Pseudomagnetic Moment in Graphene in Time-Dependent Electric Field

H. Goudarzi[a,*], M.R. Setare[b] and S. Zarrin[a]

[a]Department of Physics, Faculty of Science, Urmia University, P.O. Box 165, Urmia, Iran

[b]Department of Physics, Payame Noor University, Bijar, Iran

(Received May 10, 2010; in final form November 24, 2010)

We study the dynamics of graphene ring in the presence of time-dependent electric field, where the Dirac particles in graphene ring interact with external electromagnetic fields. Using the Dirac Hamiltonian in electromagnetic field, we obtain the pseudomagnetic moment of the Dirac particles around a graphene ring. It is shown that the appeared pseudomagnetic moment term essentially can be in control by time-dependent electric field, and also it depends on the energy gap of graphene. It seems that one can construct a logic system in graphene and also in semiconductors by the pseudomagnetic moment term.

PACS: 73.21.−b, 73.22.Pr, 72.80.Vp

1. Introduction

In the recent years, graphene has developed to one of clearly most active directions of work in today's both experimental and theoretical condensed matter physics [1–3]. As graphene is a two-dimensional monoatomic layer of graphite and also its special honeycomb lattice structure, the fermions in carbon atom in graphene are treated as relativistic. The unusual band structure of a graphene has been known for 63 years [4]. The velocity of electrons is $v \approx 10^6$ m/s. Semenoff [5], and Di Vincenzo and Mele [6] noticed that massless excitation states are governed by a wave equation, the Dirac equation, of relativistic quantum mechanics. Reduction of 3+1-dimensional Dirac equation to 2+1-dimensional equation for graphene sheet is given by the Weyl representation of the Dirac equation, and one can easily apply these equations for graphene.

One important effect which can achieve in a graphene ring geometry by relativistic formalism of quantum theory, is the pseudospin–orbit coupling producing in semiconductors and graphene [7]. By this appearing pseudospin effect in graphene, it can construct a logic system [8–11], and thus, using external electromagnetic field it is really possible to control some physical parameters of graphene as magnetic moment.

In this paper, we intend to express another effect in graphene interacting with time-dependent electric field, using the concept of pseudospin interaction in graphene. Of course, this effect concerns the magnetic properties of graphene. To provide this effect, we consider a graphene ring, around which there are the Dirac particles with high density, being subject to an external electromagnetic field, of which electric field depends on time and magnetic field is independent of time. Actually, as we will see in the next sections, this configuration of external fields leads to produce an effective magnetic field and also an effective magnetic moment (pseudo) in the graphene ring, since this effect can physically be a consequence of creating pseudospin term of the Dirac particles [12, 13]. If electric field depends on time, then one can change the pseudomagnetic moment of graphene in terms of time.

However, following equations in the next sections, it is shown that the pseudomagnetic moment in graphene depends on the energy gap of graphene, and then also on time-dependent potential of external electric field.

This paper is organized as follows. In Sect. 2 we derive the pseudomagnetic moment term of mentioned above structure in two non-relativistic and relativistic cases of the Dirac particles. In Sect. 3 we continue to determining the effective magnetization of graphene ring. It can lead us to the pseudopermeability term of graphene. Finally, Sect. 4 shows the conclusions.

2. Massive Dirac particles in graphene

It is considered a graphene ring, around which there exist many massive Dirac particles being subject to a perpendicular magnetic field. As we know, in principal the energy gap in graphene is very small, but recently, electrons found experimentally in graphene grown on SiC

* corresponding author; e-mail: `h.goudarzi@urmia.ac.ir`, `goudarzia@phys.msu.ru`

substrate behave as massive Dirac electrons with energy spectra $E = \hbar v_{\mathrm{F}}\sqrt{k_x^2 + k_y^2 + (mv_{\mathrm{F}}/\hbar)^2}$. Based on experimental data in Refs. [14, 15] the energy gap in graphene is $mv_{\mathrm{F}}^2 \approx 130$ MeV, and it plays sufficiently as the mass of the Dirac particles. However, it seems that it can be reasonable to consider the Dirac fermions as massive electrons in graphene.

