Effect of Surface Tension and Rotation on Rayleigh–Taylor Instability of Two Superposed Fluids with Suspended Particles

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The Rayleigh–Taylor instability of two superposed incompressible fluids of different densities in the presence of small rotation, surface tension and suspended dust particles is investigated. The linearized equations of the problem are constructed and the general dispersion relation is obtained using normal mode analysis by applying the appropriate boundary conditions. The effects of surface tension, the Atwood number, small rotation and suspended dust particles are studied on both conditions of Rayleigh–Taylor instability and growth rate of the unstable Rayleigh–Taylor mode. The numerical calculations have been performed to see the effect of rotation, the Atwood number, relaxation frequency and mass concentration of suspended dust particles. It is found that the growth rate of Rayleigh–Taylor instability depends upon the mass concentration and relaxation frequency of suspended dust particles. The uniform small rotation, relaxation frequency and mass concentration of suspended dust particles all have stabilizing influence on the growth rate of Rayleigh–Taylor instability. It is also found that the Atwood number has destabilizing influence on the growth rate of the considered Rayleigh–Taylor configuration.

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1. Introduction

The instability of a heavy fluid layer supported by a light one is known as the Rayleigh–Taylor (R–T) instability. The R–T instability has much importance in laser fusion, space plasma, astrophysical plasmas, laboratory plasma and in chemical engineering processes in industries [1–3]. It generally occurs under gravity whenever the heavy fluid is accelerated by the light ones. Chandrasekhar [4] and Roberts [5] have given a comprehensive account, which contains solutions of this classical problem of R–T instability under different conditions and assumptions of hydromagnetics. Menikoff et al. [6] have discussed the character of growth rate of the normal modes of the R–T instability of superposed incompressible viscous fluids in term of dimensionless parameters and derived a simple R–T instability dispersion relation. Evans et al. [7] have analyzed the growth rate of ablation-driven R–T instability in laser-driven targets. Mikaelian [8] has derived analytical formula for the growth rate of R–T instability in a number of density profiles. Recently, the R–T instability of low frequency non-uniform multi-ion species magnetoplasmas is studied by El-Shorbagy and Shukla [9]. Huang et al. [10] have discussed the effects of compressibility and the finite Larmor radius (FLR) corrections on R–T instability of z-pinch implosions. Thus the problem of R–T instability is widely discussed for many problems of different types of medium.

In the past decades, owing to the relevance of suspended dust particles in a large number of astrophysical phenomena, space situations and experimental, and laboratory problems, various workers have incorporated the effect of suspended dust particles in analysis of the R–T and other instabilities. The effect of suspended dust particles on the stability of two superposed fluids in hydrodynamics is of industrial and scientific importance in geophysical engineering. In this direction, Saffman [11] has discussed in detail a dust gas in magnetohydrodynamics. Scanlon and Segel [12] have made a thorough study of the implication of suspended particles in hydromagnetics for the Benard convection problem. Sharma and Sharma [13] have also discussed the R–T instability for a medium consisting of two superposed fluids with suspended particles and obtained the criteria determining stability and instability of the system in the presence of suspended particles. Sanghvi and Chhajlani [14] have incorporated the finite resistivity effect on the R–T configuration of stratified plasma in the presence of suspended particles and found that the dust particles have a stabilizing as well as a destabilizing influence under different conditions. Sanghvi and Chhajlani [15] have further discussed the influence of FLR corrections on the Kelvin–Helmholtz (K–H) instability of two superposed streaming fluids acted upon by a uniform magnetic field transverse to the direction of streaming in the presence of suspended particles and found that FLR corrections give the stabilizing effect in the presence of suspended particles. Recently, el-Sayed [16] has investigated the hydromagnetic instability in the context of R–T instability

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for oldroydian viscoelastic porous media taking the effects of suspended dust particles and uniform magnetic field. The effect of suspended dust particles on the R–T instability is also investigated for the Rivilin–Ericksen viscoelastic fluids [17, 18]. Thus we find that suspended dust particles play a major role in discussions of R–T instability of different types of fluids.

In addition to this, Mikaelian [19] has investigated the R–T instability and Richtmyer–Meshkov instabilities in multilayer fluids with surface tension. Reid [20] has also discussed the effects of surface tension and viscosity on the stability of two superposed fluids. He has discussed the most detailed and considerable effects arising from surface tension. Sharma [21] has studied the R–T instability in case of fully ionized superposed fluid incorporating the effect of surface tension. In the other study, Sharma [22] has also discussed the R–T instability of rotating superposed fluids through porous medium. The R–T instability of two superposed partially ionized plasma is discussed by Ogbonna and Bhatia [23] for non--rotating viscid case. Bhatia and Chhonkar [24] have studied the effect of rotation in x-direction in case of superposed fluids having the interface in the x–y plane with gravity in z-direction. Vaghela and Chhajlani [25] have analyzed the R–T instability for two superposed fluids of partially ionized rotating fluids with surface tension. Recently, Chertkov et al. [26] have investigated the effect of surface tension on immiscible R–T turbulence. Thus the effect of surface tension is widely explored to discuss the R–T instability.

In addition to above analysis, Alterman [27] has discussed the effect of surface tension and rotation on K–H instability of two superposed fluids. Sharma and Chhajlani [28] have investigated the R–T instability of two superposed magnetized plasma fluids with rotation and suspended dust particles. Sharma et al. [29] have considered the R–T instability of the Rivilin–Ericksen fluid through porous medium and included the uniform rotation in this problem. Davalos-Orozco [30, 31] has analyzed the R–T instability of a fluid system under general and parallel rotation. The problem of suspended dust particles is also discussed with rotation for the problem of convection by Sunil et al. [32]. El-Ansary et al. [33] have studied the effect of surface tension and rotation on the R–T instability.

