On the Scattering of Neutron in the Magnetic Field of 180° Bloch Wall

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The exact solution of the Pauli equation has been derived for neutron wave propagating in magnetized continuum containing the magnetization non-uniformity such as the 180° Bloch wall, whose structure corresponds to the Landau–Lifshitz model. The scattering coefficients with and without neutron spin flip are presented as functions of ratio of neutron energy to the media's magnetic induction value. The possibility of narrow ($\lesssim 100$ Å) domain wall width measurement is discussed by the example of $YFe_{11}Ti$ alloy.

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1. Introduction

The problem of magnetic interaction of neutron with the domain wall (DW) in a ferromagnet attracted the researches attention more than once. The reason is that the scattering coefficients, other parameters being equal, depend on the DW width δ whose parameter knowledge is important for physics of ferromagnetism, as the fundamental characteristic of material named the exchange stiffness constant ([1], p. 102) depends on δ value directly. Calculating, for example, transmittance coefficients for some or other wall magnetization law as functions of δ, one can evaluate the real value of δ for this ferromagnetic material, fitting those calculated values to measured transmittance data [2]. The elementary calculations were presented in [3] where the neutron was considered as a classical particle whose magnetic moment vector precesses under the influence of the DW magnetic field. In a classical approximation they can derive only transmittance coefficients for neutron with and without flip, its sum is equal to one, whereas the reflection phenomena are ignored. The quantum-mechanical description of the neutron scattering on DW (in particular on the 180° DW) which is free of this shortcoming was executed in [4]. In this work the authors, similarly as in [3], used the one-helix approximation of the wall magnetization law. By this, the magnetization vector $\boldsymbol{M}$ rotates harmonically between two parallel planes which form an imaginary slab and $\boldsymbol{M}$ is considered as a permanent vector out of this slab. Such model is rather far from the real DW structures, the different types of which depending on the material's magnetic anisotropy are considered in [1]. Authors of [5] numerically solved the Pauli equation for the Landau–Lifshitz wall magnetization law which corresponds to the DW in a ferromagnet with only the second-order uniaxial anisotropy [1]. The matrix method of the solution of Pauli equation was used in [5], which is given a detailed account in [6]. In [5] they presented the results only for the particular case $\delta = 2631$ Å and the saturation induction value $B_s = 19.7$ kGs the last value corresponds to Fe + 4 at% Si alloy at the room temperature. Thus the progress in the scattering problem solution has been developed by the numerical methods loss of simplicity. Meanwhile the simple exact analytical solution of the problem calculated in [5] exists and is presented below.

2. Formulation of the problem

Let us consider the disk cut from the uniaxial ferromagnet monocrystal so that its easy axis lies in the disk surface plane. In such a geometry the row of ideally parallel domain walls was observed [7] from which we will consider only one wall separating two domains called as the domain 1 (left) and the domain 2 (right) (Fig. 1). Ferromagnetic medium has the saturation magnetization M_s, the exchange stiffness constant A and the second-order anisotropy constant $K_1 > 0$. Let y be the coordinate orthogonal to DW plane and z — the one parallel to the easy axis. If $\theta(y)$ is the magnetization vector $\boldsymbol{M}(y)$ polar tilt angle from the axis Oz, then the magnetic induction value of a Bloch DW in the one-dimensional approximation is

$$\boldsymbol{B}(y) = 4\pi\boldsymbol{M}(y) = B_\mathrm{s}(-\sin\theta\cdot\boldsymbol{e}_1 + \cos\theta\cdot\boldsymbol{e}_3)\,. \quad (2.1)$$

Here $B_\mathrm{s} = 4\pi M_\mathrm{s}$, e_j ($j = 1, 2, 3$) are the unit vectors of a given coordinate system and $\boldsymbol{M}(y)$ polar longitude is equal to π elsewhere. In the Landau–Lifshitz model

$$\cos\theta = -\tanh(y/d)\,, \quad \sin\theta = 1/\cosh(y/d)\,, \quad (2.2)$$

where $d = \sqrt{A/K_1}$. The quantity $\delta = \pi d$ is called the domain wall width [1].

