Proceedings of the XXXVIII International School and Conference on the Physics of Semiconductors "Jaszowiec" 2009

Magnetoexcitons in Semiconductor Quantum Rings with Complicated (Kane's) Dispersion Law

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The influence of the nonparabolicity of charge carriers dispersion law (Kane's dispersion) on a magnetoexciton energy spectrum in InSb quantum rings is theoretically investigated. The analytical expression for the energy spectrum of exciton in a narrow-gap semiconductor nanoring in a magnetic field is obtained. The Aharonov–Bohm oscillations in the energy of excited states are studied.

PACS numbers: 73.21.−b, 71.35.Ji

1. Introduction

Recent advances in nanoscopic fabrication techniques have made it possible to grow self-assembled InAs nanorings [1]. Today's semiconductor nanorings can be viewed as promising candidates for application in nanoelectronics. Two of the most interesting properties of these scattering-free quantum rings are their response to external magnetic fields and their multi-particle excitation spectra. The additional non-simply connected geometry of nanorings is of great interest. The physics of the flux sensitivity of an electron on the ring is its charge, which couples to the vector potential. Correspondingly, the coupling to the flux has opposite signs for the electron and the hole. For this reason, an exciton being a bound state of the electron and the hole as a neutral entity should not be sensitive to the flux. However due to finite size of the exciton, such a sensitivity will emerge. One of the manifestation of the Aharonov–Bohm effect (ABE) in ring geometry is a periodicity of exciton energy as a function of magnetic flux Φ through the ring with the period of oscillations $\Phi_0 = hc/e$. Magnetoexcitons in quantum rings (QRs) and antidotes with parabolic dispersion are investigated in Ref. [2]. The energetics of two oppositely charged particles in an AB ring interacting through a contact potential was earlier investigated in Ref. [3] by using a Green function procedure.

The aim of this paper is to investigate the possibility of the ABE manifestation for exciton energy spectrum in nanorings with complicated (Kane's) dispersion law.

2. Theory

It is well known that the one-dimensional nonrelativistic Coulomb energy is infinite in its ground state [4] and it is logarithmically deep in a strong magnetic field [5]. At the same time, the dispersion law in semiconductors, which are widely used for fabrication of nanostructures, is essentially nonparabolic and in the two-band approximation of Kane's dispersion law (which is valid for InSb and InAs) is similar to the relativistic one [6]. The account of the nonparabolicity of the dispersion law strongly affects the energy of impurity states in quantum well wires [7]. The aim of this work is to investigate analytically the energetics of an exciton in a QR with nonparabolic (Kane's) dispersion law in a perpendicular magnetic field in ABE setting.

We consider two oppositely charged particles of equal masses moving along a circular one-dimensional ring of radius R, which is threaded by a magnetic flux Φ. The Wannier equation for particles interacting through Coulomb interaction is

$$\left[E_{\rm c}\left(\mathrm{i}\nabla_{\rm e}\right) - E_{\rm v}\left(i\nabla_{\rm e}\right) - \frac{e^2}{\varepsilon r}\right]\psi\left(r_{\rm e}, r_{\rm h}\right) = E\psi\left(r_{\rm e}, r_{\rm h}\right), \tag{1}$$

where the symbol c refers to the electron, v — to the hole.

It is well known that in narrow-gap A^3B^5 semiconductors the Kane dispersion law is realized

$$\varepsilon_{\rm c,v}(k) = -mc^2 \pm \sqrt{m^2c^4 + \hbar^2k^2c^2}, \tag{2}$$

where the conduction band bottom is taken at the origin; the parameter $c = (\varepsilon_{\rm g}/2m)^{1/2} \approx 10^6$ m/s is proportional to the matrix element of bands interaction and plays the role of the light velocity c_0; $\varepsilon_{\rm g}$ is the forbidden band width, m is the effective mass of the electron on the conduction band bottom.