Recently, El-Sayed [34] has investigated the hydromagnetic transverse instability of two highly viscous fluid particle flows with FLR corrections. He has obtained the dispersion relation for static R–T configuration taking perturbation in y-direction only. The effect of rotation and surface tension was not considered in his study but we have taken these effects and obtained the complete dispersion relation for non-viscous medium.

From all these discussions, we also conclude that various problems of R–T instability in the presence of suspended dust particles, surface tension and rotation are under current investigation but none of the authors has taken the combined effect of all these parameters viz. suspended dust particles, rotation and surface tension to investigate the R–T instability of two superposed fluids. Thus the object of the present problem is to study the effect of rotation and surface tension on the stability of two superposed fluids in the presence of suspended dust particles with three-dimensional perturbations in the analysis.

2. Linearized perturbation equations

We consider two infinite homogeneous incompressible fluids separated by a plane interface at $z = 0$, each of these regions ($z < 0$ and $z > 0$) denoted by the subscript 1 and 2 (see Fig. 1). It is assumed that the fluids are permeated with suspended dust particles of uniform shape and size. The density of fluids is greater than the density of dust particles. The fluid is assumed to be infinitely extending having the free horizontal surface in the x–y plane. The fluid has uniform rotation with an angular velocity $\boldsymbol{\Omega}$ $(0, 0, \Omega)$ about the vertical z-direction. The fluid is under the action of acceleration due to gravity $\boldsymbol{g}$ $(0, 0, -g)$. If the suspended dust particles are assumed to be of uniform size, spherical shape and have small relative velocities between the two phases, then the extra body force per unit volume $KN(\boldsymbol{v} - \boldsymbol{u})$ is added to the momentum transfer equation, where $K = 6\pi\rho\nu a$. The quantities a, ν, $\boldsymbol{u}$ and $\boldsymbol{v}$ denote the particle radius, kinematic viscosity of the clean fluid, velocity of fluids and velocity of suspended particles, respectively. The density of the fluid and number density of the dust particles are represented by ρ and N.

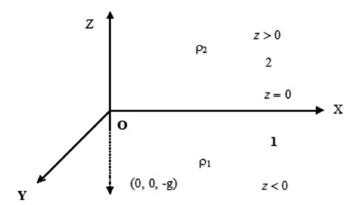

Fig. 1. Schematic diagram of the considered R–T configuration.

The suspended dust particles have homogeneous distribution in both the regions of the fluids. The effects of pressure gradient of the suspended particles and the gravity force on suspended particles are assumed negligible. The collisional force of suspended particles with the fluid components is of the order of the pressure gradient of the fluid component. For the case of superposed fluids of different densities the surface tension is effective at the interface between the fluids.

Thus the linearized perturbation equations of the problem are

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} = -\nabla\delta p + \boldsymbol{g}\delta\rho + 2\rho(\boldsymbol{u}\times\boldsymbol{\Omega}) + KN(\boldsymbol{v}-\boldsymbol{u}) + \sum_s \left[T_s\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta z_s\right]\delta(z-z_s)\,, \tag{1}$$

$$\left(\tau\frac{\partial}{\partial t}+1\right)\boldsymbol{v}=\boldsymbol{u}\,, \tag{2}$$

$$\frac{\partial\delta\rho}{\partial t}+\rho(\nabla\cdot\boldsymbol{u})=0\,, \tag{3}$$

$$\nabla\cdot\boldsymbol{u}=0\,, \tag{4}$$

$$\frac{\partial\delta z_s}{\partial t}=w_s\,, \tag{5}$$

where $\boldsymbol{u}(u,v,w)$, δp and $\delta\rho$ denote the perturbations in the velocity, pressure and density, respectively. $\tau=m/K$ represents relaxation time for the suspended dust particles, T_s is the surface tension of interfacial surface, $\delta(z-z_s)$ denotes the Dirac δ function. δz_s is the perturbation of interfacial surface and the subscript "s" indicates the interfacial surface.

The disturbance is analyzed in terms of normal mode analysis. We assume solution of the above equations in which the perturbations have the space (x,y,z) and time (t) dependence of the form

$$\exp(\mathrm{i}k_x x+\mathrm{i}k_y y+nt)\,, \tag{6}$$

where $k^2=k_x^2+k_y^2$ and n is the growth rate of the harmonic perturbations.

With these substitutions and the abbreviation D for $\mathrm{d}/\mathrm{d}z$, Eq. (1) is written using Eqs. (2)–(6) in terms of the perturbation components of velocity u, v, and w as

$$\rho\left[n+\frac{mNn}{\rho(\tau n+1)}\right]u=-\mathrm{i}k_x\delta p+2\rho\Omega v\,, \tag{7}$$

$$\rho\left[n+\frac{mNn}{\rho(\tau n+1)}\right]v=-\mathrm{i}k_y\delta p-2\rho\Omega u\,, \tag{8}$$

$$\rho\left[n+\frac{mNn}{\rho(\tau n+1)}\right]w=-\mathrm{D}\delta p+\frac{g}{n}(\mathrm{D}\rho)w -k^2\sum_s\left[\frac{T_s}{n}\delta(z-z_s)w_s\right]. \tag{9}$$

In writing these equations we have made use of the fact that ρ depends only on z.

