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Magnetic Symmetry of the Plain Domain Walls in the Plates of Cubic Ferro- and Ferrimagnets

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Magnetic symmetry of possible plane domain walls in arbitrary oriented plates of the crystal of hexoctahedral crystallographic class is considered. The symmetry classification is applied for ferro- and ferrimagnets.

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1. Introduction

For sequential examination of static and dynamic properties of domain walls (DWs) in magnetically ordered media it is necessary to take into account their magnetic symmetry [1, 2]. The complete symmetry classification of plane 180°-DWs in magnetically ordered crystals [1], similar classification of these DWs with the Bloch lines in ferromagnets and ferrites [2] and magnetic symmetry classification of plane non-180°-DWs (all possible DW types including 0°-DWs [3]) in ferro- and ferrimagnets [4] were carried out earlier. These DW symmetry classifications allows arbitrary crystallographic point symmetry group of the crystal. The influence of the spatially restricted sample surfaces on the DW magnetic symmetry was not considered in works [1–4]. The real magnetic sample restricts a spatial (3D) magnetization distribution. Therefore, it modifies the DW symmetry in general case. This paper presents the investigation of the influence of the restricted sample surfaces on the symmetry of the all possible (0°-, 60°-, 70.5°-, 90°-, 109.5°-, 120°- and 180°-DW [4, 5]) plane (i.e., DW with $r_0 \gg \delta$, where r_0 is the curvature radius of the DW [1]) DWs in an arbitrary oriented plate of the cubic (crystallographic point symmetry group $m\bar{3}m$) ferro- and ferrimagnets.

2. Domain wall symmetry in the restricted sample

The DW symmetry can be described by the magnetic symmetry classes (MSCs) G_k where k is a MSC number [1]. The MSC G_k of DW is the magnetic symmetry group including all symmetry transformations (all translations are considered as unit operations) that do not change the spatial distribution of magnetic moments in the crystal with DW. The above-mentioned group is a subgroup of the magnetic (Shubnikov) symmetry group G_P^∞ of the crystal paramagnetic phase [6]. Total number of MSCs of arbitrary type DWs (i.e., DWs with arbitrary 2α angle ($0° \le 2\alpha \le 180°$) between the unit time-odd axial vectors $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ directed along magnetization vectors $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ in neighbouring domains) in ferro- and ferrimagnets is equal to 64. General enumeration of MSCs contains 42 MSCs ($1 \le k \le 42$) of 180°-DWs [1], 10 MSCs ($7 \le k \le 13$ and $16 \le k \le 18$) of non 0°/180°-DWs and 42 MSCs ($k = 2$, $6 \le k \le 13$, $16 \le k \le 19, k = 22$, 24, 26, 30, 32, 37, 39 and $43 \le k \le 64$) of 0°-DWs [4]. The MSCs ($k = 25$, 28, 37–41, 52, 54, 61–63) with sixfold symmetry axes (including inversion axes) do not realize in the cubic crystals [4].

