Acoustic and Biomedical Engineering

Evaluation of the Active Plate Vibration Reduction by the Parameter of the Acoustic Field

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Effectiveness of the active vibration reduction of triangular plate is evaluated by way of the analysis of acoustical field. The active vibration reduction is performed with actuators. The effect is measured by the analysis of the acoustical field both far distance and near one from the plate. As the control parameter, the difference between acoustical pressures is considered. The first pressure and the second one are radiated by the plate without and with the vibration reduction, respectively. The control parameter is calculated for two reduction cases. First case, when actuators are attached at so-called quasi-optimal places and second one, when they are shifted. The numerical calculations show that the acoustical field is sensitive to change of the plate active vibration reduction. It responds to even little changes of the plate vibrations in both active reduction cases mentioned above. So that it is handy indirect control parameter of the active vibration reduction.

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1. Introduction

One of the most important technical problems is contactless measure of the transverse vibration of mechanical structures such as plates, beams, shells and so on. Such problem, for example, appears in an active reduction of the plate vibration.

The active vibration reduction is realised with actuators [1–3] and the effect depends on their distribution on the plate surface [4–8]; a survey of the optimal distribution methods of the actuators on the structure is given in [9]. As pointed out in [4–6], little changes of actuators distribution causes the change of the plate vibration. From mechanical point of view it is interesting how vibrations are transposed on the bending moment and shearing forces at the clamped side; this problem is solved in [4–6]. The chosen two parameters have to be calculated. Then one needs to find another parameter.

In the paper, the acoustic pressure radiated by the vibration plate is proposed as this control parameter more particularly as it may be easily gained experimentally; the survey of the problem is given in [2, 7, 8, 10, 11] and will not be repeated here. The acoustic pressure is analysed in both the far field and the near one. At the latter it is considered, at any small distance, in the plane parallel to the plate surface. In [12] there was already proved that the acoustic field is sensitive to change of the plate vibration, particularly in the near field. Therefore it fits the bill as the parameter of vibration change during an active vibration reduction for different distribution of actuators on the plate surface. To confirm this idea two cases are considered [4–6]. At the first case actuators are bonded at quasi-optimal places and at the second one they are a little shifted.

To compare acoustic pressure radiated by the plate without and with vibration reduction some control parameters are defined. To well fulfil the task in the near field a simple difference of acoustic pressures at some points is defined as a local control parameter. The mean algebraic value of sum of local control parameters at some points is defined as a global control parameter. The first value is the pressure radiated by the plate without the vibration reduction and the second one by the plate with the vibration reduction.

The idea proposed in the paper is verified numerically. For this purpose the triangular plate with clamped-free-free (C-F-F) boundary conditions is considered. The actuators are distributed at quasi-optimal places (for simplicity marked by Q) and they are a little shifted (marked by S); these places were found in [4–6]. Next, the local and the global control parameters are computed both in the far field and in the near field. The conclusions are derived from numerical results. In the end the conclusions were compared to those ones published in literature.

2. Active vibration reduction of the plate with actuators

The governing equation of transverse vibration motion of the plate, on the basis of Kirchhoff's classical small deflection theory, has the form [1] p. 134, [2] p. 1238, [13, 14],

$$N\Delta^2 w + \rho h D_t^2 w = F_{\mathrm{E}} + F_{\mathrm{P}}\,, \tag{1}$$

where $w = w(x,y,t)$ — transverse deflection, $F_{\mathrm{E}} = F_{\mathrm{E}}(x,y,t)$ — external excitation by plane acoustic wave, $F_{\mathrm{P}} = F_{\mathrm{P}}(x,y,t)$ — force due to actuators, ρ — mass density, h — thickness, $N = Yh^3/12(1-\nu^2)$ — flexural rigidity, Y — Young's modulus, ν — Poisson's ratio, Δ — Laplace operator, $\Delta = \mathrm{D}_x^2 + \mathrm{D}_y^2$, $\mathrm{D}_o^2(\cdot) = \partial^2(\cdot)/\partial o^2$.

The C-F-F boundary conditions imposed on the plate are defined [15], Fig. 1:

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$$w = 0\,, \quad \theta = 0\,, \tag{2}$$

$$Q_n = 0\,, \quad M_{nn} = 0\,, \tag{3}$$

where Eq. (2) and Eq. (3) relate to clamped and free sides, respectively, θ — torsion angle, Q_n — equivalent shear force, M_{nn} — bending moment, $\boldsymbol{n}$ — normal to a boundary.