The neutron wave propagating in a ferromagnet interacts with the atom nucleus and the magnetic moments

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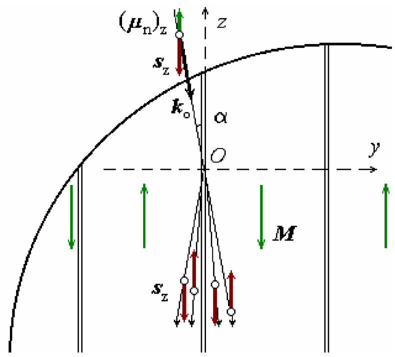

Fig. 1. The geometry of problem. $\boldsymbol{s}_z$ is the projection of the vector of neutron spin on Oz axis. $(\boldsymbol{\mu}_\mathrm{n})_z$ is the projection of the vector of neutron magnetic moment on Oz axis.

of atom's electron shells. In the neutron optics approximation the nuclear forces potential $\hat{U}_\mathrm{nucl} = U_\mathrm{n} = \mathrm{const}$ and the magnetic interaction potential

$$\hat{U}_\mathrm{magn} = -\hat{\boldsymbol{\mu}}_\mathrm{n} \cdot \boldsymbol{B}(\boldsymbol{r})\,. \tag{2.3}$$

Here $\hat{\boldsymbol{\mu}}_\mathrm{n} = \mu_\mathrm{n} \cdot \hat{\boldsymbol{\sigma}}$ is the operator of magnetic moment of neutron, $\mu_\mathrm{n} = -9.66 \times 10^{-24}$ erg/Gs is its magnitude and $\hat{\boldsymbol{\sigma}}(\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ are the Pauli spin matrices. Let us introduce the parameter $U = B_\mathrm{s}|\mu_\mathrm{n}|$. Then

$$\hat{U}_\mathrm{magn} = U \cdot \hat{\boldsymbol{\sigma}}(-\sin\theta \cdot \boldsymbol{e}_1 + \cos\theta \cdot \boldsymbol{e}_3)\,. \tag{2.4}$$

Consider the spin-down neutron of mass m with the wave vector $\boldsymbol{K}_0$ in vacuum impinging the disk at little angles to the planes Oxz and Oyz simultaneously. Then the neutron total energy $W = \frac{\hbar^2 K_0^2}{2m} = \frac{\hbar^2 K^2}{2m} - U + U_\mathrm{n} = W_1 + U_\mathrm{n}$, where $\boldsymbol{K}$ is its wave vector after the incidence on surface. By this the Pauli equation for neutron is

$$-\frac{\hbar^2}{2m}\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} + \frac{\mathrm{d}^2}{\mathrm{d}z^2}\right)\Psi(y,z) + \hat{U}_\mathrm{magn}\Psi(y,z) = W_1\,\Psi(y,z)\,, \tag{2.5}$$

where $\Psi(y,z) = \mathrm{e}^{\mathrm{i}K_z z}\psi(y) = \mathrm{e}^{\mathrm{i}K_z z}\begin{pmatrix}\varphi(y)\\ \chi(y)\end{pmatrix}$ is the wave function of neutron, $K_z = K\cos\alpha$, α is the glancing angle (Fig. 1, $\boldsymbol{K}_0$ lies in plane Oyz for simplicity). Dividing the variables in (2.5), we obtain

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}t^2}\frac{1}{d^2} + \frac{2mU}{\hbar^2}\left(\hat{\sigma}_x \cosh^{-1} t + \hat{\sigma}_z \tanh t\right)\psi = -\frac{2m}{\hbar^2}E_-\psi\,, \tag{2.6}$$

where $t = y/d$ is the dimensionless coordinate variable and $E_- = W_1 - (W_1 + U)\cos^2\alpha$ is the total energy of the normal motion in respect of DW. Putting the dimensionless variables $w = W_1/U$, $\varepsilon_- = E_-/U$, we come to the equality

$$\varepsilon_- = w\sin^2\alpha - \cos^2\alpha\,. \tag{2.7}$$

Let us introduce the dimensionless parameter of the ferromagnetic material $q = 4mU\delta^2/\hbar^2$. The y-projection k_y of the wave vector $\boldsymbol{k}$ at $y < 0$, multiplied by $d/2$, is written as

$$k_y = \sqrt{\frac{q\,(\varepsilon_- + 1)}{8\pi^2}} = \sqrt{\frac{q\,(w+1)}{8\pi^2}}\sin\alpha\,. \tag{2.8}$$

If $E_- > U$, then spin-down neutron may pass through the wall without flip and so at $y \to +\infty$ the dimensionless y-projection of the wave vector will be equal to

$$k'_y = \sqrt{\frac{q\,(\varepsilon_- - 1)}{8\pi^2}} = \sqrt{k_y^2 - \frac{q}{4\pi^2}}\,. \tag{2.9}$$

(The corresponding dimensional wave number will be signified as K'.) Using the designations introduced in this paragraph, one can write down the solution of (2.6) in a compact form.