The unified co-ordinate system $Oxyz$ is chosen as $[\boldsymbol{e}_{\tilde{x}}, \boldsymbol{e}_{\tilde{y}}, \boldsymbol{e}_{\tilde{z}}] = [\boldsymbol{a}_2, -\boldsymbol{a}_1, \boldsymbol{n}_{\mathrm{W}}]$ where $\boldsymbol{n}_{\mathrm{W}}$ is the unit polar time-even vector along the DW plane normal [4]. For the 180°-DWs the vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ are given earlier [1] as vectors $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$, respectively. For the case of $2\alpha \ne 180°$ the unit vector $\boldsymbol{a}_1$ coincides with the direction of the vector $\Delta\boldsymbol{m} - \boldsymbol{n}_{\mathrm{W}}(\boldsymbol{n}_{\mathrm{W}}\Delta\boldsymbol{m})$ (at $b_\Delta \ne 0$ and $b_\Sigma = 0$) or $[\boldsymbol{a}_2 \times \boldsymbol{n}_{\mathrm{W}}]$ (at $b_\Delta = 0$ or $b_\Sigma \ne 0$) where $\Delta\boldsymbol{m} = \boldsymbol{m}_2 - \boldsymbol{m}_1$, $b_\Delta = |[\boldsymbol{n}_{\mathrm{W}} \times \Delta\boldsymbol{m}]|$, $b_\Sigma = |[\boldsymbol{n}_{\mathrm{W}} \times \boldsymbol{m}_\Sigma]|$. Here the unit vector $\boldsymbol{a}_2$ coincides with the direction of the vector $\boldsymbol{m}_\Sigma - \boldsymbol{n}_{\mathrm{W}}(\boldsymbol{n}_{\mathrm{W}}\boldsymbol{m}_\Sigma)$ (at $b_\Sigma \ne 0$) or $[\boldsymbol{n}_{\mathrm{W}} \times \boldsymbol{a}_1]$ (at $b_\Delta \ne 0$ and $b_\Sigma = 0$), or else, with an arbitrary direction in the DW plane ($\boldsymbol{a}_2 \perp \boldsymbol{n}_{\mathrm{W}}$ at $b_\Sigma = b_\Delta = 0$) where $\boldsymbol{m}_\Sigma = \boldsymbol{m}_1 + \boldsymbol{m}_2$. The mutual orientation of the vectors $\boldsymbol{m}_1$, $\boldsymbol{m}_2$ and $\boldsymbol{n}_{\mathrm{W}}$ is determined by the parameters: $a_\Sigma = (\boldsymbol{n}_{\mathrm{W}}\boldsymbol{m}_\Sigma)$, $a_\Delta = (\boldsymbol{n}_{\mathrm{W}}\Delta\boldsymbol{m})$, $a_{\mathrm{C}} = (\boldsymbol{n}_{\mathrm{W}}\boldsymbol{m}_{\mathrm{C}})$, b_Σ and b_Δ, where $\boldsymbol{m}_{\mathrm{C}} = [\boldsymbol{m}_1 \times \boldsymbol{m}_2]$. The mutual orientation of the vectors $\boldsymbol{m}_1$, $\boldsymbol{m}_2$, $\boldsymbol{n}_{\mathrm{W}}$ and $\boldsymbol{n}_S$ is determined by parameters: $a_1 = (\boldsymbol{a}_1\boldsymbol{n}_S)$, $a_2 = (\boldsymbol{a}_2\boldsymbol{n}_S)$, $a_n = (\boldsymbol{n}_{\mathrm{W}}\boldsymbol{n}_S), b_1 = |[\boldsymbol{n}_S \times \boldsymbol{a}_1]|, b_2 = |[\boldsymbol{n}_S \times \boldsymbol{a}_2]|$ and $b_n = |[\boldsymbol{n}_S \times \boldsymbol{n}_{\mathrm{W}}]|$, where $\boldsymbol{n}_S$ is sample plane normal.

The MSC G_P of restricted sample of crystal in paramagnetic phase could be defined as $G_P = G_P^\infty \cap G_S$ where

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the sample shape MSC G_S is $\infty/mmm1'$ for volumetric plate. MSCs of DWs in volumetric plate should satisfy the condition $G_k \subset G_P$. The MSCs of the all possible plane DWs in the arbitrary oriented plate of cubic crystals of hexoctahedral class (crystallographic point symmetry group $m\bar{3}m$ in the paramagnetic phase [6]) are presented in Table. Here, symmetry axes are collinear with vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ and reflection planes are perpendicular to them. For MSCs with: $k = 24$, $k = 26$, $k = 27$; $29 \le k \le 36$; $42 \le k \le 51$, $k = 53$, $55 \le k \le 60$ and $k = 64$ only generative symmetry elements are represented.

