

Weak ^3He Pairing in ^3He – He(II) Mixtures

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In this paper a theoretical study of a possible phase transition in dilute ^3He – He(II) mixtures is presented using the Galitskii–Migdal–Feynman formalism. The effective scattering length is calculated from the Galitskii–Migdal–Feynman T -matrix, which is essentially the effective scattering amplitude dependent on the medium. It is found that at very low ^3He concentrations the s -wave effective scattering length for ^3He – He(II) varies discontinuously from positive to negative values at some critical concentration. This indicates a crossover from a regime with dimers to another with the Cooper pairs. The binding energy of the weakly-bound dimers $^3\text{He}_2$ is computed. The effective p -wave scattering lengths are calculated and compared to the effective s -wave scattering lengths at low and high concentrations. It is found that p -scattering has an important effect on the instability of these mixtures at concentrations $x > 1\%$. Finally, the transport coefficients are computed and compared to the theoretical predictions of Fu and Pethick and the experimental results of König and Pobell.

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1. Introduction

^3He – He(II) mixtures are interesting systems for several reasons. On the one hand, they are widely used as refrigerants for cooling purposes down to the mK range [1]; recent efforts for developing cycles which use liquid ^3He – ^4He mixtures as the working fluid indicate that these could be more efficient than dilution refrigeration cycles [2]. On the other hand they are model systems for testing theories of weakly-interacting fermions. Besides, they constitute “natural laboratories” for studying properties of matter in an extremely pure environment, where most of the unavoidable disturbances present at higher temperatures are almost completely “frozen out”.

At temperatures smaller than the Fermi degeneracy temperature (≈ 0.1 K at zero pressure), ^3He – He(II) mixtures are dilute weakly-interacting neutral *many-fermionic* systems. This is because of the negligible number density of Bose-type excitations (phonons and rotons) under these circumstances, and the dominance of the ^3He quasiparticles [3, 4]. ^3He – He(II) mixtures have an additional degree of freedom, which is the ^3He concentration. This enables us to study the density effect on various properties.

The basic phenomenological theory for dilute ^3He – He(II) mixtures is that of Landau and Pomeranchuk [5]. According to this theory, ^3He quasiparticles interact with each other so weakly that the systems behave as almost ideal Fermi gases. A major theoretical

development was the semi-phenomenological approach of Bardeen et al. [6]. These authors predicted the existence of a supermobility state — characterized by the ^3He atoms moving in the ^4He -background with an exceedingly long mean free path and without friction.

On the purely microscopic level, there has been the variational track — including the correlation-basis-functions theory [7]; and the perturbative track — including the Galitskii–Migdal–Feynman (GMF) and the Brueckner–Bethe–Goldstone (BBG) frameworks [8]. The perturbative track offers greater elegance and formal power; on the other hand, variational calculations appear to have enjoyed greater numerical success [9]. Jackson et al. [9] stated that “Comparisons of the two tracks have often been [quite] uneasy”.

Al-Sugheir et al. [8] studied the effect of hole–hole scattering on any possible fermion–fermion pairing in these systems by calculating the effective relative phase shifts, incorporating many-body effects based on both BBG and GMF formalisms. In the GMF formalism, the s -wave phase shift at zero relative momentum was $-\pi$ and had a cusp at the Fermi momentum; while in the BBG formalism, this phase shift had zero values up to the Fermi momentum. It was concluded that, if they exist at all, $^3\text{He}_2$ molecules in dilute ^3He – He(II) mixtures will be quite fragile; so that they could hardly exist above a certain critical temperature, which should be below $100 \mu\text{K}$.

The possibility of pairing between two ^3He quasiparticles in these mixtures raises the exciting prospect of obtaining a new quantum fluid — namely, a Fermi superfluid “suspended” in a Bose superfluid. In addition, by “tuning” the ^3He concentration x , both s -wave and

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p -wave pairing can presumably be brought into dominance [10, 11]. The first calculation of the transition temperature on the basis of an effective interaction in momentum space was achieved by Bardeen et al. [6]. Such a transition would yield a ^3He paired-fermion superfluid dissolved in the already-superfluid ^4He [He(II)] boson solvent. This would give two interpenetrating superfluids. Therefore, a four-fluid model would need to be described [11]. In Ref. [10], the superfluid transition temperatures in ^3He -He(II) mixtures (the ^3He concentration being more than 3%), with a repulsive interaction, were calculated using high-order perturbation theory corrections. This involved calculating the contributions of third- and fourth-order diagrams in the gas phase and taking into account retardation effects. Estimates of the superfluid transition temperature for ^3He in ^3He -He(II) range from 10^{-5} – 10^{-4} K for singlet pairing at low x , and 10^{-10} – 10^{-4} K for triplet pairing at higher x [10–13]. However, no experimental evidence has yet been found for such a transition.