3. The solution of the Pauli equation

Further, the quantities k_y, k'_y, ε_- and ε_+ will be written without bottom indexes where it is possible. We will solve the matrix Eq. (2.6) nearly as in Ref. [8] where the author solved the problem of the scalar particle scattering on the potential barrier $U(t) \sim \mathrm{const}(1 + \tanh t)$, although the concrete mathematical transformations are some other. Namely let $s = -\mathrm{e}^{-2t}$, $\psi(t) = (-s)^{-\mathrm{i}k}\begin{pmatrix}f(s)\\ g(s)\end{pmatrix}$, $\hat{\varDelta} = s\frac{\mathrm{d}}{\mathrm{d}s}$, $a_1 = -\mathrm{i}(k + k')$, $a_2 = -\mathrm{i}(k - k')$, $a = -2\mathrm{i}k$ and $a' = -2\mathrm{i}k'$. Substituting the bottom component of (2.6) into the upper one we will obtain

$$\Big[\hat{\varDelta}\left(\hat{\varDelta} + a_1 - 0.5\right)\left(\hat{\varDelta} + a_2 - 0.5\right)\left(\hat{\varDelta} + a\right) - s\left(\hat{\varDelta} + a_1\right)\left(\hat{\varDelta} + a_2\right)\left(\hat{\varDelta} + a + 0.5\right)\left(\hat{\varDelta} + 0.5\right)\Big] \times f(s) = 0\,. \tag{3.1}$$

The set of fundamental solutions of (3.1) is known ([9], p. 190): these are the generalized hypergeometric functions. In particular, near $s = 0$, which corresponds to the domain 2 area in Fig. 1,

$$f_1(s) = {}_4F_3\left(\begin{matrix} a_1, & a_2, & 0.5 + a, & 0.5 \\ 0.5 + a_1, & 0.5 + a_2, & 1 + a & \end{matrix}\,\middle|\, s\right), \tag{3.2}$$

$$f_2(s) = (-s)^{1/2} s^{-a_2}\tilde{f}_2(s) = (-s)^{1/2} s^{-a_2} \times {}_4F_3\left(\begin{matrix} 1 + a_1, & 1 - a_2, & 0.5 + a', & 0.5 \\ 1.5 + a_1, & 1.5 - a_2, & 1 + a' & \end{matrix}\,\middle|\, s\right), \tag{3.3}$$

$$f_3(s) = s^{-a} f_1^*(s)\,, \quad f_4(s) = s^{-a} f_2^*(s)\,, \tag{3.4}$$

where * denotes complex conjugation. To derive the second component of spinor ψ, let us return to upper component of (2.6) which gives

$$g(s) = \frac{(-s)^{-1/2}}{a_1 a_2} \times \Big[\hat{\varDelta}\left(\hat{\varDelta} + a\right) - s\left(\hat{\varDelta} + a_1\right)\left(\hat{\varDelta} + a_2\right)\Big] f(s)\,. \tag{3.5}$$

At $|s| < 1$, the generalized hypergeometric function may be represented as a convergent series by the s degrees. The substitution of four corresponding series into (3.5) gives the following series for the lower components of spinors:

$$g_1(s) = (-s)^{1/2}(AC)\tilde{g}_1(s) = (-s)^{1/2}(AC) \times {}_4F_3\left(\begin{array}{cccc} 1+a_1, & 1+a_2, & 0.5+a, & 0.5 \\ 1.5+a_1, & 1.5+a_2, & 1+a & \end{array}\middle| s\right), \quad (3.6)$$

$$g_2(s) = s^{-a_2}(AC^*)^{-1}\tilde{g}_2(s) = s^{-a_2}(AC^*)^{-1} \times {}_4F_3\left(\begin{array}{cccc} a_1, & -a_2, & 0.5+a', & 0.5 \\ 0.5+a_1, & 0.5-a_2, & 1+a' & \end{array}\middle| s\right), \quad (3.7)$$

$$g_3(s) = s^{-a}g_1^*(s)\,, \quad g_4(s) = s^{-a}g_2^*(s)\,, \quad (3.8)$$

where $A = 1/(0.5+a_1)$, $C = a_1a_2/(0.5+a_2)$. Afterwards for the area of left domain it is necessary to understand the formulae (3.6)–(3.8) as formulae for the analytical continuation of the generalized hypergeometric series.

