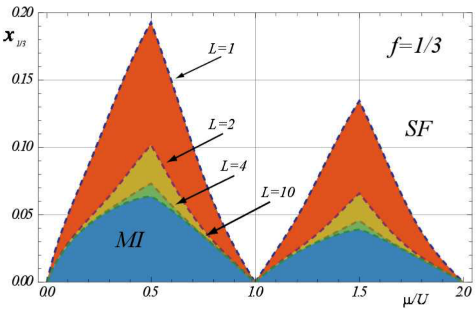

Fig. 1. The zero-temperature phase diagram of a stack of square lattice planes (number of particles per lattice site is $n_\mathrm{B} = 1$ inside the first and $n_\mathrm{B} = 2$ with magnetic field $f = 1/3$ for various number L of layers and $\eta = 1$. Within the MI phase the phase order parameter $\Psi_\mathrm{B} = 0$.

On the other hand, in the strong coupling regime ($t_\parallel \ll U$) the phase fluctuation becomes significant and the long-range order is destroyed leading to a series of MI lobes with fixed integer filling $n_\mathrm{B} = 1, 2, \dots$ [3, 6]. A single-layer system ($L = 1$) has a simple square (two--dimensional) geometry, which results in the phase diagram with characteristic narrow-edged lobes. As the number of layers is being increased, the tops of the lobes become smooth and their maxima deviate towards lower values of the chemical potential μ. As a result, the phase diagram becomes similar to the one of a cubic (three-dimensional) system. It is important to notice that also in the presence of the synthetic magnetic field, the phase diagrams of the finite L system becomes indistinguishable from the infinite (cubic) one for L as small as 10.

The phase coherent Bose gas can be also driven into the Mott insulating phase by applying the synthetic magnetic field. The effect of the SMF is presented in Fig. 2.

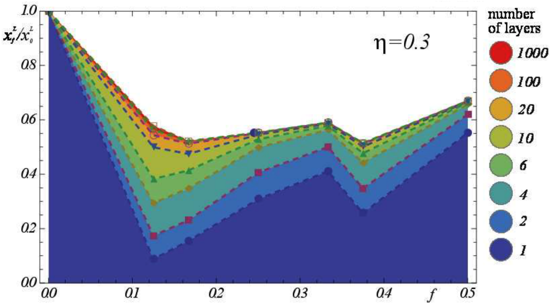

Fig. 2. Ratio of critical coupling x_0^L/x_f^L of the tip of the first lobe of the system without ($f=0$) and with synthetic magnetic field f as a function of number layers L for anisotropic system $\eta=0.3$.

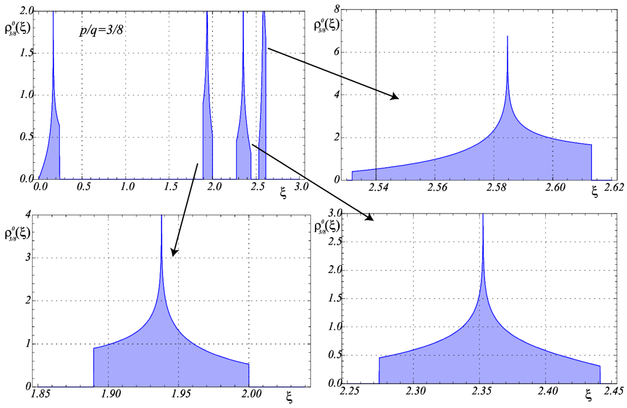

Fig. 3. Density of states with characteristic van Hove's singularities in the artificial magnetic field for $f=p/q=3/8$.

The long range order is suppressed by the phase changes imposed on the bosonic wave function and this suppression has a non-monotonic character strongly depending on the topology of the system. The Mott insulating phase becomes more stable, which is as expected since the magnetic field should localize particles. In the single-layer system, the effect of the SMF is the most pronounced and this decreases with growing number of layers L. By adding more layers the global coherence of the system is restored, because growing dimensionality entails the suppression of quantum fluctuations effects. Here, the convergence of properties of the finite system to those of the infinite (cubic) one is much slower, although also non-trivially dependent on f. When the system is anisotropic (see the bottom plot in Fig. 2), the convergence is much faster, but still depending on the specific value of f. Precise results for various values of f (see Fig. 2) where obtained owing to determination of analytical formulae for lattice density of states in the presence of synthetic magnetic fields. Some of them have never been presented before (see Fig. 3).

4. Conclusions

The physics of strongly correlated bosonic systems is the competition between two tendencies of the bosons to spread out as a wave and to localize as a particle combined with a frustration caused by synthetic magnetic field. We calculated the phase diagram using the quantum rotor approach with exactly evaluated density of states for two-dimensional layered lattices with rational magnetic flux/rotation frustration parameter $f=p/q$ for a number of values $f=p/q$. By calculating analytically the density of states for several values of the magnetic field we were able to accurately predict the evolution of the system towards the Mott phase. In systems that are in the global coherent state at $f=0$, but with the ratio t/U close to the critical value $(t/U)_{\text{crit}}$, a rotation can be used to drive the condensates into the MI. Note that the dependence of the x_f/x_0 from frustration parameter f is non-monotonical. The effect is reduced when more layers are being added, i.e. during the two- to quasi three-dimensional geometry crossover. Furthermore, we have established a correspondence between anisotropic infinite (quasi three-dimensional) and isotropic finite (slab geometry) systems that share exactly the same critical values, which can be an important clue for choosing experimental setups that are less demanding, but still leading to the identical results. Finally, we have shown that the properties of the ideal Bose gas in three-dimensional optical lattice can be closely mimicked by finite (slab) systems, when the number of two-dimensional layers is larger than ten or even less, when the layers are weakly coupled.

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