We shall now derive an equation for w by eliminating δp from (9). First by multiplying Eqs. (7) and (8) by $\mathrm{i}k_x$ and $\mathrm{i}k_y$, respectively, and adding the resulting equations, we obtain

$$k^2\delta p=-\rho\left[n+\frac{mNn}{\rho(\tau n+1)}\right](\mathrm{D}w)-2\rho\Omega\xi\,, \tag{10}$$

where

$$\xi=\mathrm{i}k_x v-\mathrm{i}k_y u. \tag{11}$$

If we multiply (7) by $-\mathrm{i}k_y$ and (8) by $\mathrm{i}k_x$ and add, we obtain

$$\xi=\frac{2\Omega(\mathrm{D}w)}{n+\frac{mNn}{\rho(\tau n+1)}}\,. \tag{12}$$

Making use of (12) into (10), we get

$$k^2\delta p=-\rho\left[n+\frac{mNn}{\rho(\tau n+1)}\right](\mathrm{D}w) -\frac{4\Omega^2\rho(\mathrm{D}w)}{n+\frac{mNn}{\rho(\tau n+1)}}\,. \tag{13}$$

Eliminating δp from (9) with the help of (13) we finally obtain

$$\left[n+\frac{mNn}{\rho(\tau n+1)}\right]\left\{\mathrm{D}\left[\rho(\mathrm{D}w)\right]-k^2\rho w\right\} +4\Omega^2\left[n+\frac{mNn}{\rho(\tau n+1)}\right]^{-1}\mathrm{D}\left[\rho(Dw)\right]+\frac{gk^2}{n}(D\rho)w -k^4\sum_s\left[\frac{T_s}{n}\delta(z-z_s)w_s\right]=0\,. \tag{14}$$

Equation (14) represents the general relation of R–T instability of two superposed fluids incorporating the effects of rotation, surface tension in the presence of suspended dust particles. In the absence of suspended particles dispersion relation (14) reduces to one that is obtained by Sharma [22] ignoring effect of magnetic field in that case. On neglecting the effect of suspended dust particles in (14), we get the same result as is obtained by Chandrasekhar [4] (cf. p. 154, Ch. X). Thus the presence of suspended dust particles modifies the dispersion relation of R–T instability in the present problem.

3. Boundary conditions and dispersion relation

Consider two uniform fluids of densities ρ_1 (lower fluid) and ρ_2 (upper fluid) separated by a horizontal boundary at $z=0$. In case of the two regions of constant density (14) reduces to

$$(\mathrm{D}^2-\chi^2)w=0\,, \tag{15}$$

where

$$\chi^2=k^2\left\{1+\frac{4\Omega^2}{\left[n+\frac{mNn}{\rho(\tau n+1)}\right]^2}\right\}^{-1}. \tag{16}$$

The general solution of (15) can be written in the form of

$$w=c_1\,\mathrm{e}^{\chi z}+c_2\,\mathrm{e}^{-\chi z}, \tag{17}$$

where c_1 and c_2 are two constants.

We assume that the common boundary, which separates the two fluids, is located in the plane $z=0$ and we also suppose that the fluids are of infinite extent above and below this interface.

Since w must vanish both when $z\to-\infty$ (in the lower fluid) and $z\to+\infty$ (in upper fluid), we must suppose that

$$\begin{cases} w_1=c_1\,\mathrm{e}^{\chi z} & \text{for } z<0,\\ w_2=c_1\,\mathrm{e}^{-\chi z} & \text{for } z>0,\end{cases} \tag{18}$$

where we have chosen the same constant c_1 in the solutions for $z>0$ and $z<0$ to ensure the continuity of w across the interface at $z=0$.

To connect these solutions at $z=0$ we require appropriate boundary conditions. Following Chandrasekhar [4] the boundary conditions across the interface are

(i) The velocity w should vanish when $z\to\infty$ (for the

upper fluid) and $z \to -\infty$ (for the lower fluid) i.e. $w = 0$.
(ii) $w(z)$ is continuous at $z = 0$.
(iii) The jump condition is obtained by integrating (14) across the boundary $z = z_s$ between $z_s - \varepsilon$ and $z_s + \varepsilon$ and then assuming $\varepsilon \to 0$. In view of the continuity of w across $z = z_s$ and the boundness of ρ this limiting process gives

$$\left(1 + \frac{4\Omega^2}{n'^2}\right) \Delta_s[\rho(\mathrm{D}w)] + \left(\frac{gk^2}{nn'}\right)\left(\Delta_s\rho - \frac{k^2 T_s}{g}\right) w_s = 0\,, \tag{19}$$

where

$$n' = n + \frac{mNn}{\rho(\tau n + 1)},$$

and $\Delta_s(f) = f(z_s + 0) - f(z_s - 0)$ is the jump, which a quantity f experiences at the interface $z = z_s$. The subscript "s" distinguishes the value of a quantity known to be continuous at an interface, which takes at $z = z_s$; Eq. (19) can be written as

$$\left(1 + \frac{4\Omega^2}{n'^2}\right)[\rho_2(\mathrm{D}w_2) - \rho_1(\mathrm{D}w_1)] + \frac{gk^2}{nn'}\left[(\rho_2 - \rho_1) - \frac{k^2 T}{g}\right] w = 0\,. \tag{20}$$

On substituting the solutions of (18) in (20), we obtain the following dispersion relation for the R–T instability of two superposed rotating fluids including suspended dust particles and surface tension

$$nn'\left(1 + \frac{4\Omega^2}{n'^2}\right)^{1/2} = gk\left[\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} - \frac{k^2 T}{g\,(\rho_2 + \rho_1)}\right]. \tag{21}$$

Equation (21) represents the dispersion relation for two rotating dusty fluids of different densities including the effect of suspended dust particles and surface tension. This equation can be easily reduced to give the standard results obtained by Chandrasekhar [4] showing the effect of surface tension and rotation taken separately on the R–T instability for two superposed fluids.