TABLE

MSCs of the plane 2α-DWs in plates of the cubic $m\bar{3}m$ crystal.

k	$\{nml\}$-sample	$\boldsymbol{m}_1, \boldsymbol{m}_2, \boldsymbol{n}_W$ and $\boldsymbol{n}_S$: mutual orientation**	$\boldsymbol{m}_1$, $\boldsymbol{m}_2$ and $\boldsymbol{n}_W$: mutual orientation	Symmetry elements***	MSC symbol
1	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_\Sigma = b_\Sigma = a_\Delta = 0$	$(1, 2_2, \bar{2}_1, \bar{2}_n) \times (1, \bar{1})$	mmm
2	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_\Delta = b_\Delta = a_\Sigma = 0$ or $a_\Sigma = b_\Sigma = a_\Delta = 0$	$1, \bar{2}'_1, \bar{2}_2, 2'_n$	$mm'2'$
3	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_\Sigma = b_\Sigma = a_\Delta = 0$	$1, 2_2, \bar{2}_1, \bar{2}_n$	$mm2$
4	$\{nml\}^*$	$a_1 = 0$ or $b_1 = 0$	$a_\Sigma = b_\Sigma = a_\Delta = 0$	$1,\ \bar{1}', 2'_1, \bar{2}_1$	$2'/m$
5	$\{nml\}^*$	$a_n = 0$ or $b_n = 0$	$a_\Sigma = b_\Sigma = a_\Delta = 0$	$1,\ \bar{1}', 2'_n, \bar{2}_n$	$2'/m$
6	$\{nml\}^*$	$a_2 = 0$ or $b_2 = 0$	$a_\Sigma = a_\Delta = a_C = 0$	$1, \bar{2}_2$	m
7	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_\Sigma = a_\Delta = 0$	$1,\ 2'_1, 2_2, 2'_n$	$22'2'$
8	$\{nml\}^*$	$a_n = 0$ or $b_n = 0$	$a_\Sigma = a_\Delta = 0$	$1,\ 2'_n$	$2'$
9	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_C = a_\Delta = b_\Sigma = 0$	$1,\ 2'_1, \bar{2}'_2, \bar{2}_n$	$mm'2'$
10	$\{nml\}^*$	$a_1 = 0$ or $b_1 = 0$	$a_\Delta = 0$	$1,\ 2'_1$	$2'$
11	$\{nml\}^*$	$a_n = 0$ or $b_n = 0$	$a_C = a_\Delta = b_\Sigma = 0$	$1,\ \bar{2}_n$	m
12	$\{nml\}^*$	$a_1 = 0$ or $b_1 = 0$	$a_C = 0$	$1,\ \bar{2}'_1$	m'
13	$\{nml\}^*$	$a_2 = 0$ or $b_2 = 0$	$a_\Sigma = 0$	$1,\ 2_2$	2
14	$\{nml\}^*$	$a_2 = 0$ or $b_2 = 0$	$a_\Sigma = b_\Sigma = 0$	$1,\ \bar{1}, 2'_2, \bar{2}'_2$	$2'/m'$
15	arbitrary	arbitrary	$a_\Sigma = b_\Sigma = 0$	$1,\ \bar{1}'$	$\bar{1}'$
16	arbitrary	arbitrary	arbitrary	1	1
17	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_C = a_\Sigma = b_\Delta = 0$	$1,\ \bar{2}'_1, 2_2, \bar{2}'_n$	$m'm'2$
18	$\{nml\}^*$	$a_n = 0$ or $b_n = 0$	$a_C = a_\Sigma = b_\Delta = 0$	$1,\ \bar{2}'_n$	m'
19	$\{nml\}^*$	$a_n = 0$ or $b_n = 0$	$b_\Delta = b_\Sigma = 0$	$1,\ 2_n$	2
20	$\{nml\}^*$	$a_n = 0$ or $b_n = 0$	$a_\Sigma = b_\Sigma = b_\Delta = 0$	$1,\ \bar{1}', 2_n, \bar{2}'_n$	$2/m$
21	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_\Sigma = b_\Sigma = b_\Delta = 0$	$1,\ 2_1, 2_2, 2_n$	222
22	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$b_\Delta = b_\Sigma = 0$	$1,\ \bar{2}'_1, \bar{2}'_2, 2_n$	$m'm'2$
23	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_\Sigma = b_\Sigma = b_\Delta = 0$	$(1, 2_1, 2_2, 2_n) \times (1, \bar{1}')$	$m'm'm'$
24	{111}	$b_n = 0$	$b_\Delta = b_\Sigma = 0$	3_n	3
26	{111}	$b_n = 0$	$b_\Delta = b_\Sigma = 0$	$3_n, \bar{2}'_1$	$3m'$
27	{111}	$b_n = 0$	$a_\Sigma = b_\Sigma = b_\Delta = 0$	$3_n, 2_1$	32
29	{111}	$b_n = 0$	$a_\Sigma = b_\Sigma = b_\Delta = 0$	$\bar{3}'_n, \bar{2}'_1$	$\bar{3}'m'$
30	{100}	$b_n = 0$	$b_\Delta = b_\Sigma = 0$	4_n	4
31	{100}	$b_n = 0$	$a_\Sigma = b_\Sigma = b_\Delta = 0$	$4_n, \bar{2}'_n$	$4/m'$
32	{100}	$b_n = 0$	$b_\Delta = b_\Sigma = 0$	$4_n, \bar{2}'_1$	$4m'm'$
33	{100}	$b_n = 0$	$a_\Sigma = b_\Sigma = b_\Delta = 0$	$4_n, 2_1$	422
34	{100}	$b_n = 0$	$a_\Sigma = b_\Sigma = b_\Delta = 0$	$4_n, \bar{2}'_1, \bar{2}'_n$	$4/m'm'm'$
35	{100}	$b_n = 0$	$a_\Sigma = b_\Sigma = b_\Delta = 0$	$\bar{4}'_n$	$\bar{4}'$
36	{100}	$b_n = 0$	$a_\Sigma = b_\Sigma = b_\Delta = 0$	$\bar{4}'_n, 2_1$	$\bar{4}'2m'$
42	{111}	$b_n = 0$	$a_\Sigma = b_\Sigma = b_\Delta = 0$	$\bar{3}'_n$	$\bar{3}'$