Returning to dimensional coordinate and wave number and multiplying for convenience functions ψ_2–ψ_4 by some complex constants, one can extract finally the solutions of (2.5) as the modulated de Broglie waves

$$\psi_1(y) = \mathrm{e}^{\mathrm{i}Ky}\begin{pmatrix} f_1(-\mathrm{e}^{-2y/d}) \\ AC\,\mathrm{e}^{-y/d}\tilde{g}_1(-\mathrm{e}^{-2y/d}) \end{pmatrix}, \quad (3.9)$$

$$\psi_2(y) = \mathrm{e}^{\mathrm{i}K'y}\begin{pmatrix} AC^*\,\mathrm{e}^{-y/d}\tilde{f}_2(-\mathrm{e}^{-2y/d}) \\ \tilde{g}_2(-\mathrm{e}^{-2y/d}) \end{pmatrix}, \quad (3.10)$$

$$\psi_3(y) = \psi_1^*(y)\,, \quad \psi_4(y) = \psi_2^*(y)\,. \quad (3.11)$$

By the physical sense of problem, the neutron wave has to satisfy the boundary conditions in the infinity region (really in the depth of domains 1 and 2):

$$\psi(-\infty) = \begin{pmatrix} r_{+-}\,\mathrm{e}^{-\mathrm{i}K'y} \\ \mathrm{e}^{\mathrm{i}Ky} + r_{--}\,\mathrm{e}^{-\mathrm{i}Ky} \end{pmatrix}, \quad (3.12)$$

$$\psi(+\infty) = \begin{pmatrix} t_{+-}\,\mathrm{e}^{\mathrm{i}Ky} \\ t_{--}\,\mathrm{e}^{\mathrm{i}K'y} \end{pmatrix}, \quad (3.13)$$

where r_{+-} is the reflection amplitude with spin-flip, t_{--} is the transmission amplitude without spin-flip and so on. Here the right subscript denotes the initial polarization. Among the derived solutions (3.2)–(3.11), the spinors ψ_1 and ψ_2 have the asymptotic behaviour at $y \to +\infty$ according to (3.13). By means of formula connected with the analytical continuation of hypergeometric function ${}_4F_3\left(\begin{array}{c}(a_4)\\(b_3)\end{array}\middle|\begin{array}{c}1\\s\end{array}\right)$ at $|s| < 1$ and the hypergeometric functions ${}_4F_3\left(\begin{array}{c}1+a_k-(b_3), a_k\\1+a_k-(a_4)'\end{array}\middle| s\right)$ (see [9], p. 179, here $k = 1\dots4$, little brackets signify the corresponding set of indexes, little brackets with stroke signify the set of indexes with omitting 1 as result of calculations of $1+a_k-a_j$, $j = 1\dots4$), one can fit the number $Q(k,\,k')$, so that the spinor

$$\psi(y) = [\psi_1(y) + Q(k,k')\psi_2(y)] \times \mathrm{const} \quad (3.14)$$

asymptotical behaviour will be corresponding to (3.12) not only to (3.13). The calculations gives

$$Q = -\frac{a_2}{\sin\pi a_2} \times \frac{\Gamma^2(-a_2)\,\Gamma(1+a)\,\Gamma(0.5+a')}{\Gamma^2(0.5-a_2)\,\Gamma(0.5+a)\,\Gamma(1+a')}\,. \quad (3.15)$$

Finally we have:

(a) the transition coefficient for neutron without spin-flip

$$T_{--} = \frac{k'}{k}\,|t_{--}|^2 = \frac{\tanh 2\pi k \tanh 2\pi k'}{\tanh^2\pi(k+k')\cosh^2\pi(k-k')}\,, \quad (3.16)$$

(b) the transition coefficient with spin-flip

$$T_{+-} = |t_{+-}|^2 = \left(\frac{\tanh\pi(k-k')\tanh 2\pi k}{\tanh\pi(k+k')}\right)^2, \quad (3.17)$$

(c) the reflection coefficient without spin-flip

$$R_{--} = |r_{--}|^2 = \left(\frac{\tanh\pi(k-k')}{\cosh 2\pi k\tanh\pi(k+k')}\right)^2, \quad (3.18)$$

(d) the reflection coefficient with spin-flip

$$R_{+-} = \frac{k'}{k}\,|r_{+-}|^2 = \frac{\tanh^2\pi(k-k')\tanh 2\pi k\tanh 2\pi k'}{\sinh^2\pi(k+k')}\,. \quad (3.19)$$

The sum of these four coefficients is equal to one identically, which may be verified by means of the elementary trigonometry methods.