In addition to the magnetic field, a time-dependent electric field perpendicular to the magnetic field affects the graphene ring. In the presence of an electromagnetic field characterized by the four-potential $A_\mu = (\phi, -\boldsymbol{A})$, where ϕ is the scalar potential and $\boldsymbol{A}$ is the vector potential, respectively, the Dirac equation can be written in covariant form as [16]:

$$(\mathrm{i}\hbar\gamma^\mu D_\mu - mc)\,\psi = 0\,, \tag{1}$$

where m is the mass of electron, γ^μ are the Dirac matrices given by

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad i = 1, 2, 3, \tag{2}$$

with σ^i being the 2×2 Pauli matrices, and $D_\mu = (\partial_\mu + \frac{\mathrm{i}\,e}{\hbar c}A_\mu)$ is the minimal coupling form of the four--momentum of the electron. From the definition of D_μ, it satisfies the following commutation relation:

$$[D_\mu, D_\nu] = \frac{\mathrm{i}\,e}{\hbar}F_{\mu\nu}\,, \tag{3}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor. One can then easily obtain the following form of Dirac equation:

$$\left(\hbar^2 D^\mu D_\nu + \frac{\hbar e}{2}\Sigma^{\mu\nu}F_{\mu\nu} + m^2c^2\right)\psi = 0\,, \tag{4}$$

where $\Sigma^{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu]$. Using Eqs. (2), (3) and (4), we derive the following:

$$\left[\boldsymbol{\Pi}^2 - \frac{(E_n - V)^2}{c^2} + \frac{e\hbar}{c}\boldsymbol{\sigma}\cdot\boldsymbol{B} + \frac{\mathrm{i}e\hbar}{c}(\boldsymbol{\alpha}\cdot\boldsymbol{E}) + m^2c^2\right] \times \psi = 0\,, \tag{5}$$

where $\boldsymbol{\Pi} = \boldsymbol{p} + \frac{e}{c}\boldsymbol{A}$ is the kinetic momentum, $V = e\phi$, and $\boldsymbol{\alpha}$ are the Dirac matrices.

Here, we intend to study pseudospin–orbital coupling effect experienced by massive Dirac particles in graphene either in non-relativistic or relativistic limit to calculating effective magnetic moment (pseudo) term of electrons. Firstly, because to the small energy gap in graphene (the order of MeV), we are able to use non-relativistic approximation for such effect. We write the bispinor ψ as a two-dimensional column vector in non-relativistic limit, i.e.

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}, \tag{6}$$

where φ and χ are both spinors. Let us consider the lower (second) of the above equation. For the conditions $|\mathrm{i}\hbar\frac{\partial\chi}{\partial t}| \ll |mc^2\chi|$ and also $|V\chi| \ll |mc^2\chi|$ (i.e. if the kinetic energy as well as the scalar potential are small compared to the rest mass energy) the lower component of Eq. (6) becomes

$$\chi = \frac{c}{2mc^2 - V}\,(\boldsymbol{\sigma}\cdot\boldsymbol{\Pi})\,\varphi\,. \tag{7}$$

Then, assuming a small energy ε and potential V, Eq. (5) reduces to the following form:

$$\left\{\frac{\Pi^2}{2m} + \frac{\hbar e}{2mc}\boldsymbol{\sigma}\cdot\boldsymbol{B} + \frac{\mathrm{i}\hbar e\,(\boldsymbol{E}\cdot\boldsymbol{\Pi})}{2m\,(2mc^2 - V)} - \frac{e\hbar\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi})}{2m\,(2mc^2 - V)} + V\right\}\varphi = \varepsilon\varphi\,, \tag{8}$$

where $\varepsilon = E_n - mc^2$ is the energy spectrum for nonrelativistic limit and $\varepsilon \ll mc^2$. Expanding denominator of third and fourth terms of Eq. (8) and neglecting the second and higher order of $\frac{V}{2mc^2}$, we have, respectively

$$\begin{aligned} &\frac{\mathrm{i}\hbar e}{4m^2c^2}\left(1 + \frac{V}{2mc^2}\right)(\boldsymbol{E}\cdot\boldsymbol{\Pi}) \\ &= \frac{\mathrm{i}\hbar e}{4m^2c^2}(\boldsymbol{E}\cdot\boldsymbol{\Pi}) + \frac{\mathrm{i}\hbar eV}{8m^3c^4}(\boldsymbol{E}\cdot\boldsymbol{\Pi})\,, \\ &\frac{\hbar e}{4m^2c^2}\left(1 + \frac{V}{2mc^2}\right)\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi}) \\ &= \frac{\hbar e}{4m^2c^2}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi}) + \frac{\hbar eV}{8m^3c^4}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi})\,, \end{aligned} \tag{9}$$

and thus Eq. (8) becomes

$$\left\{\frac{\Pi^2}{2m} + \frac{\hbar e}{2mc}\boldsymbol{\sigma}\cdot\boldsymbol{B} + \frac{\mathrm{i}\hbar e}{4m^2c^2}(\boldsymbol{E}\cdot\boldsymbol{\Pi}) - \frac{\hbar e}{4m^2c^2}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi}) + \frac{\mathrm{i}\hbar eV}{8m^3c^4}(\boldsymbol{E}\cdot\boldsymbol{\Pi}) - \frac{\hbar eV}{8m^3c^4}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi}) + V\right\}\varphi = \varepsilon\varphi\,. \tag{10}$$

