Proceedings of the 5th Symposium on Physics in Economics and Social Sciences, Warszawa, Poland, November 25–27, 2010

Short Comprehensive Report on the Non-Brownian Stochastic Dynamics at Financial and Commodity Markets

T. Ciepliński, A. Dominiczak, and R. Kutner

Institute of Experimental Physics, Faculty of Physics, University of Warsaw, Hoża 69, PL-00681 Warsaw, Poland

In this work we empirically verify the generic breaking of the Central Limit Theorem on the financial and commodity markets. We analysed the distributions of log-returns for typical indices and price of gold, for increasing time horizons. We considered Random Coarse Graining Transformation of the Continuous-Time Random Walk model, which can represent the non-Gaussian price dynamics of underlying assets and the corresponding derivatives, e.g., various options or future contracts. We confirmed that empirical data and predictions of the model quite well agree.

PACS: 89.20.–a, 89.65 Gh, 02.50.Ey, 02.50.Ga, 05.40.Fb

1. Introduction

The publishing in Physica A (1991) of the article entitled *Lévy Walks and Enhanced Diffusion in Milan Stock Exchange* by R. N. Mantegna [1, 2] can be dated as the informal beginning of modern econophysics. In his article Mantegna analyzed the historical series of MIB15 index (Milano Italia Borsa consisting of 15 different sectors). He observed that variations of the index for different time horizons can be described by the Lévy stable distribution instead of the Gaussian one, as it might be expected from the canonical L. Bachelier approach [3], R. Merton [4, 5] and F. Black theories [6], and P. Samuelson geometric Brownian motion [7] as well as from the Cox-Ross-Rubistein model [8]. The Mantegna article mainly inspired physicists, financial mathematicians and econometricists to model finacial markets beyond the Brownian motions (or Wiener processes). It stimulated scientific communities to study, for instance, Lévy processes on financial markets [9–11], rare and extreme events, [12], the influence of discrete scale invariance (leading to famous log-periodic oscillations), fully developed turbulence and financial crashes as well as many other related significant topics. Nevertheless, this Mantegna concept is continuously verifying thanks to the progress in the time-series analysis [13] (and refs. therein) and development of semi-analytical methods including different advanced detrending techniques elaborated by physicists, [13] as well as to the progress in numerical recipes [14] and computer hardware.

The main goal of this work is to empirically verify the generic breaking of the Central Limit Theorem on financial markets [2] by analysing the statistics of log-returns for typical indices (basic financial instruments) and price of gold (considered as a commodity), within various time horizons, for instance, from minutes to weeks. Therefore, we also considered the non-Gaussian pricing proposed in [15, 16] of the underlying financial instruments and corresponding derivatives, e.g., various options (in particular, the vanilla European index option considered in this article) or future contracts (such as comodities future contracts). In brief, this pricing is based on the Random Coarse Graining of the Continuous-Time Random Walk (RCG of CTRW) where jumps and time intervals beetwen them can be correlated and they are drawn from heavy tailed distributions.

2. Results

In this Section we made a thorough analysis of worldwide financial markets [17]. We choose only few typical examples.

In Figs. 1–3 we present results concerning, for instance, index S&P 500 for three typical time horizons. The distinct deviation of empirical distributions (full circles) from the Gaussian (dashed curve) is well seen. All empirical peaks, and particularly their heavy tailes, are well described by the RCG of CTRW model (solid curve). Notably, even for log-returns in weekly horizon (cf. Fig. 3) deviations from Gaussian distribution remains still unaffected that is, these deviations are sufficiently persistent.

The subsequent two Figs. 4 and 5 present the comparison between the empirical data (for DAX30 for daily and weekly horizons) and Gaussian distribution.

Again, the persistent deviations from the Gaussian are observed. The empirical data are well described by the RCG of CTRW model. The analogous situation we have in Figs. 6 and 7 for WIG20 (the index of the Warsaw Stock Exchange). Thus we proved that the non-Gaussian market behaviour occur on every markets, regardless of their sizes.

For comparison, the analogous study we performed for log-returns of gold (considered as a commodity). In Fig. 8 we present 10-minutes log-returns for 2010. Apparently, the distribution of the 10-minute horizon log-returns has

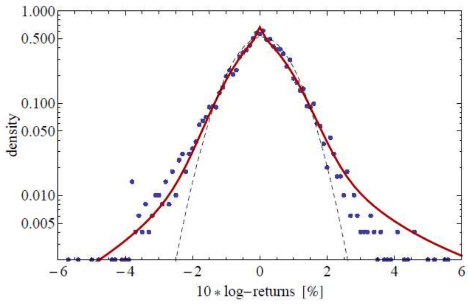

Fig. 1. The empirical distribution (histogram) of log-returns of S&P 500 index for 10-minute horizon (full circles) shown in the semi-logarithmic plot. The dashed curve is the best fit of the Gaussian distribution to the empirical data and the solid curve is the analogous fit obtained for the RCG of CTRW model. These data were recorded, for instance, within the whole 2010.