When $\varepsilon_- < 1$, the principle of conservation of energy forbids both the transmission process without flip and the reflection process with flip. It is manifested itself mathematically in the fact that the value k' became imaginary. To satisfy the normalization conditions on ψ-functions it is necessary to choose $k' = \mathrm{i}|k'|$, not $k' = -\mathrm{i}|k'|$. Then the multipliers at r_{+-} and t_{--} coefficients in (3.12)–(3.13) became equal to zero and only two coefficients of four were still necessary to describe the scattering phenomena. The substitution $k' \to \mathrm{i}|k'|$ in the formulae (3.17)–(3.18) yields

$$T_{+-} = \tanh^2 2\pi k, \quad R_{--} = 1/\cosh^2 2\pi k\,. \quad (3.20)$$

Now let the particle's initial polarization is directed along the Oz axis (spin-up neutron). The previous designations will be conserved, namely k is the neutron wave number in the left domain and k' is the neutron wave number after passing of DW without flip. So the energy of the normal motion in respect of DW will be

$$E_+ = W_1 - (W_1 - U)\cos^2\alpha\,,$$

$$\varepsilon_+ = E_+/U = w\sin^2\alpha + \cos^2\alpha\,, \quad (3.21)$$

and consequently

$$k = \sqrt{\frac{q\,(w-1)}{8\pi^2}}\,,$$

$$k'_y = \sqrt{\frac{q\,(\varepsilon_+ + 1)}{8\pi^2}} = \sqrt{k_y^2 + \frac{q}{4\pi^2}}\,, \tag{3.22}$$

and the next commutations must be executed in (3.1)–(3.9): $a_2 \to -a_2$, $a \to a'$, $a' \to a$. The wave function asymptotics for spin-up neutron in the infinity regions must be

$$\psi(-\infty) = \begin{pmatrix} \mathrm{e}^{\mathrm{i}Ky} + r_{++}\mathrm{e}^{-\mathrm{i}Ky} \\ r_{-+}\mathrm{e}^{-\mathrm{i}K'y} \end{pmatrix},$$

$$\psi(+\infty) = \begin{pmatrix} t_{++}\mathrm{e}^{\mathrm{i}K'y} \\ t_{-+}\mathrm{e}^{\mathrm{i}Ky} \end{pmatrix}. \tag{3.23}$$

To derive such the wave function one needs to compose the other linear combination of the fundamental solutions (3.9)–(3.10):

$$\tilde{\psi}(y) = [\psi_1(y) + Q'(k,k')\psi_2(y)] \times \mathrm{const}\,, \tag{3.24}$$

with a new constant Q'. After some simple although bulky calculations we obtain

$$Q'(k,k') = -\frac{1}{Q(k',k)}\,. \tag{3.25}$$

It gives for the scattering coefficients R_{++}, T_{-+}, T_{++}, R_{-+} the expressions formally having the same right parts as in (3.16)–(3.20) correspondently, but with due regard for formulae (3.22) instead of (2.8)–(2.9). One can mention that the transformation of $\boldsymbol{M}(y)$ polar longitude to zero in Fig. 1 leads to the wave function transformation $\begin{pmatrix} \varphi \\ \chi \end{pmatrix} \to \begin{pmatrix} \varphi \\ -\chi \end{pmatrix}$ which does not influence the scattering coefficients.

4. Discussion

The coefficient R_{+-} being considered as a function of the two variables ε and q, has an absolute maximum $R_{+-}(\varepsilon) = 0.074$ at $\varepsilon = 1.09$, $q = 2.8$ (Fig. 2). This seems to be too little value to observe the phenomenon in the experiment at any parameters of ferromagnet and will not be discussed further.