4. Discussion of the dispersion relation

We consider the case of two rotating dusty fluids of different densities superposed upon each other in a gravitational field. We have divided this section into some subsections to consider the individual and combined influence of all the parameters viz. rotation, suspended dust particles and surface tension. In all cases we have treated the problem under the assumption that two infinite fluid layers are separated by an interface with surface tension. Many authors, including Chandrasekhar [4], have adopted free boundaries because they allow for exact solutions of the problem.

4.1. Combined effects of rotation, suspended dust particles and surface tension

Equation (21) is the dispersion relation of the problem of R–T instability of two rotating superposed incompressible fluids in the presence of suspended dust particles and surface tension. In the case of slow rotating fluids to discuss the R–T instability and stability we assume parametric limit $(4\Omega^2/n'^2 \ll 1)$ and expand (21). Thus we get

$$n\left[n + \frac{mNn}{\rho(\tau n + 1)}\right]\left\{1 + \frac{2\Omega^2}{\left[n + \frac{mNn}{\rho(\tau n+1)}\right]^2}\right\} = gk\left[\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} - \frac{k^2 T}{g(\rho_2 + \rho_1)}\right]. \tag{22}$$

We have adopted this type of expansion for simplifying the problem for obtaining some results for R–T instability.

On solving (22) the dispersion relation takes the form

$$\begin{aligned} &n^4\tau^2 + n^3 2\tau(1+\alpha) + n^2\Bigg\{(1+\alpha)^2 + 2\Omega^2\tau^2 \\ &-gk\tau^2\left[\frac{\rho_2 - \rho_1}{\rho_1 + \rho_2} - \frac{k^2 T}{g(\rho_2 + \rho_1)}\right]\Bigg\} + n\Bigg\{4\Omega^2\tau \\ &-(2+\alpha)\tau gk\left[\frac{\rho_2 - \rho_1}{\rho_1 + \rho_2} - \frac{k^2 T}{g(\rho_2 + \rho_1)}\right]\Bigg\} + 2\Omega^2 \\ &+gk(\alpha + 1)\left[\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} + \frac{k^2 T}{g(\rho_2 + \rho_1)}\right] = 0\,. \end{aligned} \tag{23}$$

Equation (23) represents the general dispersion relation of R–T instability incorporating the effects of surface tension, slow rotation and suspended dust particles. In the absence of rotation and surface tension this dispersion relation (21) reduces to El-Sayed [34] excluding dynamic viscosity in that case. Hence the present results are the improvement of the problem of R–T instability of two superposed fluids with suspended dust particles and surface tension. The condition of R–T instability can be obtained easily from the constant term of (23) and given as

$$2\Omega^2 < gk(\alpha + 1)\left[\frac{\rho_2 - \rho_1}{\rho_1 + \rho_2} - \frac{k^2 T}{g(\rho_2 + \rho_1)}\right]. \tag{24}$$

Thus the system remains unstable for all the values of rotation smaller than the value given by condition (24).

In order to perform the numerical interpretation on the growth rate of R–T instability of two superposed slow rotating plasmas with suspended dust particles we write the dispersion relation (23) in dimensionless form and we get

$$\begin{aligned} &n^{*^4} + 2n^{*^3} f_s^*(1+\alpha) + n^{*^2}\Bigg[f_s^{*^2}(1+\alpha)^2 + 2\Omega^{*^2} \\ &-\frac{k}{k_\mathrm{c}} A\left(1 - \frac{k^2}{k_\mathrm{c}^2}\right)\Bigg] \\ &+n^*\left[4\Omega^{*^2} f_s^* - (2+\alpha) f_s^* A\frac{k}{k_\mathrm{c}}\left(1 - \frac{k^2}{k_\mathrm{c}^2}\right)\right] \end{aligned}$$

$$+\left[2\Omega^{*^2} - \frac{k}{k_\mathrm{c}}(\alpha+1)A\left(1-\frac{k^2}{k_\mathrm{c}^2}\right)\right] = 0\,, \qquad (25)$$

where $A = (\rho_2 - \rho_1)/(\rho_1 + \rho_2)$ is the Atwood number, $f_s = 1/\tau$ is the relaxation frequency of suspended dust particles, $\alpha = mN/(\rho_1 + \rho_2)$ is the mass concentration of suspended dust particles, $k_\mathrm{c} = [g(\rho_2 - \rho_1)/T]^{1/2}$ is the critical wavenumber.

$$n^* = n/(gk_\mathrm{c})^{1/2},\ f_s^* = f_s/(gk_\mathrm{c})^{1/2},$$

$$\Omega^* = \Omega/(gk_\mathrm{c})^{1/2}\,, \qquad (26)$$

are the dimensionless parameters.