The position of each particle on a ring is described by angular variables φ_1 and φ_2. In terms of center of mass $\Phi_{\rm c} = (\varphi + \varphi_2)/2$ and relative $\varphi = \varphi - \varphi_2$ variables, we have

$$\frac{\partial}{\partial\varphi_1} = \frac{\partial}{\partial\varphi} + \frac{1}{2}\frac{\partial}{\partial\Phi_{\rm c}}, \quad \frac{\partial}{\partial\varphi_2} = -\frac{\partial}{\partial\varphi} + \frac{1}{2}\frac{\partial}{\partial\Phi_{\rm c}}. \tag{3}$$

In the more interesting case for the optical absorption, when $\Phi_{\rm c} = 0$, the energy of the 1D magnetoexciton in

a QR is found from the Klein–Gordon equation for the pair of the interacting particles with the equal effective masses (in this case the half flux quantum $\Phi_0/2 = h/2ec_0$ corresponds to such pair):

$$\left[\left(2\mathrm{i}\frac{\hbar}{R}\frac{\partial}{\partial\varphi}+\frac{\Phi}{\Phi_0/2}\right)^2 c^2+\varepsilon_{\mathrm{g}}^2\right]\psi(\varphi) = \left(E+\frac{e^2}{2R\varepsilon\,|\sin(\varphi/2)|}\right)^2\psi(\varphi), \tag{4}$$

where the magnetic flux Φ through the ring is connected with the vector potential of the magnetic field by the relation $A = \Phi/2\pi R$ ($\boldsymbol{A}$ is a vector tangential to every point of the ring); ε is the dielectric constant. Following the standard approach [2, 3] we initially set

$$\psi(\varphi) = \exp(\mathrm{i}f\varphi)\chi(\varphi), \tag{5}$$

where $f = \Phi/\Phi_0$, so that $\chi(\phi)$ satisfies the equation

$$4\left[-\frac{\hbar^2c^2}{R^2}\frac{\partial^2}{\partial\varphi^2}+m^2c^4\right]\chi(\varphi) = \left(E+\frac{e^2}{2R\varepsilon\,|\sin(\varphi/2)|}\right)^2\chi(\varphi). \tag{6}$$

To determine the allowed solutions of Eq. (6) we have to take into account that the total wave function is independently periodic in φ_1 and φ_2 with period 2π, which coincides with the period of the Coulomb potential $U(\varphi) = e^2/2R\varepsilon|\sin(\varphi/2)|$. Because of the periodicity of $U(\varphi)$ Eq. (6) has formally the Bloch-type solutions [2, 3]:

$$\chi(\varphi) = \exp(\mathrm{i}q\varphi)u(\varphi), \quad -\frac{1}{2} < q \le \frac{1}{2}, \tag{7}$$

where $u(\varphi)$ is periodic function of φ with the same period 2π. For a vanishing center of mass angular momentum we find that

$$f + q = n_1, \quad -f - q = n_2, \tag{8}$$

where n_1 and n_2 are arbitrary integers, and

$$q = n/2 - f, \quad n = n_1 - n_2. \tag{9}$$

In the view of the excitonic state, the potential should be strong enough to bind the electron–hole pair, and then the relative azimutal motion is strongly localized in each potential well. The wave function in a small vicinity to the point $\varphi \approx 0$ (as well as at $\varphi \approx \pm 2\pi, \pm 4\pi$) satisfies the equation

$$\frac{\partial^2\chi(\varphi)}{\partial\varphi^2}+\frac{R^2}{\hbar^2c^2}\Big(\frac{E^2}{4}+\frac{e^2E}{2\varepsilon R|\varphi|} +\frac{e^4}{4\varepsilon^2R^2\varphi^2}-m^2c^4\Big)\chi(\varphi) = 0. \tag{10}$$

Introducing the dimensionless variables

$$\eta = \frac{R}{\hbar c}\left(m^2c^4-\frac{E^2}{4}\right)^{1/2}, \quad \rho = 2\eta|\varphi|, \quad \lambda = \frac{e^2}{\varepsilon\hbar c}E\left(m^2c^4-\frac{E^2}{4}\right)^{-1/2}, \quad \alpha = \frac{e^2}{\varepsilon\hbar c}, \tag{11}$$

where α is the analog of the fine structure constant, we find

$$\left(\frac{\partial^2}{\partial\rho^2}-\frac{1}{4}+\frac{\lambda}{4\rho}+\frac{\alpha^2}{4\rho^2}\right)\chi(\rho) = 0. \tag{12}$$