In two-dimensional systems, there have been predictions of a transition to superfluid behavior at low temperature [13–15]. Pobell's group [16] found no evidence for superfluidity in 2D solutions for $T \geq 9$ mK in zero magnetic field with ^3He coverage in the range 0.1–1.0 monolayer. Bashkin et al. [17] expected that in 2D $^3\text{He}_2$ dimers may form in the limit of small ^3He concentration, and suggested that $^3\text{He}_2$ dimers may cross over from a Bose gas of dimers to a 2D Fermi fluid (with strong pair correlations) upon increasing the ^3He concentration. The quantitative calculation of the dimer binding energy was based on a semi-empirical effective interaction in which the bare potential was the Lennard–Jones potential and the single-particle wave functions were obtained within a density-functional approach [17].

In this paper, the GMF formalism [18, 19] will be applied to ^3He -He(II) mixtures to calculate the effective phase shifts, which, in turn, are used to evaluate the effective, density-dependent s -wave and p -wave scattering lengths. This formalism is suitable for ^3He -He(II) mixtures, thanks to the relatively dilute and weakly-interacting nature of these mixtures. From the value and sign of scattering length, it can be determined whether the two ^3He atoms at very low temperatures prefer to form weakly-bound dimers $^3\text{He}_2$ or Cooper-like pairs.

The basic quantity in this formalism is the T -matrix whose instability is an indication of the formation of fermion pairs. This matrix includes hole–hole scattering inside the Fermi sea as well as particle–particle scattering outside the sea. The GMF formalism has recently been applied to both spin-polarized ^3He -He(II) mixtures and spin-polarized deuterium [20, 21]. It has been concluded that hole–hole scattering plays a crucial role in any possible fermion–fermion pairing in these systems.

The rest of the paper is organized as follows. Section 2 contains only a brief account of the GMF formalism for ^3He -He(II) mixtures, since this theory is well-described elsewhere [19]. The results are summarized and discussed

in Sect. 3. Finally, in Sect. 4, the paper closes with some concluding remarks.

2. GMF formalism for ^3He -He(II) mixtures

For a neutral Fermi system, the GMF T -matrix is given by [19]:

$$T(\mathbf{p}, \mathbf{p}'; s, \mathbf{P}) = u(|\mathbf{p} - \mathbf{p}'|) + (2\pi)^{-3} \int d\mathbf{k} u(|\mathbf{p} - \mathbf{k}|) \times [g_0(k, s) Q(\mathbf{k}, \mathbf{P}, \beta') - g_0^+(k, s) \bar{Q}(\mathbf{k}, \mathbf{P}, \beta')] \times T(\mathbf{k}, \mathbf{p}'; s, \mathbf{P}). \quad (1)$$

Here \mathbf{p} and \mathbf{p}' are the relative incoming and outgoing momenta; the parameter s is the total energy of the interacting pair in the center-of-mass frame and is given by

$$s \equiv 2 \mu_3^* \left(2P_0 - \frac{P^2}{m_3^*} \right); \quad (2)$$

P_0 is the total energy of the pair; P^2 is the energy carried by the center of mass. Throughout this work we use units such that $\hbar = 2m_3 = k_B = 1$, k_B being Boltzmann's constant and m_3 the mass of the ^3He atom. The operator $u \equiv \frac{2\mu_3^*}{\hbar^2} V \equiv \frac{b}{2} V$, where V is the Fourier transform of a static central two-body potential and μ_3^* is the effective reduced mass of the ^3He interacting pair: $\mu_3^* = \frac{1}{2} m_3^* = \frac{b}{2} m_3$ and $\beta' = \frac{\beta}{b}$, β being the inverse temperature. The conversion factor is $\frac{\hbar^2}{2m_3} = 8.0425 \text{ K } \text{\AA}^2$. Using our system of units, we have