In Fig. 2 we have depicted the dimensionless growth rate of R–T instability against the dimensionless wavenumber to show the effect of small rotation together with relaxation frequency and mass concentration of suspended dust particles and the Atwood number. The curves are plotted for various values of small rotation parameter $\Omega^* = 0.0, 0.15, 0.25$ and 0.35, respectively. The values of constant parameters like relaxation frequency of dust particles, the Atwood number and mass concentration of dust particles are taken to be 1.0, 5.0 and 0.5, respectively. We find that the growth rate of unstable R–T mode increases with increase in wavenumber and by attaining a maximum peak value it gets decreased. The increase in rotation parameter causes a decrease in growth rate of R–T instability. Also the peak value of unstable R–T mode is found to be larger for the minimum value of rotation parameter. The growth rate of R–T instability is found to be maximal for the case of no rotation. Hence presence of rotation has stabilizing influence on the growth rate of R–T instability. It is also noticed that the presence of the Atwood number, mass concentration and relaxation frequency affect growth rate of R–T instability significantly. We have shown the effect of these parameters on the growth rate of R–T instability later in Figs. 3, 4, 5 and 8.

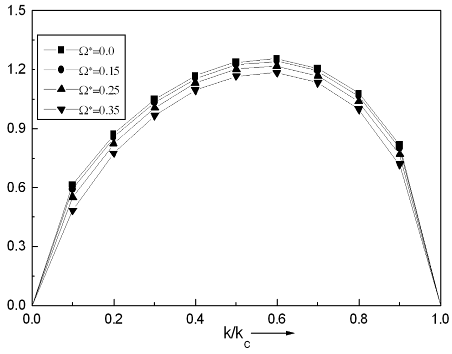

Fig. 2. The dimensionless growth rate of unstable R–T mode versus dimensionless wavenumber for different values of small rotation parameter (Ω^*) with $f_s^* = 1.0, A = 5.0$, and $\alpha = 0.5$.

In Fig. 3 we have shown the effect of relaxation frequency of suspended dust particles on the growth rate of R–T instability. The values of f_s^* are taken to be 1.25, 2.25, and 3.35, respectively. The values of constant parameters are considered as $\Omega^* = 0.2$ and $A = \alpha = 0.5$. We find that the presence of suspended dust particles has similar behavior to that of the rotation on the growth rate of R–T instability. Thus relaxation frequency of suspended dust particles has also stabilizing influence on the growth rate of the unstable R–T mode. The growth rate in this case is also affected by the presence of small rotation, mass concentration and the Atwood number.

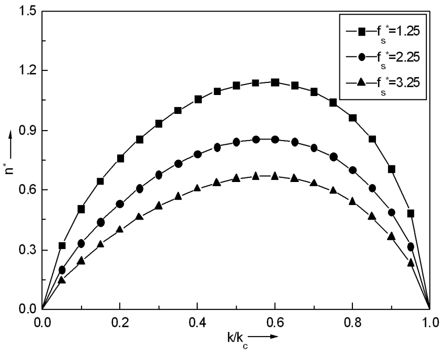

Fig. 3. The dimensionless growth rate of unstable R–T mode versus dimensionless wavenumber for different values of relaxation frequency parameter (f_s^*) with $\Omega^* = 0.2$ and $A = \alpha = 0.5$.

4.2. *Absence of rotation only ($\Omega = 0$)*

In the present subsection we deal with the case of superposed hydrodynamic fluids of different densities including suspended particles and surface tension. In order to discuss implications of the presence of particles, we analyze the case for vanishing rotation $\Omega = 0$. In this case the dispersion relation (23) can be written as

$$n^4 + n^3 2f_s(1+\alpha) + n^2\left\{f_s^2(1+\alpha)^2 - gk\left[\frac{\rho_2-\rho_1}{\rho_2+\rho_1} - \frac{k^2T}{g(\rho_2+\rho_1)}\right]\right\} + n\left\{gkf_s(2+\alpha)\left[\frac{\rho_1-\rho_2}{\rho_2+\rho_1} + \frac{k^2T}{g(\rho_2+\rho_1)}\right]\right\} + gkf_s^2(1+\alpha)\left[\frac{\rho_1-\rho_2}{\rho_2+\rho_1} + \frac{k^2T}{g(\rho_2+\rho_1)}\right] = 0\,. \qquad (27)$$

The condition of R–T instability is obtained from the constant term of (27), which is given as

$$\frac{k^2T}{g(\rho_1+\rho_2)} - \frac{\rho_2-\rho_1}{\rho_1+\rho_2} < 0, \quad \text{or} \quad k < k_\mathrm{c}\,. \qquad (28)$$

Thus with comparison of (28) to (24) we find that condition of R–T instability gets modified due to the presence of rotation in the form of (24). Also it is clear that the relaxation frequency of suspended dust particles does not play any role in condition of R–T instability but it affects the growth rate of unstable R–T mode due to the multiplication in constant term.

We can consider two different cases:

(a) Stable configuration ($\rho_1 > \rho_2$)

In this condition when the lower fluid is heavier than the upper fluid (i.e. $\rho_1 > \rho_2$), relation (27) does not admit any real positive or complex root with real positive part giving stability (necessary condition of Hurwitz's criterion) of the system. Thus the stable configuration remains stable even in the presence of suspended dust particles and surface tension.

(b) Unstable configuration ($\rho_1 < \rho_2$)

In this case when the upper fluid is heavier than the lower one (i.e. $\rho_2 > \rho_1$), then (27) will necessarily possess one real positive root (n_0), which leads to instability of the system.