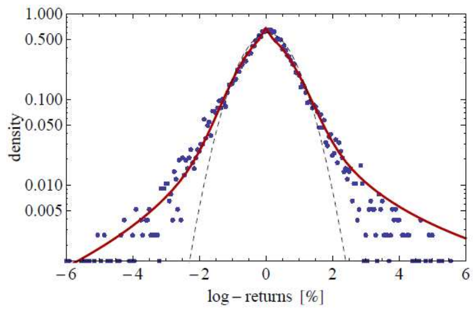

Fig. 2. The empirical distribution (histogram) of log-returns of S&P 500 index for a daily horizon (full circles) shown in the semi-logarithmic plot. The dashed curve is the best fit of the Gaussian distribution to the empirical data and the solid curve is the analogous fit obtained for the RCG of CTRW model. These data were recorded from 1951 to 2011.

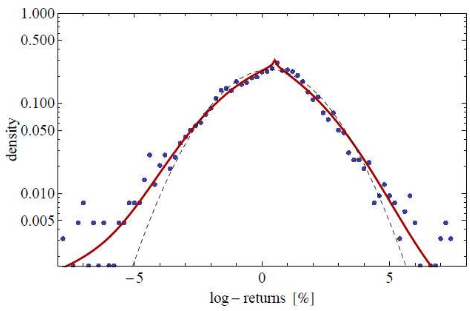

Fig. 3. The empirical distribution (histogram) of log-returns of S&P 500 index for a weekly horizon (full circles) shown in the semi-logarithmic plot. The dashed curve is the best fit of the Gaussian distribution to the empirical data and the solid curve is the analogous fit obtained for the RCG of CTRW model. These data were recorded from 1951 to 2011.

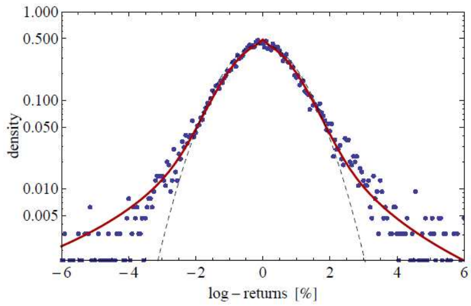

Fig. 4. The empirical distribution (histogram) of log-returns of DAX30 index for a daily horizon (full circles) shown in the semi-logarithmic plot. The dashed curve is the best fit of the Gaussian distribution to the empirical data and the solid curve is the fit obtained for the RCG of CTRW model. These data were recorded from 1981 to 2011.

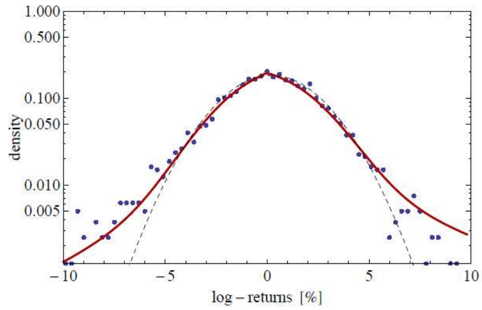

Fig. 5. The empirical distribution (histogram) of log-returns of DAX30 index for a weekly horizon (full circles) shown in the semi-logarithmic plot. The dashed curve is the best fit of the Gaussian distribution to the empirical data and the solid curve is the analogous fit obtained for the RCG of the CTRW model. These data were recorded from 1959 to 2011.

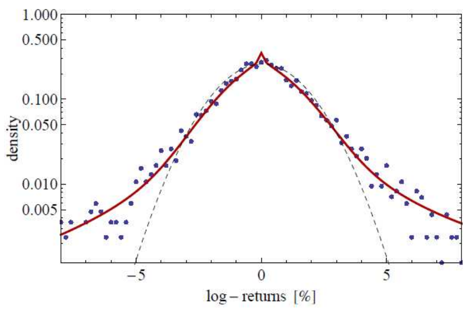

Fig. 6. The empirical distribution (histogram) of log-returns of WIG20 index for a daily horizon (full circles) shown in the semi-logarithmic plot. The dashed curve is the best fit of the Gaussian distribution to the empirical data and the solid curve is the analogous fit obtained for the RCG of CTRW model. These data were recorded from 1959 to 2011.

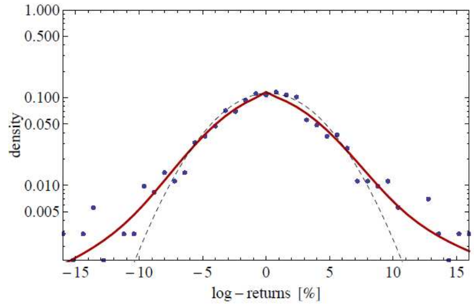

Fig. 7. The empirical distribution (histogram) of log-returns of WIG20 index for a weekly horizon (full circles) shown in the semi-logarithmic plot. The dashed curve is the best fit of the Gaussian distribution to the empirical data and the solid curve is the analogous fit obtained for the RCG of CTRW model. These data were recorded from 1993 to 2011.

