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The Analytical Calculation of Temperature-Dependent Propagation Constant, *V* No. and Waveguide Dispersion in Super-Gaussian Diode-Pumped Fiber Lasers

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In this paper, the dependence of some important parameters in fiber lasers such as propagation constant, normalized frequency or V No. and waveguide dispersion with temperature has been investigated analytically. We showed that the thermal effects cannot be neglected in the above parameters especially in middle and high power regime.

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1. Introduction

Fiber lasers are under considerable attention due to their high efficiency and good beam qualities [1–6]. In high power regimes, the heat generation due to pump beam [7] can influence the laser operation and beam propagation characteristics. The heat deposition in end--pumped fiber lasers changes the step-index fiber to graded-index, so the propagation parameters like V numbers, propagation constant and dispersion will be varied. In this paper, we consider a fiber laser pumped by a super-Gaussian profile [8] diode laser which is more realistic profile for high-power and multimode diode lasers with respect to top hat profile. After solving the heat conductive equation, the variation of such parameters for end-pumped Yb-doped fiber laser has been studied.

2. The analytical model

To investigate the variation of propagation parameters with input power, the temperature distribution in the fiber must be calculated. We begin by finding the radial temperature distribution of a fiber whose core and cladding radii are a and b, respectively. The steady state heat equation can be written as

$$\nabla^2 T = -\frac{Q}{k}, \tag{1}$$

where k and $Q = Q_0 \exp(-2r^4/\omega_0^4)$ are the medium thermal conductivity and heat power density of the source, respectively. The length of fiber is much greater than the diameter, and so is the z derivative. Using the following boundary conditions including the continuity of the temperature distribution and radial derivatives on the boundary means $T_1|_{r=a} = T_2|_{r=a}$ and $\frac{\mathrm{d}T_1}{\mathrm{d}r}|_{r=a} = \frac{\mathrm{d}T_2}{\mathrm{d}r}|_{r=a}$. Moreover, the Newtonians boundary condition on the surface is $\frac{\mathrm{d}T_2}{\mathrm{d}r}|_{r=b} = \frac{h}{k}(T_\mathrm{C} - T|_{r=b})$. The temperature distribution in the core, $T_1(r)$, and outside, $T_2(r)$ will be [8]:

$$T_1(r) = -\frac{Q}{4k}\,{}_2F_2\left(\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{3}{2};-\frac{r^4}{\omega_0^4}\right) r^2 + T_0,\quad 0 \le r \le a, \tag{2}$$

$$\begin{aligned}T_2(r) = T_\mathrm{C} &- \frac{Q}{4k}a^2\left[2\,{}_2F_2\left(\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{3}{2};-\frac{a^4}{\omega_0^4}\right) - \frac{4}{9}\left(\frac{a}{\omega_0}\right)^4 {}_2F_2\left(\frac{3}{2},\frac{3}{2},\frac{5}{2},\frac{5}{2};-\frac{a^4}{\omega_0^4}\right)\right]\\ &\times\left[\frac{k}{bh} + \ln\left(\frac{b}{a}\right)\right] - \frac{Q}{4k}a^2\left[2\,{}_2F_2\left(\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{3}{2};-\frac{a^4}{\omega_0^4}\right) - \frac{4}{9}\left(\frac{a}{\omega_0}\right)^4 {}_2F_2\left(\frac{3}{2},\frac{3}{2},\frac{5}{2},\frac{5}{2};-\frac{a^4}{\omega_0^4}\right)\right]\\ &\times \ln\left(\frac{r}{a}\right),\quad a \le r \le b,\end{aligned} \tag{3}$$

where T_0 is the temperature of the center and is described as

$$\begin{aligned}T_0 = {}&\frac{Q}{9k}a^2\left[\frac{k}{bh} + \ln\left(\frac{b}{a}\right)\right]\left(\frac{a}{\omega_0}\right)^2 \times {}_2F_2\left(\frac{3}{2},\frac{3}{2},\frac{5}{2},\frac{5}{2};-\frac{a^4}{\omega_0^4}\right)\\ &+\frac{Q}{4k}a^2\,{}_2F_2\left(\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{3}{2};-\frac{a^4}{\omega_0^4}\right) \times\left[1 + 2\left(\frac{k}{bh} + \ln\left(\frac{b}{a}\right)\right)\right] + T_\mathrm{C}.\end{aligned} \tag{4}$$

T_C is the coolant temperature, h is the heat convection

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coefficient and ${}_kF_l(a_1, a_2, \ldots a_k, b_1, b_2, \ldots b_l; x)$ is the hypergeometric function defined by [9]:

$$ {}_kF_l\left(a_1, a_2, \ldots a_k, b_1, b_2, \ldots b_l; x\right) = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \ldots (a_k)_n}{(b_1)_n (b_2)_n \ldots (b_l)_n} \frac{x^n}{n!}. \tag{5} $$

The temperature-dependent change of refractive index then can be calculated as [10]:

$$ \Delta n(r) = \left[T(r) - T_{\mathrm{C}}\right] \frac{\mathrm{d}n}{\mathrm{d}t}. \tag{6} $$