$$s = bP_0 - P^2. \quad (3)$$

The operator Q (\bar{Q}) is the product of particle–particle (hole–hole) occupation probabilities. In momentum space, the hole occupation probability is just the Fermi–Dirac distribution, which reduces to the unit step function at zero temperature. When subtracted from unity, this yields the particle-occupation probability. Q (\bar{Q}) is equal to one if both particles (holes) are outside (inside) the Fermi-sea. The angle-averaged functions Q and \bar{Q} are given by

$$Q(k, \beta') = \frac{1}{\exp(-\beta'(k^2 - k_F^2)) + 1} \times \frac{1}{\exp(-\beta'(k^2 - k_F^2)) + 1}, \quad (4)$$

$$\bar{Q}(k, \beta') = \frac{1}{\exp(\beta'(k^2 - k_F^2)) + 1} \times \frac{1}{\exp(\beta'(k^2 - k_F^2)) + 1}, \quad (5)$$

k_F being the Fermi momentum

$$k_F = (3\pi^2 \rho_3)^{1/3}. \quad (6)$$

The number density of ^3He particles in ^3He -He(II) mixtures ρ_3 is given by

$$\rho_3 = \frac{0.6022x}{27.58(1 + \alpha x)} \left[\text{\AA}^{-3} \right]. \quad (7)$$

The factor α is the volume differential coefficient, representing the difference between the volume occupied by the ^3He atom and that occupied by the heavier ^4He atom. Although α decreases with increasing pressure P_r , yet as P_r increases, the background atoms become more and more localized; so that m_3^* becomes as large as $\sim 2.9m_3$ at $P_r \approx 12$ atm. Table I shows α and m_3^* at different values of P_r [22].

TABLE I
 α and m_3^* at different values of P_r .

P_r [atm]	α	m_3^*/m_3
0	0.30	2.33
6	0.23	2.50
9	0.21	2.70
12	0.20	2.90

The free two-body Green function $g_0(s)$ is defined as:

$$g_0(\mathbf{k}, s) \equiv \frac{1}{k^2 - s - i\eta}. \quad (8)$$

The system of interacting real particles is described in terms of weakly-interacting quasiparticles; this justifies the use of free Green functions. The quantity η is a positive infinitesimal in the scattering region and zero otherwise.

The Fourier–Bessel transform of the interatomic potential was calculated using a program originally constructed by Ghassib and coworkers for interhelium potentials [23]. The effective interaction in configuration space between two ^3He quasiparticles embedded in He(II) is the sum of three physical effects [24]. The first is the direct $^3\text{He}\text{-}^3\text{He}$ interaction, $V_{33}(r)$. To this end, one of the most highly-acclaimed interatomic helium potentials, the so-called HFDHE2 [25, 26], has been used. The second effect is the *induced* interaction between the ^3He atoms and the He(II) background, $V_{34}(r)$. The third effect is associated with the *induced* $^4\text{He}\text{-}^4\text{He}$ interaction, $V_{44}(r)$. The total effective interatomic potential between two ^3He atoms is, therefore,

$$V(r) = V_{33}(r) + V_{34}(r) + V_{44}(r). \quad (9)$$

The effective phase shifts can be determined by parameterizing the on-energy-shell T -matrix, computed by putting $p = p' = \sqrt{s}$, as follows [19]:

$$T_\ell(p, p; s, P, \beta') \equiv T_\ell(p; P, \beta'),$$

$$T_\ell(p; P, \beta') = -\frac{4\pi \exp(i\delta_\ell^E(p; P, \beta')) \sin(\delta_\ell^E(p; P, \beta'))}{p Q(p; P, \beta') + \bar{Q}(p; P, \beta')}; \quad (10)$$

so that

$$\tan(\delta_\ell^E(p; P, \beta')) \equiv \frac{\text{Im}T_\ell(p; P, \beta')}{\text{Re}T_\ell(p; P, \beta')}. \quad (11)$$

$\text{Im}T_\ell(p; P, \beta')$ and $\text{Re}T_\ell(p; P, \beta')$ denoting, respectively, the imaginary and real parts of $T_\ell(p; P, \beta')$.

The effective ℓ -wave scattering length a_ℓ at low energy is defined as [27, 28]:

$$a_\ell^{2\ell+1} \equiv -\lim_{k \rightarrow 0} \frac{\tan(\delta_\ell^E(k))}{k^{2\ell+1}}. \quad (12)$$

The word “effective” indicates that the quantities involved pertain to a many-body medium; they depend on the density, temperature and pressure.