At $\varepsilon_- = 1$ the formulae (3.20) give $T_{+-}(\varepsilon_- = 1) = \tanh^2\sqrt{q}$. If $\sqrt{q} \gg 1$ ("thick" wall), the reflection of neutron is practically lacked. Such a situation is realized in the alloy Fe + 4%at. Si where $\delta > 1000$ Å for 180° walls [2] and so $\sqrt{q}$(r.t.) ≈ 10 (r.t. = room temperature). The equations $T_{+-}(\varepsilon_-) = T_{--}(\varepsilon_-)$ and $T_{++}(\varepsilon_+) = T_{-+}(\varepsilon_+)$ in that case both have the approximate solution $\varepsilon \approx q/8$, which defines the energy interval for which DW effectively rotates the spin of passing neutron (Fig. 3).

If $\sqrt{q} \ll 1$ ("narrow" wall) then $T_{+-}(\varepsilon_- = 1) = q$, $R_{--}(\varepsilon_- = 1) = 1 - q$. Both quantities are sensitive to DW width but the reflection coefficient is considerably higher.

The coefficients $R_{--}(\varepsilon)$ and $R_{++}(\varepsilon)$ are presented in Fig. 4. At $\varepsilon > 1$ both are decrease quickly with the ε increasing, by this the speed of decreasing depends of δ value slightly.

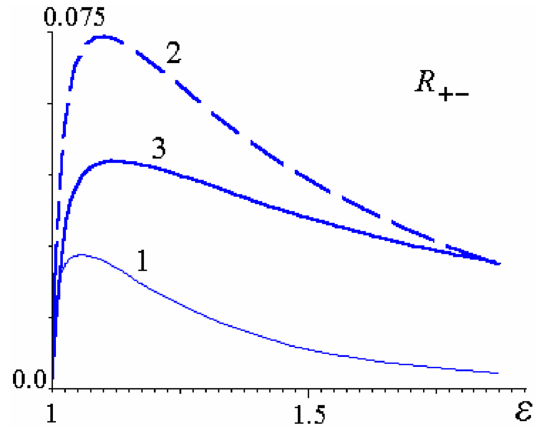

Fig. 2. The dimensionless energy dependence of reflection coefficient with spin-flip (B_s = 19700 Gs in all figures here and further). 1 — $\delta = 100$ Å, 2 — $\delta = 155$ Å, 3 — $\delta = 250$ Å. $R_{+-}(\varepsilon) = R_{-+}(\varepsilon)$.

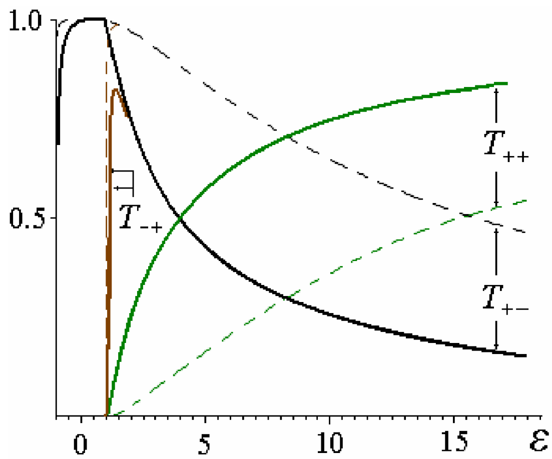

Fig. 3. The dimensionless energy dependence of transmission coefficients. The broken lines correspond to $\delta = 1000$ Å. The continuous ones correspond to $\delta = 500$ Å. When $\varepsilon > 3$, graphics of $T_{-+}(\varepsilon)$ and $T_{+-}(\varepsilon)$ practically coincide with each other. $T_{--}(\varepsilon) = T_{++}(\varepsilon)$.

As a specific example let us consider the alloy $YFe_{11}Ti$. In that alloy according to [10] the Curie temperature $T_c = 540$ K, K_1(r.t.) $= 0.85 \times 10^7$ erg/cm^3 and K_2(r.t.) $< 0.1K_1$(r.t.). For neutrons with $\lambda = 10$ Å, $\varepsilon_- = 1$ when $\alpha = 45'$ what is seems to be the observable value. Let us evaluate DW width very roughly from the formula $\delta = \mathrm{const} \times \sqrt{\mathrm{kT_c/aK_1}}$ (a = middle interatomic distance between the adjacent Fe atoms inside the crystal lattice) which is followed from the formula $\delta = \pi \times \sqrt{A/K_1}$ and from consideration of dimensionality. Comparing the two crystal lattices $YFe_{11}Ti$ and $Fe_{14}Nd_2B$ for second of which all the parameters are known [11] and considering that constanta in formula for δ is just the same we obtain for $YFe_{11}Ti$ $\delta \approx 100$ Å. Then $\sqrt{q} = 0.80$ and $R_{--}(\varepsilon_- = 1) = 0.56$. Wish it will be, for ex-