In Eq. (10), second, fourth and sixth terms appear respectively as follows:

$$\frac{\hbar e}{2mc}\boldsymbol{\sigma}\cdot\boldsymbol{B} = \mu_{\mathrm{B}}\boldsymbol{\sigma}\cdot\boldsymbol{B},$$

$$\frac{\hbar e}{4m^2c^2}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi}) = \frac{\hbar e}{4mc}\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\mathrm{eff}} = \frac{\mu_{\mathrm{B}}}{2}\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\mathrm{eff}}\,,$$

$$\frac{\hbar eV}{8m^3c^4}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi}) = \frac{\hbar eV}{8m^2c^3}\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\text{eff}} = \frac{V\mu_{\text{B}}}{4mc^2}\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\text{eff}}, \tag{11}$$

where $\boldsymbol{B}_{\text{eff}} = \frac{\boldsymbol{\Pi}\times\boldsymbol{E}}{mc}$ is the effective (pseudo) magnetic field.

From three above equations, one can obtain

$$\frac{\mu_{\text{B}}}{2}\left(1+\frac{V}{2\Delta}\right)\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\text{eff}} + \mu_{\text{B}}\boldsymbol{\sigma}\cdot\boldsymbol{B} = \left(\mu_{\text{B}_{\text{eff}}}\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\text{eff}} + \mu_{\text{B}}\boldsymbol{\sigma}\cdot\boldsymbol{B}\right), \tag{12}$$

where $\mu_{\text{B}} = \frac{e\hbar}{2mc}$ is the Bohr magneton, and $\Delta = 2mc^2$ is energy gap between a particle and a hole [1], and $\mu_{\text{B}_{\text{eff}}} \cong \frac{\mu_{\text{B}}}{2}(1+\frac{V}{2\Delta})$ is the pseudomagneton.

As it is noticed in Eq. (12), $\mu_{\text{B}_{\text{eff}}}$ depends on potential of external electric field and energy gap of graphene. So, one can change pseudomagnetic moment in terms of external potential. Thus, in addition to intrinsic magnetic moment of particles in graphene defined as

$$\mu = \langle\sigma\mu_{\text{B}}\rangle\,, \quad \mu_{\text{B}} = \frac{eh}{2mc}\,, \tag{13}$$

it is derived an another term of magnetic moment by applying higher order corrections, so called pseudomagnetic moment as following:

$$\mu_{\text{eff}} = \langle\sigma\mu_{\text{B}_{\text{eff}}}\rangle \cong \left\langle\sigma\mu_{\text{B}}\left(\frac{1}{2}+\frac{V}{4\Delta}\right)\right\rangle. \tag{14}$$

Since potential energy is less than energy gap of graphene, then one can obtain the following relation for the pseudomagnetic moment in graphene in nonrelativistic limit

$$\mu_{\text{eff}} \cong \left\langle\frac{\sigma\mu_{\text{B}}}{2}\right\rangle. \tag{15}$$

Continuing same procedure performed above, we again consider Eq. (4) for relativistic case, and using two--dimensional column vector ψ, Eq. (4) reduces to the following Hamiltonian:

$$\left\{\frac{\Pi^2}{2m} + \frac{e\hbar}{2mc}\boldsymbol{\sigma}\cdot\boldsymbol{B} + \frac{\mathrm{i}e\hbar\,(\boldsymbol{E}\cdot\boldsymbol{\Pi})}{2m\,(cp_0+mc^2-V)} - \frac{e\hbar\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi})}{2m\,(cp_0+mc^2-V)} + \frac{mc^2}{2} - \frac{(cp_0-V)^2}{2mc^2}\right\} \times\varphi = 0\,, \tag{16}$$

where $p_0 = 1/cE_n$. Let us notice that external potential is not small now with comparison to the energy spectrum E_n, i.e. $cp_0 - V < mc^2$. Expanding denominator of third and fourth terms of Eq. (16) and ignoring higher order of $\frac{V-cp_0}{mc^2}$, we have

$$\frac{\mathrm{i}e\hbar}{2m^2c^2}\left(1+\frac{V-cp_0}{mc^2}\right)(\boldsymbol{E}\cdot\boldsymbol{\Pi}) = \frac{\mathrm{i}e\hbar}{2m^2c^2}(\boldsymbol{E}\cdot\boldsymbol{\Pi}) + \frac{\mathrm{i}e\hbar\,(V-cp_0)}{2m^3c^4}(\boldsymbol{E}\cdot\boldsymbol{\Pi})\,,$$