The sign of the scattering length determines the overall repulsive or attractive nature of the interaction: a positive scattering length corresponds to an effective repulsive interaction; whereas a negative scattering length represents an effective attractive interaction. If the effective attraction is made more and more attractive, the effective scattering length becomes larger and larger in absolute value; then the sign of the scattering length will change from large negative to large positive. This change is related to the generation of bound states. Increasing the attraction further causes the scattering length to decrease in magnitude [27].

The question of how a phase transition manifests itself in $^3\text{He}\text{-He(II)}$ mixtures depends strongly on the microscopic structure. Fermions in a cold Fermi gas tend to form diatomic molecules (dimers) when the scattering length $a_0 \gg 0$ [29]; the ground state in this case is a Bose–Einstein condensate (BEC) of dimers. If $a_0 < 0$, the fermions prefer to form Cooper-like pairs; the ground state in this case is a condensate of the Cooper pairs.

3. Results and discussion

The effective s - and p -wave scattering lengths for $^3\text{He}\text{-He(II)}$ mixtures were calculated from Eq. (12) as a function of ^3He -concentration x at different values of pressure P_r . The GMF T -matrix, regarded here as our effective interaction in momentum space, was determined by a matrix-inversion technique, as in previous work [19]. In Gaussian quadrature, a 96-point mesh was used. This was large enough to give “stable” and accurate results.

3.1. s -wave scattering

Tables II–VI show the effective s -wave scattering length a_0 as a function of x at different values of P_r . The behavior of a_0 is as follows: at very low x , a_0 is positive. By increasing x , the value of a_0 increases until it reaches a maximum at some critical concentration x_c . A further increase in x causes the sign of a_0 to change from large positive to large negative and then to negative values. This behavior of a_0 is expected in quantum mechanics [28], since the effective attractive interaction decreases by increasing x . The sign change resulting from increased attraction is related to the generation of bound states. Our results show that the ^3He dimer regime occurs when $x \leq x_c$ where $a_0 \gg r_0$, r_0 is the effective

range of the potential $\approx 2.45 \text{ \AA}$. At $x > x_c$, the effective interaction is *less attractive*, $a_0 < 0$, i.e., we have an overall weak attractive potential. The magnitude of a_0 decreases with increasing x , thanks to increasing repulsive many-body effects. Then, the two fermions prefer to form Cooper-like pairing.

TABLE II

a_0 for different x at zero-external pressure P_r .

x	$a_0 [\text{\AA}]$
free-scattering limit	12.34
0.01%	35.9
0.02%	58.5
0.04%	422.6
0.05%	-94.8
0.1%	-41.2
0.5%	-9.9
1%	-7.4

TABLE III

a_0 for different x at external pressure $P_r = 6 \text{ atm}$.

x	$a_0 [\text{\AA}]$
free-scattering limit	11.87
0.01%	32.7
0.02%	50.8
0.04%	194.1
0.05%	-120.2
0.1%	-46.7
0.5%	-10.1
1%	-7.6

TABLE IV

a_0 for different x at external pressure $P_r = 9 \text{ atm}$.

x	$a_0 [\text{\AA}]$
free-scattering limit	9.43
0.01%	19.7
0.02%	25.6
0.04%	36.9
0.05%	70.1
0.10%	1598.3
0.15%	-72.1
0.20%	-34.7
0.30%	-22.4
0.50%	-12.9
1%	-8.97

From Table VI, it is noted that a_0 decreases with increasing P_r at low x ($x < x_c$), and a_0 increases in magnitude with increasing P_r at high x ($x > x_c$). This is because as P_r increases α decreases, and hence the minimum and the long-range part of the effective potential

TABLE V

a_0 for different x at external pressure $P_r = 12 \text{ atm}$.

x	$a_0 [\text{\AA}]$
free-scattering limit	7.91
0.01%	13.90
0.02%	16.48
0.04%	20.58
0.05%	29.30
0.1%	48.95
0.15%	166.8
0.2%	-109.4
0.3%	-39.93
0.5%	-17.25
1%	-10.83

TABLE VI

a_0 as a function of x at different values of P_r .

x	$a_0 [\text{\AA}]$			
	$P_r = 0 \text{ atm}$	$P_r = 6 \text{ atm}$	$P_r = 9 \text{ atm}$	$P_r = 12 \text{ atm}$
free-scattering limit	12.34	11.87	9.43	7.91
0.01%	35.9	32.7	19.7	13.90
0.02%	58.5	50.8	25.6	16.48
0.05%	-9.9	-10.1	-12.9	-17.25
0.10%	-7.4	-7.6	-8.97	-10.83

become shallower. Our results indicate that, in spite of the decrease in the overall attraction of the effective potential, the relatively large ^3He effective mass makes possible the formation of $^3\text{He}_2$ dimers in dilute ^3He - $^3\text{He(II)}$ mixtures.