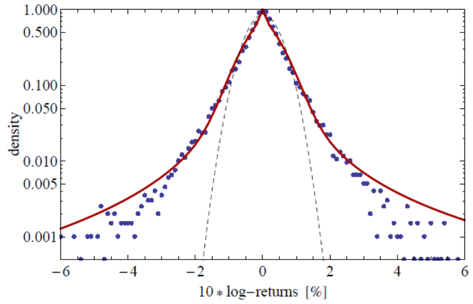

Fig. 8. The empirical distribution (histogram) of gold log-returns for 10-minute horizon (full circles) shown in the semi-logarithmic scale. The dashed curve is the best fit of the Gaussian distribution to the empirical data while the solid curve is the analogous fit obtained for the RCG of CTRW model.

kurtosis similar to those of stock markets. The results for daily (Fig. 9) and weekly (Fig. 10) log-returns are even the more leptokurtosis than those for stock markets outstanded the above.

We suggest that the commodity markets can be also well characterized by the RCG of CTRW model.

Finally, Fig. 11 shows the comparison beetwen real option price and Black-Scholes as well as the RCG of CTRW model option prices, The option is based on WIG20 as underlying financial instrument. We presented the comparison within the duration time when trading was the most intensive. One can see that the real option price is almost equal to the RCG of CTRW option except short time interval between 52 and 45 traiding days. There is also surprisingly good agreement between real prices and Black-Scholes ones although, the latter regards yet the Wiener process.

3. Conclusions

In our study we presented historical log-returns in various characteristic time scales for financial and commodity

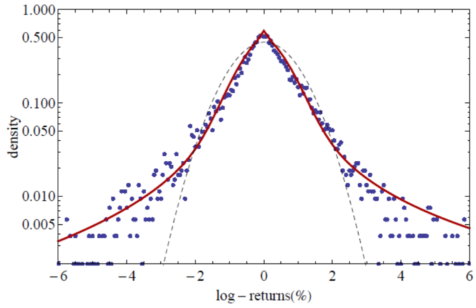

Fig. 9. The empirical distribution (histogram) of log-returns of gold (in US dollar price) for a daily horizon (full circles) shown in the semi-logarithmic plot. The dashed curve is the best fit of the Gaussian distribution to the empirical data and the solid curve is the analogous fit obtained for the RCG of CTRW model.

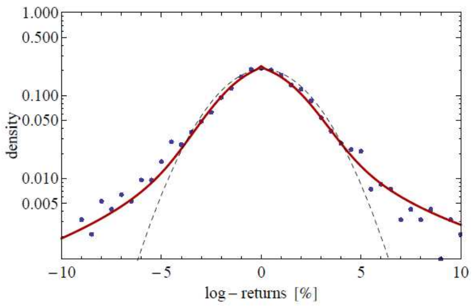

Fig. 10. The empirical distribution (histogram) of log-returns of gold (in US dollar price) for a weekly horizon (full circles) shown in the semi-logarithmic plot. The dashed curve is the best fit of the Gaussian distribution to the empirical data and the solid curve is the analogous fit obtained for the RCG of CTRW model.

world markets. Our results firmly confirm that:

(i) Markets of moderate and high liquidity cannot be considered as Gaussian, independently of their capitalizations.

(ii) The RCG of CTRW model is the modern approach

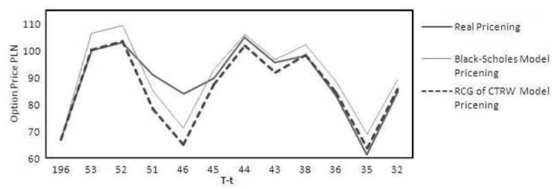

Fig. 11. Comparision between real (thick solid curve), Black-Scholes model (thin solid curve) and the RCG of CTRW model (dashed curve) option prices based on WIG20 index with strike price 2700 PLN, priced beetwen 2010.09.03 and expiration date (T) 2011.03.19. The option price (vertical axis) is given in Polish zloty (PLN) and the horizontal axis shows the time left to T.

that could be, in principle, used for description of price distributions and dynamics.

(iii) The Central Limit Theorem is broken that is, even for relatively large (weekly) time scale the distributions deviate from the Gaussian.

(iv) As gains and losses are slightly asymmetric, they should be separately considered.

There are left at least two significant open questions, for instance, why the Black-Scholes model gives option prices so close to real market option prices though, the basic financial instrument is non-Gaussian? How to improve the agreement between the RCG of CTRW model and the empirical data? The answers for these questions are of practical importance for market investors.

Acknowledgements

This work was partially supported by the Grant No. 119 awarded to one of us (R.K.) within the First Competition of the Economic Institute, organised by the National Bank of Poland.

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