TABLE (continued

k	$\{nml\}$-sample	$\boldsymbol{m}_1, \boldsymbol{m}_2, \boldsymbol{n}_{\rm W}$ and $\boldsymbol{n}_S$: mutual orientation.	$\boldsymbol{m}_1$, $\boldsymbol{m}_2$ and $\boldsymbol{n}_{\rm W}$: mutual orientation	Symmetry elements	MSC symbol
43	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_\Delta = b_\Delta = a_\Sigma = 0$	$(1, 2_1', \bar{2}_2, \bar{2}_n') \times (1, \bar{1})$	$mm'm'$
44	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_\Delta = b_\Delta = a_\Sigma = 0$	$1, 2_1', \bar{2}_2, \bar{2}_n'$	$mm'2'$
45	$\{nml\}^*$	$a_2 = 0$ or $b_2 = 0$	$a_\Delta = b_\Delta = a_\Sigma = 0$	$1,\ \bar{1}, 2_2, \bar{2}_2$	$2/m$
46	$\{nml\}^*$	$a_n = 0$ or $b_n = 0$	$a_\Delta = b_\Delta = a_\Sigma = 0$	$1,\ \bar{1}, 2_n', \bar{2}_n'$	$2'/m'$
47	$\{nml\}^*$	$a_1 = 0$ or $b_1 = 0$	$a_\Delta = b_\Delta = 0$	$1,\ \bar{1}, 2_1', \bar{2}_1'$	$2'/m'$
48	arbitrary	arbitrary	$a_\Delta = b_\Delta = 0$	$1,\ \bar{1}$	$\bar{1}$
49	$\{nml\}^*$	$a_n = 0$ or $b_n = 0$	$a_\Delta = b_\Delta = b_\Sigma = 0$	$1,\ \bar{1}, 2_n, \bar{2}_n$	$2/m$
50	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_\Delta = b_\Delta = b_\Sigma = 0$	$1,\ 2_1', 2_2', 2_n$	$22'2'$
51	{100}, {110}	$b_n = 0$ or $b_1 = 0$	$a_\Delta = b_\Delta = b_\Sigma = 0$	$(1, 2_1', 2_2', 2_n) \times (1, \bar{1})$	$mm'm'$
53	{111}	$b_n = 0$	$a_\Delta = b_\Delta = b_\Sigma = 0$	$3_n, 2_1'$	$32'$
55	{111}	$b_n = 0$	$a_\Delta = b_\Delta = b_\Sigma = 0$	$\bar{3}_n, \bar{2}_1'$	$\bar{3}m'$
56	{100}	$b_n = 0$	$a_\Delta = b_\Delta = b_\Sigma = 0$	$4_n, \bar{2}_n$	$4/m$
57	{100}	$b_n = 0$	$a_\Delta = b_\Delta = b_\Sigma = 0$	$4_n, 2_1'$	$42'2'$
58	{100}	$b_n = 0$	$a_\Delta = b_\Delta = b_\Sigma = 0$	$4_n, \bar{2}_1', \bar{2}_n$	$4/mm'm'$
59	{100}	$b_n = 0$	$a_\Delta = b_\Delta = b_\Sigma = 0$	$\bar{4}_n$	$\bar{4}$
60	{100}	$b_n = 0$	$a_\Delta = b_\Delta = b_\Sigma = 0$	$\bar{4}_n, 2_1'$	$\bar{4}2'm'$
64	{111}	$b_n = 0$	$a_\Delta = b_\Delta = b_\Sigma = 0$	$\bar{3}_n$	$\bar{3}$