We obtain $\mathrm{d}n_0/\mathrm{d}f_s$ (growth rate with increasing relaxation frequency of the suspended dust particles) from (27):

$$\begin{aligned}\frac{\mathrm{d}n_0}{\mathrm{d}f_s} &= -\big[2n_0^3(1+\alpha) + 2n_0^2 f_s(1+\alpha)^2 \\ &\quad -n_0 gka_0(2+\alpha) - 2gka_0 f_s(1+\alpha)\big] \\ &\quad /\big\{4n_0^3 + 6n_0^2 f_s(1+\alpha) + 2n_0\big[f_s^2(1+\alpha)^2 \\ &\quad -gka_0\big] - gka_0 f_s(2+\alpha)\big\}\,, \qquad (29)\end{aligned}$$

where

$$a_0 = \left[\frac{\rho_2-\rho_1}{\rho_2+\rho_1} - \frac{k^2 T}{g(\rho_2+\rho_1)}\right].$$

Equation (29) gives the following two inequalities:

$$\begin{aligned}&\big[2n_0^3(1+\alpha) + 2n_0^2 f_s(1+\alpha)^2\big] \\ &\quad \gtrless [n_0 gka_0(2+\alpha) - 2gka_0 f_s(1+\alpha)]\,, \qquad (30)\end{aligned}$$

and

$$\begin{aligned}&\big[4n_0^3 + 6n_0^2 f_s(1+\alpha) + 2n_0 f_s^2(1+\alpha)^2\big] \\ &\quad \gtrless [2n_0 gka_0 + gka_0 f_s(2+\alpha)]\,. \qquad (31)\end{aligned}$$

If both upper signs of inequalities given in (30) and (31) are satisfied simultaneously, then the growth rate $\mathrm{d}n_0/\mathrm{d}f_s$ is negative. In the other case, if the upper and lower signs or vice versa hold, then the growth rate turns out to be positive. We therefore conclude that the growth rate of unstable R–T modes is decreased with increasing relaxation frequency of the suspended dust particles. This means that under the restriction (30) and (31), the suspended dust particles have stabilizing influence on the considered system with surface tension.

For numerical interpretation we write the general dispersion relation (21) in the presence of suspended dust particles excluding the effect of rotation as

$$\begin{aligned}n^3 + n^2 f_s(1+\alpha) &= gkAn\left(1-\frac{k^2}{k_{\mathrm{c}}^2}\right) \\ &\quad + gkAf_s\left(1-\frac{k^2}{k_{\mathrm{c}}^2}\right). \qquad (32)\end{aligned}$$

Equation (32) can be written in the non-dimensional form by using (26) as

$$\begin{aligned}&{n^*}^3 + {n^*}^2 f_s^*(1+\alpha) - (k/k_{\mathrm{c}})An^*\left(1-\frac{k^2}{k_{\mathrm{c}}^2}\right) \\ &\quad - (k/k_{\mathrm{c}})Af_s^*\left(1-\frac{k^2}{k_{\mathrm{c}}^2}\right) = 0\,. \qquad (33)\end{aligned}$$

Equation (33) is solved numerically for various values of the non-dimensional parameters. The results are presented in Figs. 4–6. The stabilizing effect of the relaxation frequency of suspended dust particles is shown in Fig. 4 where we have plotted the real positive root of growth rate (leading to instability) against dimensionless wavenumber (k/k_{c}) for various values of the dimensionless relaxation frequency parameter (f_s^*) taking $A = 0.4$ and $\alpha = 0.6$. We find that the growth rate is suppressed with increase of relaxation frequency and mass concentration of suspended dust particles. The growth rate of R–T instability is also affected by the presence of mass concentration and the Atwood number as shown in Figs. 5 and 8.

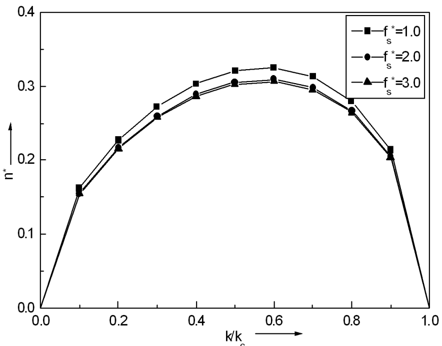

Fig. 4. The growth rate of R–T instability versus dimensionless wavenumber for various values of relaxation frequency of suspended dust particles (f_s^*).

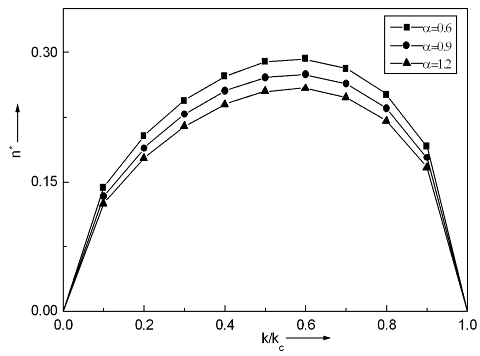

Fig. 5. The growth rate of R–T instability versus dimensionless wavenumber for various values of mass concentration of suspended dust particles (α).

In Fig. 5, the curves show effect of mass concentration (α) on the growth rate of R–T instability. The value of constant parameters are taken to be $A = 0.3$ and $f_s^* = 0.4$. We find that on increasing the value

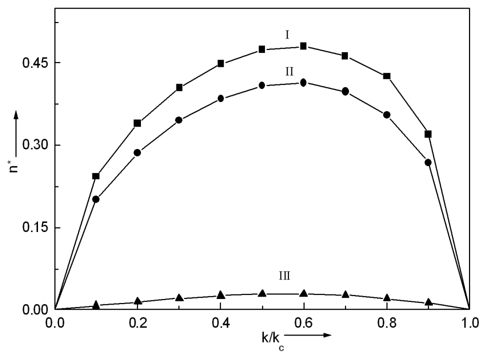

Fig. 6. The growth rate of R–T instability versus dimensionless wavenumber for various parameters. Curve I for $f_s^* = \Omega^* = \alpha = 0$ and $A = 0.6$, curve II for $f_s^* = 0.4$, $A = 0.6$, $\alpha = 0.7$ and $\Omega^* = 0.0$, and curve III for $\Omega^* = 0.4$ and $A = 0.6$.