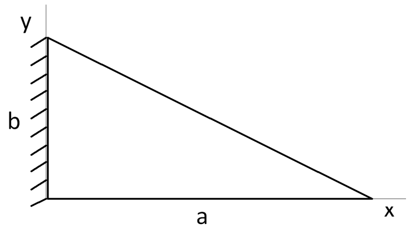

Fig. 1. Triangular plate with C-F-F boundary conditions.

Approximate solution of the vibration equation of the triangular plate based on finite element method (FEM) is discussed in detail in [16]. The FEM is the base of ADINA computer code and this program is applied in calculations. The solution of Eq. (1) with (2) and (3) boundary conditions, in steady harmonic state, is the function $w(x,y)$ and this function is taken into account in the following considerations.

If the actuator excited by the voltage applied in the poling direction, the interaction appears between actuator and the plate [1, 2]. Here, the interaction means the reduction of the plate vibrations.

3. Indirect control parameters of active vibration reduction

The acoustic field radiated by the plate vibrating with any mode located in an infinite and rigid baffle is calculated using the Huygens–Rayleigh integral [12, 17]. In the harmonic steady state

$$\Phi(x,y) = \frac{1}{2\pi}\int_S v(x,y)\frac{\exp(-\mathrm{i}kr)}{r}\,\mathrm{d}S\,, \tag{4}$$

where $\Phi(x,y) = \Phi$ — velocity potential, $v(x,y,t) = \mathrm{D}_t w(x,y,t)$ — vibration velocity of the driving surface, k — dimensionless wave number; the rest of symbols are depicted in Fig. 2.

The acoustic pressure is given by

$$p(x,y) = \mathrm{i}\rho_0\omega\,\Phi(x,y)\,, \tag{5}$$

where ρ_0 — air density, ω — annular frequency.

In the following, based on the acoustic pressures radiated by vibrating plate without and with the vibration reduction, two control parameters are defined.

Let $p = p(\theta,\varphi)$ be an acoustic pressure radiated by the vibrating plate without vibration reduction. In the following two acoustic pressures are defined. First of them is given by plate with vibration reduction when actuators are quasi-optimally distributed and it is marked by

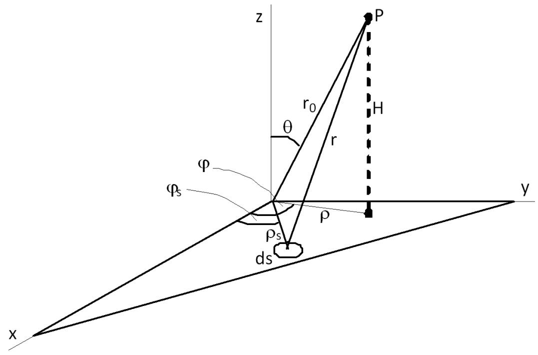

Fig. 2. Geometry of the problem.

$p_Q = p_Q(\theta,\varphi)$. Second — when actuators are shifted and it is marked by $p_S = p_S(\theta,\varphi)$. For simplicity, the $p_Q(\theta,\varphi)$ and $p_S(\theta,\varphi)$ are commonly marked by $p_R = p_R(\theta,\varphi)$ and it is calculated from Eq. (5).

Next, the sequence of the $p(\theta,\varphi)$, $p_R(\theta,\varphi)$ should be calculated at the set of the points $P_i = P(\theta_i,\varphi_i)$, $i = 1,2,\ldots,n_i$. As a result, n_i-element sequences $p_i = p(\theta_i,\varphi_i)$, $p_{R;i} = p_R(\theta_i,\varphi_i)$ are obtained, respectively. One defines the function

$$d_R = (1/n_i)\sum_i d_{R;i} = (1/n_i)\sum_i (p_i - p_{R;i})\,. \tag{6}$$

The d_R is named the global control parameter, $d_{R;i}$ — the local one.

The far field is considered at a large distance from the plate. In this case the Fraunhofer approximation is applied

$$1/r \approx 1/r_0\,, \quad r \approx r_0 - \rho\cos(\rho_0, r_0)$$
$$= r_0 - (x\sin\theta\cos\varphi + y\sin\theta\sin\varphi)\,. \tag{7}$$

In the harmonic steady state the vibration velocity $v(x,y)$ depends on the plate mode $w(x,y)$ and inserting (7) into (4) one obtains

$$\Phi(\theta,\varphi) = \frac{\exp(-\mathrm{i}kr_0)}{2\pi r_0}\int_S w(x,y)\exp\big(\mathrm{i}k(x\sin\theta\cos\varphi + y\sin\theta\sin\varphi)\big)\,\mathrm{d}S\,, \tag{8}$$

where (r_0,θ,φ) — spherical coordinates.