* (nml)-plates with arbitrary Miller indexes except non-zero values $|n| \neq |m| \neq |l| \neq |n|$.

** At $(\boldsymbol{n}_{\rm W}\boldsymbol{a}_1) = (\boldsymbol{n}_{\rm W}\boldsymbol{a}_2) = 0$.

*** The possible symmetry elements are rotations around twofold symmetry axes 2_n, $2_n'$ or 2_1, $2_1'$ or else 2_2, $2_2'$ that are collinear with the unit vectors $\boldsymbol{n}_{\rm W}$ or $\boldsymbol{a}_1$ or else $\boldsymbol{a}_2$, respectively, reflections in planes $\bar{2}_n$, $\bar{2}_n'$ or $\bar{2}_1$, $\bar{2}_1'$ or else $\bar{2}_2$, $\bar{2}_2'$ that are normal to the above mentioned vectors, respectively, rotations around three-, four-fold symmetry axes 3_n, 4_n that are collinear with the vector $\boldsymbol{n}_{\rm W}$, rotations around three-, four-fold inversion symmetry axes $\bar{3}_n, \bar{3}_n', \bar{4}_n, \bar{4}_n'$ that are collinear with the vector $\boldsymbol{n}_{\rm W}$, inversion in the symmetry center $\bar{1}$, $\bar{1}'$ and identity 1. Here an accent at symmetry elements means a simultaneous use of the time reversal operation [6].

General enumeration of MSCs of 0°-DWs contains MSCs with: $k = 2$, $6 \le k \le 13$, $16 \le k \le 19$, $k = 22$, 24, 26, 30, 32; $43 \le k \le 51$, $k = 53$; $55 \le k \le 60$ and $k = 64$. The 60°- and 120°-DWs are represented by MSCs with $k = 10$, 16, 18 and $k = 11$, 13, 16, respectively. The MSCs of the 70.5°-, 90°- (both for $\langle 100\rangle$ and $\langle 110\rangle$ like easy magnetization axis [5]) and 109.5°-DWs are the MSCs with $7 < k < 13$, $16 < k < 18$. The general list of 180°-DWs includes MSCs with $1 \le k \le 42$, except for $k = 25$, 28, 37–41.

3. Conclusions

The complete collection of (nml)-plates with all possible orientations includes the full list of MSCs of 2α-DWs in cubic $m\bar{3}m$ crystal. For separate (nml)-plates with fixed combination of the Miller indexes this list is limited. Such limitation depends on plate orientation. It is minimal and maximal for the samples with high-symmetry (such as (100)-, (110)- or (111)-plates) and low-symmetry (the (nml)-plates, where indexes are non-zero and have different absolute values) developed surface, respectively. Maximal quantity of MSCs of 2α-DWs is for (100)-plates. The MSC with $k = 16$ is the MSC of all above-mentioned 2α-DWs in arbitrary oriented plate of the cubic $m\bar{3}m$ crystal.

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