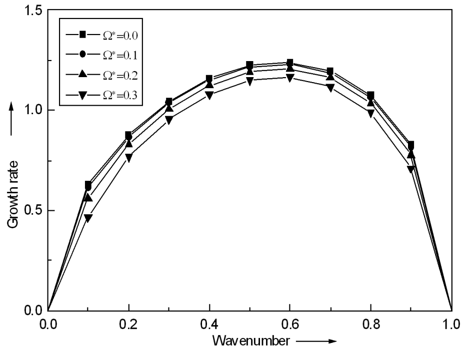

Fig. 7. The growth rate of R–T instability versus dimensionless wavenumber for various values of small rotation parameter (Ω^*).

of α the growth rate of R–T instability decreases, hence thereby stabilizes influence of mass concentration on the growth rate of unstable R–T mode. The effect of the Atwood number and relaxation frequency is also discussed on the growth rate of R–T instability. We find that relaxation frequency of suspended dust particles stabilizes the growth rate of R–T instability. In Fig. 6, various curves show the variation of growth rate for different values of parameters taken. Curve I represents the classical results for $\alpha = 0.6$, curve II and III represent the effects of suspended dust particles and rotation on the growth rate of unstable R–T mode, respectively. These calculations are based on the Mikaelian [19]. The change in the growth rate of R–T instability due to small rotation and suspended dust particles is presented and these effects reduce the growth rate of classical R–T instability as given by Mikaelian paper [19].

4.3. Absence of suspended dust particles only ($\tau = \alpha = 0$)

In order to see the effect of slow rotation on the growth rate of R–T instability, we write the general dispersion relation (23) in absence of suspended dust particles and

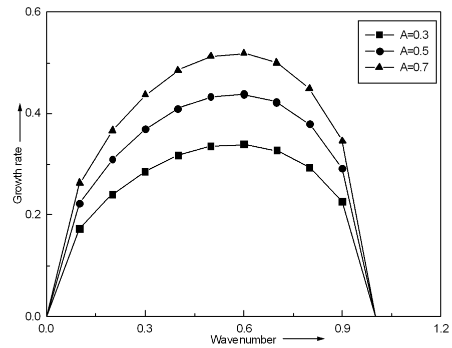

Fig. 8. The growth rate of R–T instability versus dimensionless wavenumber for various values of the Atwood number (A).

we get

$$n^2 + 2\Omega^2 + gk\left[\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} + \frac{k^2 T}{g(\rho_1 + \rho_2)}\right] = 0\,. \tag{34}$$

Equation (34) is slightly different from Chandrasekhar's [4] due to expansion of terms existing by the presence of small rotation in our problem. We get the condition of R–T instability in this case as

$$2\Omega^2 < gk\left[\frac{\rho_2 - \rho_1}{\rho_1 + \rho_2} - \frac{k^2 T}{g(\rho_2 + \rho_1)}\right]. \tag{35}$$

We find that condition (35) differs from condition given by (24) due to the presence of mass concentration of suspended dust particles in (24). Thus we conclude that the presence of suspended dust particles modify the R–T instability criterion.

Equation (34) is written in non-dimensional form using (26) and we get

$$n^{*^2} + 2\Omega^{*^2} - \frac{k}{k_c}A\left(1 - \frac{k^2}{k_c^2}\right) = 0\,. \tag{36}$$

In Fig. 7, various curves show the growth rate of R–T instability for different values of uniform small rotation parameter $\Omega^* = 0.0, 0.1, 0.2$, and 0.3. The numerical value of the Atwood number (A) is taken to be 4.0. The results of these calculations are presented in Fig. 7, where the growth rate (positive real part of n^*) is plotted against the wavenumber k/k_c. It can be clearly seen from the figure that as Ω^* increases, the growth rate of R–T instability (n^*) decreases, showing the stabilizing influence of rotation. The growth rate is found to be maximal for the case of no rotation.

4.1.3. Absence of both rotation and suspended dust particles ($\Omega = \tau = \alpha = 0$)

In the absence of rotation and suspended dust particles ($\Omega = \tau = N = 0$), the dispersion relation (23) reduces to

$$n^2 - gk\left[\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} - \frac{k^2 T}{g(\rho_2 + \rho_1)}\right] = 0\,. \tag{37}$$

Equation (37) incorporates the effect of surface tension on the classical R–T instability of two superposed fluids. This result is identical to Chandrasekhar [4] and

Mikaelian [8]. The condition of R–T instability is identical to that as given in (28). Thus, from comparison of (24) and (28) we find that condition of R–T instability gets modified due to the presence of rotation and suspended particles in the form of (24).

Equation (37) can be written as

$$n^2 = gkA\left(1 - \frac{k^2}{k_c^2}\right). \tag{38}$$

We get the same results as obtained by Chandrasekhar [4].

