Acoustic and Biomedical Engineering

Uncertainty Assessment of Index *M*

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Administrative decisions, related to the selection of the priority of activities minimising acoustic effects in the analysed areas, are determined by the index M values. This index is also known as the noise protection agents demand. A form of its function is determined by variables describing the number of residents and the level of exceeding the permissible noise in these inhabited regions. Uncertainty of decisions resulting from such assessments is conditioned by numerical measures: inaccurate measurements of exceeding permissible noise indicators as well as errors in the estimation of the number of residents exposed to them, formulated in probabilistic categories and interpreted in an interval way. Thus, the uncertainty assessment of index M, being the function determined by means of those variables, requires the determination of the density of probability distribution function of its occurrence. The method of its determination constitutes the contents of the presented hereby paper. The study concentrates on the derivation of the density of probability distribution function of index M, at the assumption that the distribution of variable errors is in a form of the normal cut distribution. The computational algorithm, allowing to perform necessary numerical calculations assigned to the uncertainty assessment process of index M, is presented. The obtained results are illustrated by the example.

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1. Introduction

The Environment Protection Act of 27th April 2001 [1] imposes a duty of preparing acoustic maps allowing to assess the environment acoustic state (Art. 118). In the act, it is also required to work out environment noise protection programs for zones where the allowable values are exceeded (Art. 119).

A realisation of the environment noise protection program is a complex task, often limited by technical, legal and economic conditions. In such a program, one has to take into consideration ecological and economical effectiveness of its tasks, in mutual relations with the dates of successive realization of the tasks and financial sources.

The time schedule of operations related to the realisation of the environment protection program in inhabited regions is determined by the value exceeding the permissible noise level ΔL.

The noise protection agents demand value M which decides on the sequence of individual tasks of the program

$$M = 0.1K(10^{0.1\Delta L} - 1), \quad (1)$$

where ΔL — value of exceeding the permissible noise level [dB] in the analysed zone, and K — number of residents living in the zone where the permissible noise level ΔL was exceeded.

The sequence of the tasks realisation in inhibited regions is established in such a way that it starts from regions of the highest index M value, and gradually follows to the regions of the lowest value of this index.

Calculations of index M are not complete without the uncertainty estimation. The uncertainty estimation, which means the possible error of the index M, determines the knowledge of its probability distribution. This distribution is determined on probability distribution characteristic assumed for the initial data calculation: ΔL and K. It allows to calculate: the expected value $E(M)$, standard deviation $\sigma(M)$ and the distribution quantile, assumed — for the required probability that the right solution will be within a certain expanded interval. The problem of uncertainty assessment is crucial for each decision related to control processes [2].

The special procedures were worked out for its estimation. They are published in the Guide to the Uncertainty in Measurements [3–6], under the aegis of the International Office of Measures (BIPM).

The following problems are, first, development of terminology related to uncertainty expressions. Second, building of models of the assessment of the controlled variable uncertainty and related initially measured values and recommended computational algorithms allowing to

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achieve the assumed calculation accuracy. Finally, giving methods of validation of the obtained results.

There are two possible proceedings concerning the solutions proposed in the Guide. The first one corresponds to the traditional concept based on the statistic error analysis (error approach, traditional approach or true value approach) and its propagation [6]. The second one to the uncertainty approach realized — via the mathematical model of the measurement — with the use of the Monte Carlo simulation [3–5]. Both approaches assume that the determination of the control assessment uncertainty is done on the basis of the probability distribution of the measurement results.

In the second solution [3–5], the measure of the control result assessment is its probability distribution related to the density probability distribution of the measurement results determining it.

Computations are done by the method of the distributions propagation attributed to the initial values via the mathematical model of the assessed output value. The recommended algorithm, leading to achievement of the assumed calculation accuracy of the expanded interval, is being determined by means of the Monte Carlo simulation. The uncertainty measure of the deciding variable — being actually assessed — is the length of the expanded interval of the initial distribution calculated for the determined expanded probability (usually equal to 95%).

The development of this approach constitutes solutions dedicated to the uncertainty assessment of the averaged sound levels in acoustic estimations of environment hazards [7]. We see it also in calculations of uncertainty of long-term noise indicators, taking into account differences in sound level emissions at various periods of the calendar year [8]. Their essence are the accurate estimations of the analytical form of the density probability distribution of the estimated variable. In this approach we take the assumption that the probability distribution of values determining this estimated control value is known.

An expansion of such approach into the estimation problem of index M and assessment of its uncertainty is the subject of the hereby paper. The probability distribution for the estimated value is also determined in this paper.