The binding energy of the weakly-bound $^3\text{He}_2$ dimers, if they exist at all, can be determined using the equation ($a_0 \gg r_0$) [30]:

$$E_B = \frac{\hbar^2}{m_3^* a_0^2}. \quad (13)$$

Our results for x_c and E_B are summarized in Tables VII and VIII. Table VII shows x_c and the corresponding E_B at different values of P_r . It is noted that x_c increases with increasing P_r . In Table VIII, E_B is shown at $x \leq x_c$ for different values of P_r . The increase of E_B at the same concentration with increasing P_r is expected, thanks to the enhanced localization of the interacting ^3He pair. Clearly, the ^3He - ^3He binding in this system is very fragile.

The question then arises: If, for argument's sake, molecular binding *does* occur in our system, how many $^3\text{He}_2$ the molecules will result? The answer is that this number will be so as to conform to requirements of self-saturation — i.e., $^3\text{He}_2$ will continue to form until it becomes energetically unfavorable to further convert single quasiparticles to molecules [31].

TABLE VII

 x_c and E_B at different values of P_r .

P_r [atm]	x_c	E_B [μK]
0	0.046%	38.7
6	0.046%	170.8
9	0.1%	2.33
12	0.16%	200

TABLE VIII

 E_B as a function of x at different values of P_r .

x	E_B [mK]			
	$P_r = 0$ atm	$P_r = 6$ atm	$P_r = 9$ atm	$P_r = 12$ atm
0.01%	5.36	6.04	15.33	28.7
0.02%	2.02	2.48	9.07	20.4
0.04%	0.04	1.71	4.33	13.1
0.05%	no dimers	no dimers	1.21	6.45
0.10%	no dimers	no dimers	2.33×10^{-3}	2.31
0.15%	no dimers	no dimers	no dimers	0.20

Another point worth noting concerns the possibility of clustering in the system, that is, the formation of n -mers ($n \geq 3$). Theoretical considerations [32] suggest that the formation of trimers and higher-order clusters cost less energy than dimers. However, this point goes beyond the scope of the present work.

3.2. p -wave scattering

The effective p -wave scattering length a_1 is displayed in Table IX as a function of x at $P_r = 12$ atm. For comparison purposes, a_0 is also shown. It is noted that both s - and p -waves are characterized by purely attractive interactions, i.e., $a_0 < 0$ and $a_1 < 0$. Therefore our system is unstable against Cooper-pair formation, where both a_0 and a_1 are less than zero [33]. Clearly, a_1 has only a weak dependence on concentration. The magnitude of a_1 is in the range 3–4 Å, which corresponds to a very weak attractive interaction. At low x ($x < 1\%$), the magnitude of a_0 is much larger than the magnitude of a_1 , which means that the effective s -wave scattering dominates the effective p -wave scattering, i.e., p -scattering can be ignored. At higher x ($x > 1\%$), a_0 and a_1 are comparable, which means that the effective p -wave scattering cannot be ignored; both s -wave and p -wave pairing may then take place. To determine which type of pairing may occur, the singlet and triplet transition temperatures $T_c^{(s)}$ and $T_c^{(p)}$ should be computed. The type of pairing which has the higher transition temperature may take place. It has been predicted that dilute ${}^3\text{He}\text{-He(II)}$ mixtures favor s -wave pairing at $x < 3\%$ [10, 34]. In this range the scattering length corresponds to attraction; so that s -wave pairing may occur. On the other hand, the interaction tends to produce p -wave coupling at $x > 3\%$. Therefore the instability of our system exists thanks to the Kohn–Luttinger mechanism at $x > 3\%$ [10, 35]. The

triplet transition temperature $T_c^{(p)}$ can be calculated using [12]:

$$T_c^{(p)} = T_F \exp(-5\pi^2 / (8(2\ln 2 - 1)k_F^2 a_1^2)).$$

Our results for $T_c^{(p)}$ are in the range $\approx 10^{-9}$ – 10^{-7} K, as shown in Table X. These are comparable to the results of [12, 36, 37].