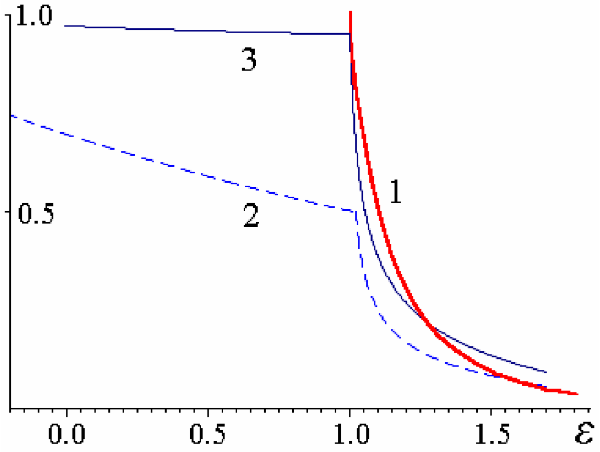

Fig. 4. The dimensionless energy dependence of reflection coefficients without spin-flip. 1 — $R_{++}(\varepsilon)$: $\delta = 320$ Å, 2 — $R_{--}(\varepsilon)$: $\delta = 80$ Å, 3 — $R_{--}(\varepsilon)$: $\delta = 20$ Å.

ample, $\delta = 80$ Å, then $\sqrt{q} = 0.65$ and consequently $R_{--}(\varepsilon_- = 1) = 0.67$. The simplest calculation shows that the presence of domains with magnetization directed parallel to neutron spin in the specimen leads to only small correction in the formula for reflection coefficient for neutron beam $R(\delta) = (R_{--}(\varepsilon) + R_{++}(\varepsilon))/2$. Thus these facts may be the state of experiment in which the δ value is defined not from transmission data as in [2] but using the neutron reflection data from DW.

5. Conclusion remarks

To what degree the derived formulae are useful for the experimental physics? First of all, accordingly (2.3) the scattering will be noticeable for the ferromagnets with the magnetization as high as possible. The iron intermetallic alloys far from the Kurie point $T \ll T_c$ contrary, for example, to the uniaxial ferrites comply this requirement. However many of such alloys have the values of high anisotropy coefficients comparable with K_1 at $T \ll T_c$ namely. It makes impossible the application to them the Landau–Lifshits model. The alloy $YFe_{11}Ti$ is the unique exception from that rule. According to [10] there exist the temperature interval 300 K $< T <$ 400 K where it's magnetization is still comparable with the zero-temperature one ($B_s = 11.2$ kGs) but all the high anisotropy coefficients drops to zero. So we have at least one model material for the exact determination of the domain wall width measuring the reflection of neutrons from DW. In addition there is no the neutron absorption for $YFe_{11}Ti$ contrary to pure Co and the metodics of monocrystal growing has been develop for it contrary to ordered FePt.

Finally there are uniaxial ferromagnetic alloys such as $YFe_{11}Ti$ for which the results of present article offer in principle to stand the experiment for evaluation the exchange stiffness constant in such alloys, measuring the reflection of neutrons from DW.

As for alloys with the complex anisotropy, the Landau–Lifshitz model and consequently the formulae (3.16)–(3.20) are the approximation to be compete the one-helix model of DW used in this branch of physics earlier. Namely, it may be more applicable if the surface DW energy value σ_1 minimizing the energy functional of ferromagnet on the set of trial functions (2.2) with δ as a free parameter is closer to its exact value than the σ_2 one minimizing it on the set of trial functions $\theta(|y| < \delta/2) = \pi/2 - \pi y/\delta$, $\theta(|y| \geq \delta/2) = \text{const}$. For the alloy Fe + 4%at. Si used in [2], for which the energy functional takes place

$$\sigma = \int_{-\infty}^{+\infty} \left(A \left(\frac{\mathrm{d}\theta}{\mathrm{d}y} \right)^2 + K \sin^2\theta \cos^2\theta \right) \mathrm{d}y\,, \tag{5.1}$$

([1], p. 105; the magnetostriction energy governing the δ value in FeSi practically does not disturb the surface energy value) the simplest calculation gives $\sigma_1 > \sigma_2$ and the one-helix approximation of DW structure used there seems to be optimal.

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