From the acoustic point of view, in the far field, it is very useful to express the discrepancy between acoustic pressures.

There is the near field in the vicinity of the plate. In this zone the acoustic field is more sensitive to change of the plate vibration. Not to lose any details, the acoustic field is analysed in the plane parallel to the surface of the plate at the constant distance H, where H is determined in [17], see Fig. 2. Now, Eq. (4) should be used without any approximation. Then the r-distance is calculated exactly. From Fig. 2 follows that:

$$r^2 = H^2 + x^2 + y^2 + x_{\mathrm{P}}^2 + y_{\mathrm{P}}^2 - 2\,(xx_{\mathrm{P}} + yy_{\mathrm{P}})\,, \tag{9}$$

where $x_{\mathrm{P}}, y_{\mathrm{P}}$ — Cartesian coordinates of the P point.

4. Numerical calculations, results

To demonstrate the validation and effectiveness of control parameters defined in the acoustic field, the numerical experiments are performed. For this purpose the plate made of aluminium with thickness $h = 1.59$ mm is taken into account. The clamped side $b = 254$ mm and the free side $a = 381$ mm. The material constants used for calculations are assumed as [15]: the density $\rho = 7169$ kg/m^3, Young's modulus $Y = 71.7$ GPa, Poisson's ratio $\nu = 0.32$, respectively. For the PZTs, the piezoelectric strain constant $d_{31} = -190 \times 10^{-12}$ m/V and thickness $h_{\mathrm{a}} = 0.48$ mm, density $\rho_{\mathrm{a}} = 7800$ kg/m^3, Young's modulus $Y_{\mathrm{a}} = 66$ GPa and Poisson's ratio $\nu_{\mathrm{a}} = 0.34$, respectively.

The loudspeaker provides the excitation. It is in form of plane acoustic wave acting perpendicularly to the plane, harmonic in time and with a given frequency. So, the external loading is uniformly distributed on the plate. The plate is forced to the vibration with the first, second and third separate mode and frequency $f_1 = 17.4$, $f_2 = 73.1$, $f_3 = 106.9$ Hz, respectively.

Under circumstances assumed in [4–6], there are areas of the plate with big curvatures; they are clearly depicted in Fig. 3a–c. In each area the points with the biggest curvature are determined; they are quasi-optimal points and they are in Fig. 3 marked with dark grey rectangle. Just at these places actuators are bonded (Q — quasi-optimal places). For comparison of results, the actuators are a little shifted (S — shifted places) as shown with light grey rectangle.

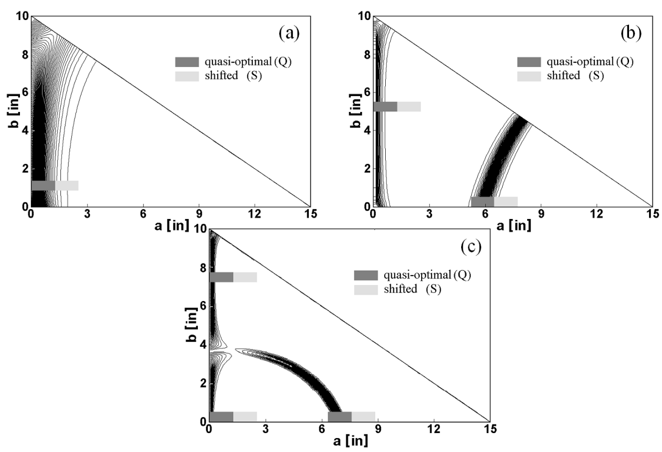

Fig. 3. The areas with big curvatures calculated in x-direction: dark grey — Q-actuators distribution, light grey — S-actuators distribution: (a) first mode, (b) second mode, (c) third mode.

The effect of acting PZTs was analysed in [4–6] with coefficient amplitude reduction $\alpha_R = (A - A_R)/A \times 100\%$ where A and A_{P} — vibration amplitude without and with reduction, respectively; for comparison the results are repeated in Fig. 4. Herein, the effect is investigated indirectly by global control parameter in the far field and by global and local control parameters in the near field. In the case of near field, the control surface is parallel to the plate one and they are moved each other away at the distance $H = \{0.0079, 0.033, 0.048\}$ m for $f = \{17.4, 73.1, 106.9\}$ Hz, respectively. All results are normalised.