The worked out model formalism of the uncertainty assessment of long-term sound levels, together with the assigned algorithm, is given. The proposed solution was illustrated by the examples of calculations of sound levels uncertainties in acoustic assessments of environmental hazards.

2. Procedure of determination of the probability density for index M

The fundamental assumption of the model of the uncertainty of calculations of the noise protection agents demand index M, Eq. (1), is knowledge of probability distributions of ΔL and K. We assumed that the initial data ΔL and K are burdened with random error.

The source of determining their form can be: inaccurately performed process of acoustic hazards modelling and not complete personal data concerning number of residents living in regions where the level of acoustic hazards was calculated.

The assumption of randomness of initial data ΔL and K in calculations of the noise protection agents demand index M, determines the form of the random variable probability distribution given by Eq. (1).

A point of departure for its determination is, of course, the reliable data collection as well as the proper estimation of their probabilistic characteristics.

The number of residents K staying within the zone where the permissible values of noise indicators are exceeded by ΔL, can be modelled by a random variable of the trimmed-normal distribution, determined in the interval: $[a,b]$, where $a \geq 1$.

Analysis of data originating from simulation investigations, as well as from measurements of exceeding the permissible levels of noise indicators ΔL in zones subjected to the acoustic protection, allows to attribute to this random variable the trimmed-normal distribution determined in the interval $[0,c]$, $c>0$.

We assume that random variables K, ΔL are independent of each other. In order to determine the probability distribution of random variable M we will apply the method used for the transformation of random variables [9]. However, in order to do this, we have to determine the random variables M and V:

$$\begin{cases} M = 0.1K(10^{0.1\Delta L}-1), \\ V = K. \end{cases} \tag{2}$$

Let us write the transformation for random variables described in Eq. (2), and its inversion

$$\begin{cases} m = 0.1k(10^{0.1l}-1), \\ v = k, \end{cases} \tag{3a}$$

and the inversion

$$\begin{cases} l = 10\log_{10}\left(10\frac{m}{v}+1\right), \\ k = v. \end{cases} \tag{3b}$$

Let us mark

$$M_0 = 0.1b(10^{0.1c}-1). \tag{4}$$

The limits of variables ΔL and K are in the following form: $V \in [a,b]$, $\Delta L \in [0,M_0]$. Thus, the Jacobian of the transformation given by Eq. (5) equals

$$J(k,l) = \begin{vmatrix} \frac{\mathrm{d}k}{\mathrm{d}v} & \frac{\mathrm{d}k}{\mathrm{d}m} \\ \frac{\mathrm{d}l}{\mathrm{d}v} & \frac{\mathrm{d}l}{\mathrm{d}m} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ \frac{10^2}{\ln 10}\frac{-m}{V(10m+v)} & \frac{10^2}{\ln 10}\frac{1}{10m+v} \end{vmatrix} = \left|\frac{10^2}{\ln 10}\frac{1}{10m+v}\right|. \tag{5}$$

In order to have the transformation (4) its well defined Jacobian must exist (5) and $J(k,l) \neq 0$. Thus, it follows from Eq. (4) that the random variables M and V values cannot be equal to zero simultaneously, which at our assumptions is satisfied: $a \geq 1$.

From Eqs. (2), (4) and (5), and from independence of variables K, ΔL it follows that the combined distribution of the vector $(M,\ V)$ is of the form

$$f_{V,M}(v,m)=\frac{10^2}{\ln 10}\frac{1}{10m+v}f_K(v)f_{\Delta L}$$
$$\times\Big(10\log_{10}\Big(10\frac{m}{v}+1\Big)\Big). \qquad (6)$$

After integrating out the random variable V (from Eq. (6)), on the determination region of $V\in[a,b]$, we obtain the equation for the random variable M:

$$f_M(m)=\int_a^b\frac{10^2}{\ln 10}\frac{1}{10m+v}f_K(v)f_{\Delta L}$$
$$\times\Big(10\log_{10}\Big(10\frac{m}{v}+1\Big)\Big)\,\mathrm{d}v. \qquad (7)$$