Using the ansatz $\chi(\rho) = C\rho^s f(\rho)\exp(-\rho/2)$ we obtain the following equation for $f(\rho)$:

$$\rho\frac{\partial^2 f}{\partial\rho^2}+(2s-\rho)\frac{\partial f}{\partial\rho}-\left(s-\frac{\lambda}{4}\right)f = 0, \tag{13}$$

where $s_{1,2} = (1 \pm \sqrt{1-\alpha^2})/2$. Equation (13) has the general solution

$$f(\rho) = C_1F\left(s-\frac{\lambda}{4}, 2s, \rho\right) + C_2F\left(1-s-\frac{\lambda}{4}, 2-2s, \rho\right)\rho^{1-2s}, \tag{14}$$

where $F(a, b, z)$ is the confluent hypergeometric function [8], C_1 and C_2 are arbitrary constants. The infinite power series expansion will be terminated for $F(s-\lambda/4, 2s, \rho)$, if $\lambda = 4(n+s)$ and it will be terminated for $F(1-s-\lambda/4, 2-2s, \rho)$, if $\lambda = 4(n+1-s)$. Let us note that these two solutions are not linearly independent: the first solution for the small values of $s = s_1 \approx \alpha^2/4$ is identical to the second for larger value of $s_2 \approx 1 - \alpha^2/4$. Therefore we can, as in [9], take $C_2 = 0$ and use the first solution of $\chi(\rho)$ with two values of s. Using the value $\lambda = 4(n + s)$ we find the expression for the energy of the "relativistic" exciton

$$E_n = \varepsilon_{\mathrm{g}}\frac{2\,(n+s)}{\sqrt{4(n+s)^2+\alpha^2}}. \tag{15}$$

Let us note that the electron–hole pair is localized now at the distance equal to the effective Compton radius $a_{\mathrm{C}}^* = \alpha a_{\mathrm{B}}^*$ with a_{B}^* being the effective Bohr radius.

The normalization constant $C = C_1$ for the region $\varphi \ge 0$ (as well as at $\varphi \ge \pm 2\pi, \pm 4\pi \ldots$); $C = C_1$ and $C = -C_1$ for even and odd solutions respectively in the region $\varphi < 0$ (as well as at $\varphi < \pm 2\pi, \pm 4\pi \ldots$).

In the tight-binding approximation any solution of the type (7) corresponds to the energy [2, 10]

$$\tilde{E} = E_n - t_{0n} - 2t_{1n}\cos\left(2\pi\frac{\Phi}{\Phi_0}+\pi\nu\right), \quad \nu = 0, 1, 2\ldots, \tag{16}$$

$$t_{0n} = \int_{-\infty}^{+\infty}\mathrm{d}\varphi\chi_{n0}(\varphi)\left(\frac{e^2}{2\varepsilon R\,|\sin\varphi/2|}-\frac{e^2}{\varepsilon R|\varphi|}\right)\times\chi_{n0}(\varphi), \tag{17}$$

$$t_{1n}^{\theta} = \int_{-\infty}^{+\infty}\mathrm{d}\varphi\chi_{n0}(\varphi)\left(\frac{e^2}{2\varepsilon R\,|\sin\varphi/2|}-\frac{e^2}{\varepsilon R|\varphi|}\right)\times\chi_{n\theta}\left(\varphi-2\pi\right), \tag{18}$$

where the value of the index $\theta = 0$ describes the even solution of the wave function and $\theta = 1$ describes the odd solution.

3. Results and discussion

It is known that in the case of the Coulomb interaction in nanoring with parabolic dispersion, there is no ground excitonic state due to intrinsic divergence of the ground state energy of 1D exciton [4]. Taking into account the

realistic nonparabolic dispersion law given by Eq.(2) we found that the ground state of 1D exciton exists in InSb nanoring and we obtained the energy value $E_0 = 15.41R^*$ (R^* is the effective Rydberg energy).