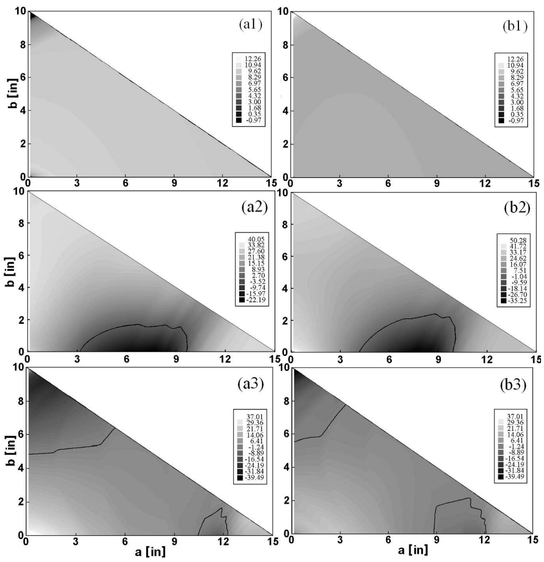

Fig. 4. Distribution of the coefficient amplitude reduction α_R: (a) Q-distribution, (b) S-distribution, (a1), (b1) 1 mode, (a2), (b2) 2 mode, (a3), (b3) 3 mode.

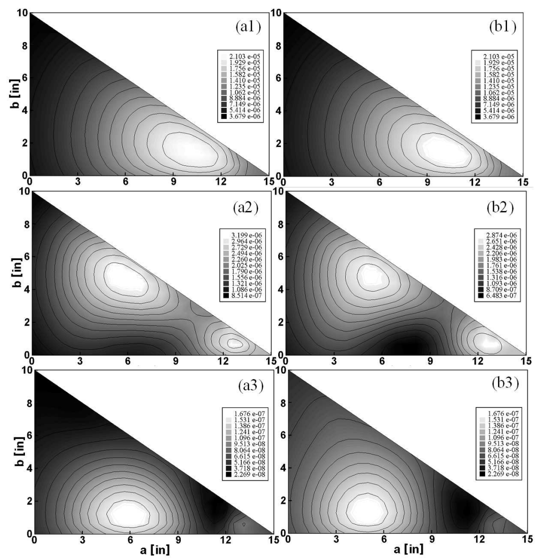

Fig. 5. Distribution of the local control parameters $d_{R;i}$: (a) $d_{Q;i}$, (b) $d_{S;i}$; (a1), (b1) 1 mode, (a2), (b2) 2 mode, (a3), (b3) 3 mode.

Examining Fig. 4a one observes considerable decreases of amplitude in whole area. But in some area (marked inside the dark line) the amplitude somewhat increases. If actuators are little shifted compared to quasi-optimal places, the results are somewhat worse, Fig. 4b. The area (inside the dark line, too) in which the amplitude increases is larger. However, globally in both cases the decrease of amplitude is bigger than its increase. This conclusion confirms the analysis of the acoustic field.

TABLE

Near and far field global control parameters.

	d_Q		d_S	
Mode number	Near field	Far field	Near field	Far field
1	1.1129×10^{-5}	1.0836×10^{-6}	9.3692×10^{-6}	9.1301×10^{-7}
2	1.9332×10^{-6}	1.8190×10^{-7}	1.6654×10^{-6}	1.5773×10^{-7}
3	7.6517×10^{-8}	6.9204×10^{-9}	7.2394×10^{-8}	6.3788×10^{-9}

In the far field the control parameters d_Q and d_S are presented in Table. Since the values of d_Q are bigger than d_S then they confirm the conclusion given above. Additionally local control parameters $d_{Q;i}$ and $d_{S;i}$ are presented in Fig. 5. They clearly confirm this conclusion. Furthermore, comparing Figs. 4a and 5a and next Figs. 4b and 5b it is observed that the local control parameter not only confirmed the above conclusion but always pointed out the places decreasing and increasing of vibration amplitude provided that the vibration amplitude is sufficiently large. It seems that the local control parameter is the best one to contactless control parameter of active vibration reduction.

5. Conclusions

Theory and numerical calculations lead to the following conclusions:

1. An acoustic field radiated by the plane is sensitive to change of the plate vibration. So, any acoustic control parameter may be indirect control parameter of an active vibration reduction.

2. The global control parameter pointed out that the quasi-optimal distribution of the actuators reduces better the vibration amplitude than other their distribution. It confirms the conclusion published in literature.

3. The local control parameter confirms the conclusion given above. Furthermore it reflects the places decreasing and increasing of vibration amplitude, too, provided that the vibration amplitude is sufficiently large.

4. The control parameters formulated in acoustic field are handy, contactless, indirect control parameters of an active vibration reduction of the plate.

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