Equation (9) can be applied for a pair of probability distribution of K and ΔL if only K is determined in the interval $[a,b]$ while ΔL in $[0,c]$. However, introducing — in accordance with the assumption — the trimmed-normal distributions, we obtain Eq. (8) which determines the density function of index M:

$$f_M(m)=\frac{1}{[F_M(b)-F_M(a)][F_{\Delta L}(c)-F_{\Delta L}(0)]}$$
$$\times\frac{1}{2\pi\sigma_1\sigma_2}\int_a^b\frac{10^2}{\ln 10}\frac{1}{10m+v}$$
$$\times\exp\Bigg(-\frac{(v-\mu_1)^2}{2\sigma_1^2}$$
$$-\frac{\big(10\log_{10}(10\frac{m}{v}+1)-\mu_2\big)^2}{2\sigma_2^2}\Bigg)\mathrm{d}v, \qquad (8)$$

where $m\in[0,M_0]$ and

$$F_{\mathrm{K}}(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}\sigma_1}\exp\Big(-\frac{(t-\mu_1)^2}{2\sigma_1^2}\Big)\,\mathrm{d}t,$$
$$F_{\Delta L}(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}\sigma_2}\exp\Big(-\frac{(t-\mu_2)^2}{2\sigma_2^2}\Big)\,\mathrm{d}t \qquad (9)$$

are the distribution of trimmed normal variable K and ΔL.

3. Numerical example

Let the random variable $K\sim N_{[1200,1400]}(1250.25)$ while $\Delta L\sim N_{[0,15]}(10.1)$. That time variable M is determined in the interval $[0;4287.2]$ and the random variable $K_1\sim N_{[400;600]}(500;25)$ and $\Delta L_1\sim N_{[0;25]}(18;1)$, M_1 is determined on $[0;18913.67]$. Characteristics of process M and M_1 are in Table.

The observed random variables are characterised by a significant right-hand asymmetry, what we see on Fig. 1, which is displayed by shifting the average value to the right from the distribution mode.

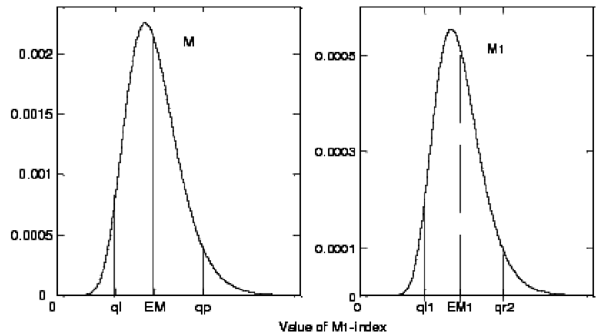

Fig. 1. The diagrams of the density functions of index $M=0.1K(10^{0.1\Delta L}-1)$ and $M_1=0.1K_1(10^{0.1\Delta L_1}-1)$.

TABLE

The list of characteristics of process M and M_1.

Expected value EM	Left quantile ql	Right quantile qr	Standard deviation σ_M	End of the interval of M M_0
685.64	415.01	1028.90	189.66	4287.19
Expected value EM_1	Left quantile ql_1	Right quantile qr_1	Standard deviation σ_{M1}	End of the interval of $M1$ M_{01}
3189.50	2089.00	4591.20	358.02	18913.67

In both examples the attention should be directed towards the long right-hand tails of the function of the density of indicators M and M_1.

On the basis of the above examples it can be seen, from data in table, how the index of the noise protection agents demand is constructed. This index has much higher average value for much smaller number of inhabitants endangered by the higher exceeding of the permissible noise values (M_1) than for much larger number of inhabitants endangered by the smaller exceeding of the permissible noise values (M).

4. Final remarks

The selection of rational operations, preventing from environment acoustic hazards occurring in the given zone, can be satisfactorily solved in case of the proper calculations of the noise protection agents demand index M. For that purpose in addition to the index values there must be calculated assessments of its computational uncertainty generated by uncertain (random) input data. In order to solve such problems the probabilistic bases of modelling such tasks are given in the paper. The method of the probability distribution determination of index M, its expected value, and the estimation of its uncertainty given by the variance and the assumed quantile of the determined distribution is described.

The presented results, as well as conclusions drawn on their bases, can constitute the base of solutions verifying the rightness of decisions undertaken in the development of environment protection programs — with taking into account the assumed confidence level of the calculations. The outlined computational idea can be extended to other forms of input variables distribution of the analysed model.

The possible directions of looking for the solution of the problem were formulated by indication of useful model formalisms. The empirical illustration of this application was shown. It is worth to mention that the presented problem can have significant consequences in the administrative decision process related to the management of the acoustic environment protection.

In further studies, the authors will try to achieve more general models with improved properties. It can be done by using the tools of statistical analysis that do not require knowledge of the distributions of probabilities of input data, such as the bootstrap method.

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