$$\frac{e\hbar}{2m^2c^2}\left(1+\frac{V-cp_0}{mc^2}\right)\boldsymbol{\sigma}\cdot(\boldsymbol{E}\cdot\boldsymbol{\Pi}) = \frac{e\hbar}{2m^2c^2}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\cdot\boldsymbol{\Pi}) + \frac{e\hbar\,(V-cp_0)}{2m^3c^4}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\cdot\boldsymbol{\Pi})\,, \tag{17}$$

and then Eq. (16) can be expressed as

$$\left\{\frac{\Pi^2}{2m} + \frac{e\hbar}{2mc}\boldsymbol{\sigma}\cdot\boldsymbol{B} + \frac{\mathrm{i}e\hbar}{2m^2c^2}(\boldsymbol{E}\cdot\boldsymbol{\Pi}) + \frac{\mathrm{i}e\hbar\,(V-cp_0)}{2m^3c^4}(\boldsymbol{E}\cdot\boldsymbol{\Pi}) + \frac{mc^2}{2} - \frac{(cp_0-V)^2}{2mc^2} - \frac{e\hbar}{2m^2c^2}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi}) - \frac{e\hbar\,(V-cp_0)}{2m^3c^4}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi})\right\} \times\varphi = 0\,. \tag{18}$$

In Eq. (18), we can rewrite the second, seventh and eighth terms to the following form, respectively:

$$\frac{e\hbar}{2mc}\boldsymbol{\sigma}\cdot\boldsymbol{B} = \mu_{\text{B}}\boldsymbol{\sigma}\cdot\boldsymbol{B}\,,$$

$$\frac{e\hbar}{2m^2c^2}\boldsymbol{\sigma}\cdot(\boldsymbol{\Pi}\times\boldsymbol{E}) = \frac{e\hbar}{2m^2c^2}\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\text{eff}} = \mu_{\text{B}}\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\text{eff}}\,,$$

$$\frac{e\hbar\,(V-cp_0)}{2m^3c^4}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{\Pi}) = \frac{e\hbar\,(V-cp_0)}{2m^2c^3}\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\text{eff}} = \frac{(V-cp_0)}{mc^2}\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\text{eff}}\,. \tag{19}$$

Adding three above equations, it leads us to derive the following relation for coupling of pseudospin with external magnetic field

$$\mu_{\text{B}}\left[1+\frac{2\,(V-cp_0)}{\Delta}\right]\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\text{eff}} + \mu_{\text{B}}\boldsymbol{\sigma}\cdot\boldsymbol{B} = \left(\mu_{\text{B}_{\text{eff}}}\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\text{eff}} + \mu_{\text{B}}\boldsymbol{\sigma}\cdot\boldsymbol{B}\right), \tag{20}$$

and then the effective (pseudo) magneton Bohr is determined as follows:

$$\mu_{\text{B}_{\text{eff}}} \cong \mu_{\text{B}}\left[1+\frac{2\,(V-cp_0)}{\Delta}\right]. \tag{21}$$

Consequently, corresponding effective (pseudo) magnetic moment can be expressed as

$$\mu_{\text{eff}} = \langle\sigma\mu_{\text{B}_{\text{eff}}}\rangle \cong \left\langle\sigma\mu_{\text{B}} + \frac{2\sigma\mu_{\text{B}}\,(V-cp_0)}{\Delta}\right\rangle. \tag{22}$$

Now, we would like to exhibit behavior of effective pseudomagnetic moment of the Dirac particles in graphene in terms of energy gap Δ and potential V:

1. if $\Delta \gg cp_0 - V$, one can obtain

$$\mu_{\text{eff}} = \langle \sigma \mu_{\text{B}} \rangle \,, \tag{23}$$

and then, as it is seen, this is the same as obtained for intrinsic magnetic moment.

2. if $cp_0 \gg V$, then we have

$$\mu_{\text{eff}} \cong \left\langle \frac{\sigma \mu_{\text{B}} \Delta}{2 p_0 c} \right\rangle , \tag{24}$$

and this term is more smaller than intrinsic magnetic moment of the Dirac particles.

3. if $cp_0 - V \gg \Delta$, then effective magnetic moment becomes

$$\mu_{\text{eff}} \cong \left\langle \frac{\sigma \mu_{\text{B}} \Delta}{2\,(cp_0 - V)} \right\rangle , \tag{25}$$

which is sufficiently small and one can neglect it. As it is shown in above, one can control effective (pseudo) magnetic moment of electrons by applying external fields and in particular by tuning band-gap in graphene.