In order to see the effect of density difference between the fluids (i.e. the Atwood number) on the growth rate of R–T instability we write (38) in non-dimensional form using (26) as

$$n^{*^2} = \frac{k}{k_c}A\left(1 - \frac{k^2}{k_c^2}\right). \tag{39}$$

Equation (39) is solved numerically for various values of the Atwood number. In Fig. 8, we have plotted the dimensionless growth rate (positive real root n^*) of R–T instability against the dimensionless wavenumber (k/k_c). From the curves it is clear that the instability region increases with increasing the Atwood number. Thus we find that the increase in the difference of densities of the fluids has a destabilizing influence on the considered R–T configuration.

We now examine the dispersion relation (23) for the role of rotation with suspended dust particles and surface tension. The dispersion relation (23) for this case becomes

$$\begin{aligned} &n^4 + n^3\left[2f_s(1+\alpha)\right] + n^2\left[f_s^2(1+\alpha)^2 + 2\Omega^2 - gka_0\right] \\ &+ n\left[4f_s\Omega^2 - gkf_sa_0(2+\alpha)\right] \\ &+ 2\Omega^2 f_s^2 - gkf_s^2a_0(1+\alpha) = 0\,. \end{aligned} \tag{40}$$

When the lower fluid is heavier than upper one, then (40) can be written as

$$\begin{aligned} &n^4 + n^3\left[2f_s(1+\alpha)\right] + n^2\left[f_s^2(1+\alpha)^2 + 2\Omega^2 + gka_0\right] \\ &+ n\left[4f_s\Omega^2 + gkf_sa_0(2+\alpha)\right] \\ &+ 2\Omega^2 f_s^2 + gkf_s^2a_0(1+\alpha) = 0\,. \end{aligned} \tag{41}$$

We note that Eq. (41) does not admit any real positive or complex root with real positive part implying stability (necessary condition of Hurwitz's criterion). Thus stable configuration remains stable even in the presence of rotation.

Now we calculate the derivative of the growth rate of unstable R–T mode (n_0) with relaxation frequency of the suspended particles and rotation. We get from (40)

$$\begin{aligned} \frac{\mathrm{d}n_0}{\mathrm{d}f_s} = &-\big[2n_0^3(1+\alpha) + 2n_0^2f_s(1+\alpha)^2 + 4n_0\Omega^2 \\ &+4f_s\Omega^2 - n_0gka_0(2+\alpha) - 2gkf_sa_0(1+\alpha)\big] \\ &/\big[4n_0^3 + 6n_0^2f_s(1+\alpha) + 2n_0f_s^2(1+\alpha)^2 + 4n_0\Omega^2 \\ &+4f_s\Omega^2 - 2n_0gka_0 - gkf_sa_0(2+\alpha)\big]. \end{aligned} \tag{42}$$

Equation (42) gives two inequalities

$$\begin{aligned} &\left[2n_0^3(1+\alpha) + 2n_0^2f_s(1+\alpha)^2 + 4n_0\Omega^2 + 4f_s\Omega^2\right] \\ &\quad \gtrless \left[n_0gka_0(2+\alpha) + 2gkf_sa_0(1+\alpha)\right], \end{aligned} \tag{43}$$

and

$$\begin{aligned} &4n_0^3 + 6n_0^2f_s(1+\alpha) + 2n_0f_s^2(1+\alpha)^2 + 4n_0\Omega^2 + 4f_s\Omega^2 \\ &\quad \gtrless \left[2n_0gka_0 + gkf_sa_0(2+\alpha)\right]. \end{aligned} \tag{44}$$

If both upper signs of inequalities given in (43) and (44) are satisfied simultaneously, then the growth rate $\mathrm{d}n_0/\mathrm{d}f_s$ is negative. In the other case if the upper and lower signs or vice versa hold then the growth rate turns out to be positive. Therefore, we conclude that the growth rate of unstable R–T mode is decreased with increase in relaxation frequency of suspended dust particles. This means that under the restriction (43) and (44), the suspended dust particles have a stabilizing influence in the presence of rotation.

5. Conclusion

In the present paper, we have analyzed the R–T instability of two superposed fluids taking the effect of rotation, suspended dust particles and surface tension. The general dispersion relation for R–T instability is obtained, which is further reduced for some special cases to see the individual effects of rotation and suspended dust particles on the condition of R–T instability. The effects of slow rotation, surface tension and suspended dust particles are studied on the condition of R–T instability as well as stability. It is found that the arrangement remains unstable for long wavelength perturbations. For the case of R–T configuration with combined effect of surface tension, suspended dust particles and small rotation the system remains unstable for all values of uniform rotation, less than the particular value as given by new condition. The growth rate of the R–T instability vanishes for critical wavenumber $(k = k_c)$ and it is maximum for the case of zero wavenumber. Below the critical wavelength $(\lambda < \lambda_c)$ the growth becomes negative and therefore perturbations of wavelength shorter than the cut-off wavelength λ_c are stable. The inclusion of rotation and suspended dust particles stabilizes the growth rate of R–T instability.

From the graphical illustrations, we conclude that rotation, relaxation frequency and mass concentration of suspended dust particles all have stabilizing influence on the growth rate of unstable R–T mode. The peak value of growth rate is found to be minimum for larger values of rotation, relaxation frequency and mass concentration of suspended dust particles. It is also found that the density difference between the fluids (the Atwood number) has destabilizing influence on the growth rate of the system. The peak value of growth rate remains maximum for larger values of the Atwood number.

Thus, we have investigated the effect of surface tension and slow rotation parameters on the R–T instability of two superposed fluids in the presence of suspended dust particles.

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