In order to access measurable ABE, the bound electron and hole should have a possibility to tunnel in the opposite directions and meet each other on the opposite side of the nanoring ($\phi = \pi$). Because of the strong localization of electron and hole at the distance equal to the effective Compton radius $a_{\rm C}^* = \alpha a_{\rm B}^*$ we do not observe AB oscillations for the ground state. We have already found the AB oscillations in the energy of first excited states (Fig. 1).

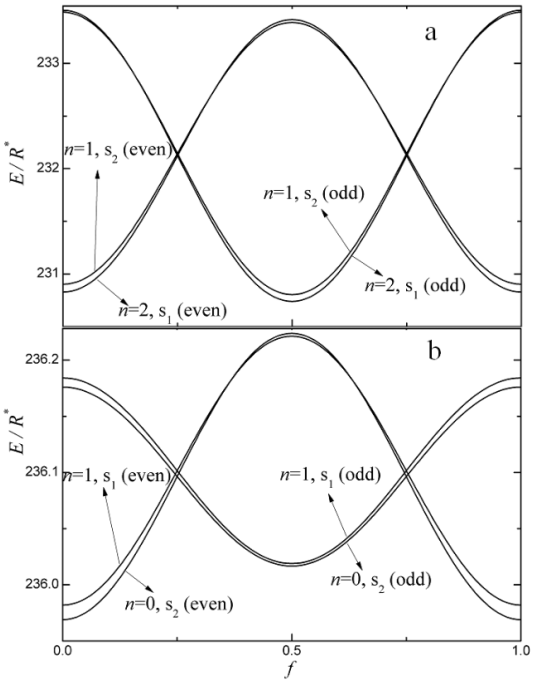

Fig. 1. The oscillations of the exciton excited energy levels in InSb nanoring with the radius $R = 2a_{\rm B}^*$ as a function of the magnetic flux with the fundamental period hc/e: (a) for first (second) even and odd excited states with $n = 1$, $s = s_2$ ($n = 2$, $s = s_1$); (b) for third (fourth) even and odd excited states with $n = 0$, $s = s_2$ ($n = 1$, $s = s_1$).

The energies and lengths are given in the units of effective Rydberg energy R^* and Bohr radius $a_{\rm B}^*$. For our calculations we use material parameters of the InSb: $m_{\rm e} \approx m_{\rm lh} = 0.014m_0$, $\varepsilon = 16.8$, $\varepsilon_{\rm g} = 0.17$ eV [11]; for these parameters $c = 10^8$ cm/s, $R^* = 0.68$ meV, $a_{\rm B}^* = 631$ Å, $\alpha = 0.13$ and $a_{\rm C}^* \approx 82$ Å. Figure 1 shows the dependence of first four (even and odd) excited states energies of 1D magnetoexciton in InSb nanoring on magnetic flux.

The dependence of the excited energies of 1D magnetoexciton on the ring radius is given in Fig. 2 for vanishing value of the flux. In the limiting case $R \to \infty$ the curves tend to the corresponding curves for 1D "relativistic" exciton in InSb quantum wire.

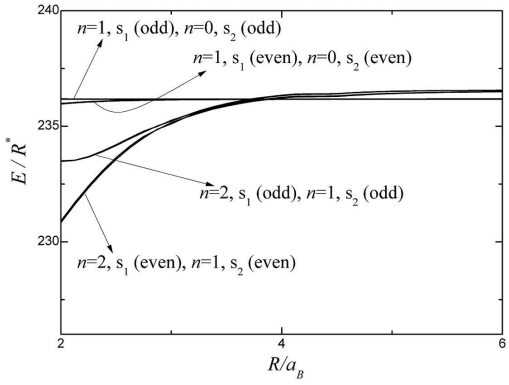

Fig. 2. The dependence of the excited energies of 1D magnetoexciton in a InSb nanoring on the ring radius for $f = 0$.

The theory of trembling motion (Zitterbewegung ZB) of charge carriers in various narrow-gap materials is reviewed in [12] and a similarity of ZB in such materials is mentioned. The analogy between ABE manifestation in single exciton spectrum in QRs of various narrow-gap materials such as InSb-type semiconductors, bilayer graphene (with the gap induced by electric field) [13] etc. is expected.

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