3. Pseudopermeability in graphene

Using effective pseudomagnetic moment obtained in the previous section, one can calculate a relation for the total magnetization in both relativistic and non--relativistic limit. We have

$$X = \frac{\mu J B}{k_{\text{B}} T} + \frac{\mu_{\text{eff}} J B}{k_{\text{B}} T} \,, \tag{26}$$

where X is the magnetization, J is total angular momentum of carbon atom in graphene, k_{B} and T are the Boltzmann constant and temperature, respectively. Now one can obtain the magnetism parameter for graphene with taking into account effect of effective magnetic moment as

$$M = N\,(\mu + \mu_{\text{eff}}) \tanh X \,, \tag{27}$$

where N is the summation of number up ($m = \frac{1}{2}$) and down ($m = -\frac{1}{2}$) spin levels of the Dirac particles in around graphene ring. If $X < 1$ and assuming that $X \cong \frac{\mu J B}{k_{\text{B}} T}$, then Eq. (27) reduces to the following form:

$$M \cong N X\,(\mu + \mu_{\text{eff}}) \,. \tag{28}$$

Finally, one can determine the quantity of pseudopermeability as follows:

$$\chi_m = \frac{M}{B} \cong \frac{N\,(\mu + \mu_{\text{eff}})\,X}{k_{\text{B}} T} \,. \tag{29}$$

As it is shown in Eq. (28), the contribution of created effective (pseudo) magnetic moment in graphene at the presence of external electromagnetic field is explicitly indicated. These results can be considered as interesting technical effects in some fields of experimental physics and of course, they can play very important role in nanophysics and semiconductors industry.

4. Conclusion

As a result, here there was demonstrated theoretically the possibility of producing an effective pseudomagnetic moment for the Dirac particles in a graphene ring by applying a variable external electromagnetic field. Also by this created effective magnetic moment in graphene, one can obtain a new permeability parameter for such system. In particular, such physical parameters as effective magnetic moment and permeability can be controlled by external potential and band-gap of graphene, that its importance has been shown in the very important paper in [17].

References

[1] K.S. Novoselov, A.K. Geim, S.V. Morozov, D. Jiang, Y. Zhang, S.V. Dubonos, I.V. Grigorieva, A.A. Firsov, *Science* **306**, 666 (2004).

[2] A.K. Geim, K.S. Novoselov, *Nature Mater.* **6**, 183 (2007).

[3] A.H. Castro Neto, F. Guinea, N.M.R. Peres, K.S. Novoselov, A.K. Geim, arXiv:0709.1163 [cond--mat.other].

[4] P.R. Wallace, *Phys. Rev.* **71**, 622 (1947).

[5] G.W. Semenoff, *Phys. Rev. Lett.* **53**, 2449 (1984).

[6] D.P. Di Vincenzo, E.J. Mele, *Phys. Rev. B* **29**, 1685 (1984).

[7] S.G. Tan, M.B.A. Jalil, D.E. Koh, H.K. Lee, Y.H. Wu, arXiv:0806.1568 [cond-mat].

[8] S. Datta, B. Das, *Appl. Phys. Lett.* **46**, 665 (1989).

[9] R. Winkler, *Spin–Orbit Coupling Effects in Two--Dimensional Electron and Hole Systems*, Springer, Berlin 2003.

[10] S.Q. Shen, *Phys. Rev. Lett.* **95**, 187203 (2005).

[11] S.Q. Shen, *Phys. Rev. Lett.* **78**, 1335 (2004).

[12] A.K. Geim, K.S. Novoselov, *Nature Mater.* **6**, 183 (2003).

[13] S.G. Tan, M.B.A. Jalil, Xiong-Junliu, arXiv: 0705.3502 [cond-mat].

[14] H.B. Heersche, P. Jarillo-Herrero, J.B. Oostinga, L.M.K. Vandersypen, A.F. Morpurgo, *Nature* **446**, 56 (2007).

[15] S.Y. Zhou, G.H. Gweon, A.V. Fedorov, P.N. First, W.A. de Heer, D.-H. Lee, F. Guinea, A.H. Castro Neto, A. Lanzara,, *Nature Mater.* **6**, 770 (2007).

[16] W. Greiner, *Relativistic Quantum Mechanics*, 3rd ed., Springer, Berlin 2000.

[17] Y. Zhang, T.-Ta Tang, C. Girit, Z. Hao, M. C. Martin, A. Zettl, M. F. Crommie, Y. Ron Shen, F. Wang, *Nature* **459**, 820 (2009).