TABLE IX

 a_0 and a_1 for different values of x at $P_r = 12$ atm.

x	a_0 [Å]	a_1 [Å]
0.2%	−109.4	−3.92
0.3%	−39.93	−3.90
0.5%	−17.25	−3.86
1%	−10.83	−3.78
2%	−6.93	−3.62
3%	−5.63	−3.48
4%	−5.17	−3.41
5%	−4.8	−3.32
6%	−4.49	−3.24

TABLE X

 $T_c^{(p)}$ for different values of x at $P_r = 12$ atm.

x	$T_c^{(p)}$ [K]
3.0%	2.16×10^{-9}
4.0%	3.35×10^{-8}
5.0%	1.60×10^{-7}
6.0%	4.86×10^{-7}

The fact remains that so far there has been no experimental evidence to the formation of a more ordered state in the system below some critical temperature [34, 38, 39]. Further work, both experimental and theoretical, is needed in this respect, especially at ultralow temperatures and relative high pressures. In passing, we note that the above effective phase shifts can be used to calculate the corresponding effective cross sections — including the total, diffusion and viscosity cross-sections [40].

3.3. Transport properties at very low temperatures

The predicted temperature dependence of Landau’s Fermi liquid parameters, namely, the diffusion coefficient ($D \sim T^{-2}$), the thermal conductivity ($\kappa \sim T^{-1}$) and the viscosity ($\eta \sim T^{-2}$) at $T \ll T_F$, was confirmed by Anderson et al. [41], Abel et al. [42] and König and Pobell [43].

König and Pobell investigated ${}^3\text{He}\text{-}{}^4\text{He}$ mixtures at ${}^3\text{He}$ concentrations $0.98\% \leq x \leq 9.5\%$ in the temperature range $1 \text{ mK} \leq T \leq 100 \text{ mK}$ and at pressures $0 \leq P_r \leq 20$ bar. Their results [38, 43] showed that ηT^2 tended to a constant value below ≈ 10 mK. It was found that $\eta T^2 = 1.8 \times 10^{-9}$ at $x = 0.98\%$, which was nearly independent of the pressure.

The transport phenomena at very low temperatures ($T \approx 1 \mu\text{K}$) are entirely determined by the ^3He atoms whose mean free path is limited by ^3He - ^3He collisions. Bashkin found the following expressions for κ , η , and D [44, 45]:

$$\kappa T = 0.021 \frac{p_F^3}{(m_3^* a_0)^2} (1 - 0.34\lambda), \quad (14)$$

$$\eta T^2 = 0.0022 \frac{p_F^5}{(m_3^* a_0)^2} (1 + 0.74\lambda), \quad (15)$$

$$DT^2 = 0.255 \left(\frac{\hbar}{m_3^*} \right)^3 \left(\frac{p_F}{a_0} \right)^2 (1 - 2.18\lambda), \quad (16)$$

where $\lambda \equiv \frac{p_F a_0}{\pi \hbar}$ and $p_F = \hbar k_F$.

TABLE XI

Transport coefficients κ , η , and D for different values of x at $P_r = 12 \text{ atm}$.

x	κT [$\text{J m}^{-1} \text{s}^{-1}$]	ηT^2 [Pa s] using Eq. (15)	ηT^2 [Pa s] using Eq. (17)	DT^2 [$\text{m}^2 \text{s}^{-1} \text{K}^2$]
4.0%	11.1×10^{-5}	3.23×10^{-9}	6.12×10^{-9}	9.44×10^{-9}
5.0%	16.0×10^{-5} [19×10^{-5}] ^a	5.43×10^{-9} [22.9×10^{-9}] ^a	10.3×10^{-9} [22.9×10^{-9}] ^a	127×10^{-10} [69.7×10^{-10}] ^a
6.0%	21.9×10^{-5}	8.42×10^{-9} [27×10^{-9}] ^{b,c}	15.8×10^{-9} [27×10^{-9}] ^{b,c}	1.63×10^{-8}
9.5%	43.1×10^{-5}	21.7×10^{-9} [36×10^{-9}] ^c	41.8×10^{-9} [36×10^{-9}] ^c	2.80×10^{-8}

^a [47]; ^b [43]; ^c [38]

Also, η can be calculated by [45, 46]:

$$\eta = \frac{\hbar}{\pi} \frac{\rho_3}{(3\pi^2 \rho_3 a_0^3)^{2/3}} \left(\frac{T_F}{T} \right)^2. \quad (17)$$

Our results for κ , η [using Eqs. (15) and (17)], and D are summarized in Table XI. This shows a comparison of our results to the theoretical predictions of Fu and Pethick [47] and the experimental results of König and Pobell [38, 43]. Fu and Pethick [47] studied the concentration dependence of the transport properties by using the Bardeen, Baym and Pines (BBP) potential as a bare interaction between two ^3He atoms, then using perturbation theory to calculate the scattering amplitudes as functions of the ^3He -concentration. They showed that, although the strength of the two-particle interaction is weak, the expansion parameter in perturbation theory is large (≈ 0.3 for $x = 1.3\%$). This approach leads to quite good agreement between theory and experiment for κ and η , but is less successful in yielding agreement with the experiments of Murdock et al. [48] for the spin diffusion in 5% mixtures. It is noted that our results are of the same order of magnitude as the theoretical predictions and experimental results. κ and D show good agreement with the theoretical predictions [47]; whereas η shows good agreement with the experimental results [38, 43].

4. Conclusion

In this paper, the GMF T -matrix was used to calculate the effective s - and p -wave scattering lengths for

low and high ^3He -concentrations in ^3He - He(II) mixtures. The basic achievements of the paper are: (1) pointing out the possibility of ^3He dimer-pairing crossover in these mixtures at very low concentrations; (2) calculation of the binding energy of the weakly-bound ^3He dimers; (3) pointing out the possibility of p -wave pairing at higher concentrations ($x > 1\%$); and (4) calculation of the triplet transition temperature and the transport coefficients, namely, the diffusion coefficient, the thermal conductivity and the viscosity, which are comparable to previous theoretical predictions as well as experimental results. For s -wave, below a critical concentration, ^3He atoms may tend to form dimers. Above this critical concentration, ^3He atoms may form Cooper-like pairs. The effect of the pressure is to increase both the critical concentration and the binding energy of $^3\text{He}_2$ dimers.

Our results can be summarized as follows: at very low concentration, ^3He atoms tend to form diatomic molecules ($^3\text{He}_2$ dimers). By increasing the concentration, the strength of dimer interaction becomes weaker and weaker. Then at some critical concentration, which depends on the external pressure, a crossover from a regime with dimers to Cooper-pairing regime may occur. At low concentration ($x < 1\%$), p -wave pairing can safely be ignored. At higher concentration ($x > 1\%$), however, this is not the case. It is concluded that p -scattering has an important effect on the stability of these mixtures at high x ($x > 1\%$) and should not be neglected, which is in agreement with Ref. [12].

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References

- [1] D.C. Kelly, *Thermodynamics and Statistical Physics*, Academic, New York 1973.
- [2] F.K. Miller, J.G. Brisson, *J. Low Temp. Phys.* **147**, 559 (2007).
- [3] J. Wilks, *The Properties of Liquid and Solid Helium*, Clarendon, Oxford 1967.
- [4] H.B. Ghassib, *Z. Phys. B. Condens. Matter* **6**, 91 (1984).
- [5] D.O. Edwards, M.S. Pettersen, *J. Low Temp. Phys.* **87**, 473 (1992).
- [6] J. Bardeen, G. Baym, D. Pines, *Phys. Rev.* **196**, 207 (1967).
- [7] E. Feenberg, *Theory of Quantum Fluids*, Academic, New York 1967.
- [8] M.K. Al-Sugheir, H.B. Ghassib, B.R. Joudeh, *Int. J. Mod. Phys. B* **20**, 2491 (2006).
- [9] A.D. Jackson, A. Lande, R.A. Smith, *Phys. Rep.* **86**, 55 (1982).
- [10] D.V. Efremov, M.S. Mar’enko, M.A. Baranov, M.Yu. Kangan, *J. Exp. Theor. Phys.* **90**, 861 (2000).
- [11] G.R. Pickett, in: *Proc. Conf. on Spin-Polarized Quantum Systems*, Ed. S. Stringari, World Sci., Singapore 1988, p. 411.
- [12] E. Østgaard, E. Bashkin, *Physica B* **178**, 134 (1992).
- [13] R.B. Hallock, *Physica B* **329-333**, 154 (2003).
- [14] M.Y. Kagan, A. Chubukov, *JETP Lett.* **50**, 517 (1989).
- [15] M.Y. Kagan, *Sov. Phys.-Usp.* **37**, 69 (1994).
- [16] K. Shirahama, F. Pobell, *Physica B* **194**, 863 (1994).
- [17] E. Bashkin, N. Pavloff, J. Treiner, 1995 *J. Low. Temp. Phys.* **99**, 659 (1995).
- [18] A.L. Fetter, J.D. Walecka, *Quantum Theory of Many-Particle Systems*, McGraw-Hill, New York 1971.
- [19] H.B. Ghassib, R.F. Bishop, M.R. Strayer, *J. Low Temp. Phys.* **23**, 393 (1976).
- [20] A.S. Sandouqa, M.K. Al-Sugheir, H.B. Ghassib, *Phys. Scr.* **74**, 5 (2006).
- [21] B.R. Joudeh, A.S. Sandouqa, M.K. Al-Sugheir, H.B. Ghassib, *Physica B* **404**, 1847 (2009).
- [22] B.R. Joudeh, M.Sc. Thesis, University of Jordan, 1998 (available upon request).
- [23] S.N. Ali, M.Sc. Thesis, University of Jordan, 1997 (available upon request).
- [24] L.J. Campbell, *Phys. Rev. Lett.* **19**, 156 (1967).
- [25] R.A. Aziz, V.P.S. Nain, J.S. Carley, W.L. Taylor, G.T. McConville, *J. Chem. Phys.* **70**, 4330 (1979).
- [26] A.R. Janzen, R.A. Aziz, *J. Chem. Phys.* **103**, 9626 (1995).
- [27] R. Landau, *Quantum Mechanics II*, 2nd ed. Wiley, New York 1996.
- [28] J.J. Sakuri, *Modern Quantum Mechanics*, Benjamin, Singapore 1987.
- [29] D.S. Petrov, C. Salomon, G.V. Shlyapnikov, *Phys. Rev. Lett.* **93**, 090404 (2004).
- [30] G. Gutiérrez, M. de Llano, W.C. Stwalley, *Phys. Rev. B* **29**, 5211 (1984).
- [31] E. Polturak, R. Rosenbaum, *J. Low. Temp. Phys.* **43**, 477 (1981).
- [32] H.B. Ghassib, *Z. Phys. B, Condens. Matter* **56**, 91 (1984).
- [33] R. Roth, H. Feldmeier, *Phys. Rev. A* **64**, 043603 (2001).
- [34] J. Tuoriniemi, J. Martikainen, E. Pentti, A. Sebedash, S. Boldarev, G. Pickett, *J. Low Temp. Phys.* **129**, 531 (2002).
- [35] W. Kohn, J.M. Luttinger, *Phys. Rev. Lett.* **15**, 524 (1965).
- [36] E.P. Bashkin, A.E. Meyerovich, *Adv. Phys.* **30**, 1 (1981).
- [37] P.G. van de Haar, G. Frossati, K.S. Bedell, *J. Low Temp. Phys.* **77**, 35 (1989).
- [38] R. König, F. Pobell, *J. Low. Temp. Phys.* **79**, 287 (1994).
- [39] R. König, A. Betat, F. Pobell, *J. Low. Temp. Phys.* **97**, 311 (1994).
- [40] B.R. Joudeh, A.S. Sandouqa, H.B. Ghassib, M.K. Al-Sugheir, *J. Low. Temp. Phys.* **161**, 348 (2010); A.S. Sandouqa, H.B. Ghassib, B.R. Joudeh, *Chemical Physics Letters* **490**, 172 (2010); B.R. Joudeh, *Modern Applied Science* **5**, 25 (2011).
- [41] A.C. Anderson, D.O. Edwards, W.R. Roach, R.E. Sarwinski, J.C. Wheatley, *Phys. Rev. Lett.* **17**, 367 (1966).
- [42] W.R. Abel, R.T. Johnson, J.C. Wheatley, *Phys. Rev. Lett.* **18**, 737 (1967).
- [43] R. König, F. Pobell, *Phys. Rev. Lett.* **71**, 2761 (1994).
- [44] E.P. Bashkin, *JETP* **46**, 972 (1977).
- [45] J.R. Bradley, *Rep. Prog. Phys.* **60**, 1173 (1997).
- [46] A.E. Meyerovich, *J. Low. Temp. Phys.* **124**, 461 (2001).
- [47] H.H. Fu, C.J. Pethick, *Phys. Rev. B* **14**, 3837 (1976).
- [48] E.S. Murdock, K.R. Mountfield, L.R. Corruccini, *J. Low Temp. Phys.* **31**, 581 (1978).