By substituting Eq. (2) into (6) we see the radial dependence of refractive index. Therefore, the heat inducting will change the step-index fiber to graded-index. This change of refractive index influences the fiber parameters. The normalized frequency or V number is [11]:

$$ V = \frac{2\pi a}{\lambda} n_1 \sqrt{2(n_1 - n_2)}, \tag{7} $$

where n_1 and n_2 are the maximum core and cladding refractive indices, respectively. By the derivation of (6), the variation of V No. can be found as

$$ \Delta V = \frac{2\pi a}{\lambda} \Big[\Delta n_1 (2(n_1 - n_2))^{\frac{1}{2}} + n_1 (\Delta n_1 - \Delta n_2)(2(n_1 - n_2))^{\frac{3}{2}}\Big], \tag{8} $$

where

$$ \Delta n_1 = \frac{\mathrm{d}n}{\mathrm{d}t} (T_0 - T_{\mathrm{C}}), \tag{9} $$

$$ \Delta n_2 = \frac{\mathrm{d}n}{\mathrm{d}t} \left[T(a) - T_{\mathrm{C}}\right]. \tag{10} $$

In graded-index fiber, the propagation constant, β, and wave guide dispersion, W_{D}, are as follows [11]:

$$ \beta = \left(\frac{V^2}{2a^2\delta} - \frac{6V}{a^2}\right)^{\frac{1}{2}}, \tag{11} $$

$$ W_{\mathrm{D}} = -\frac{V^2}{2\pi c} \frac{\mathrm{d}^2\beta}{\mathrm{d}V^2}, \tag{12} $$

where $\delta = -\frac{n_1 - n_2}{n_1}$.

So, according to the dependence of V No. on the temperature, the waveguide dispersion and propagation constant will be temperature dependent.

3. Results and discussion

The variations of V No., β and W_{D} with input power are discussed here. The following parameters are used in the calculations:

$$ a = 4.6\ \mu\mathrm{m}, \quad b = 400\ \mu\mathrm{m}, \quad \lambda = 1.06\ \mu\mathrm{m}, $$

$$ k = 0.0138\ \frac{\mathrm{W}}{\mathrm{cm\ K}}, \quad h = 0.001\ \frac{\mathrm{W}}{\mathrm{cm^2\ K}}, $$

$$ T_{\mathrm{C}} = 300\ \mathrm{K} \quad \text{and} \quad \frac{\mathrm{d}n}{\mathrm{d}T} = 10^{-6\circ}\mathrm{C}^{-1}. $$

Figure 1 shows the variation of V No. with input power. As seen from this figure, the thermal effect due to the pump power can change the mode propagation of the fiber and, approximately, for $P > 380$ W, $V > 2.405$.

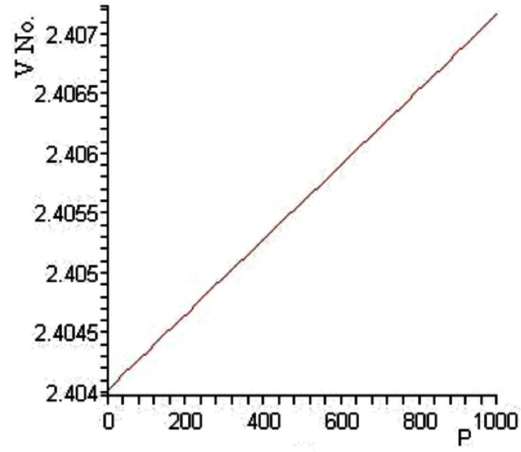

Fig. 1. The variation of V no. with input power.

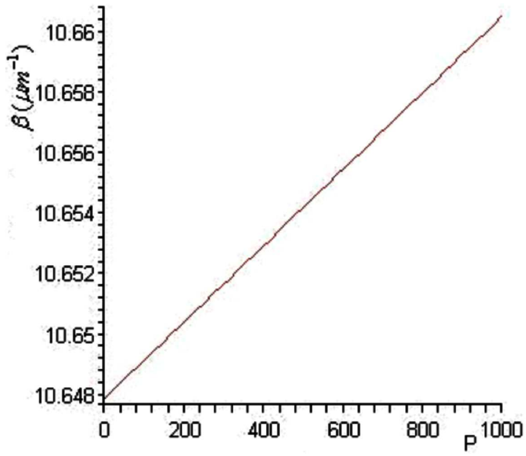

Fig. 2. The variation of propagation constant with input power.

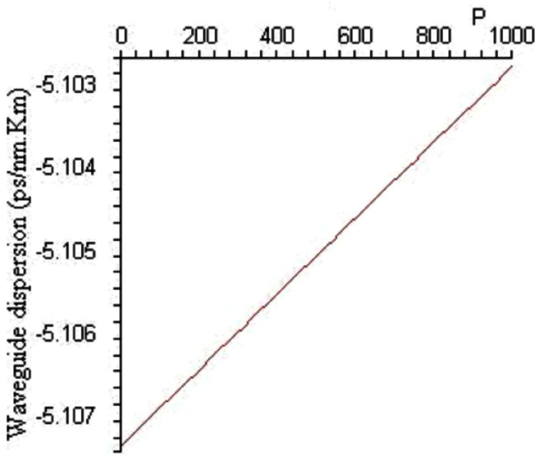

Fig. 3. The variation of waveguide dispersion versus input power.

The variation of propagation constant with input power is plotted in Fig. 2. The propagation constant increases by increasing the input power.

In Fig. 3, the waveguide dispersion versus input power is plotted. As can be seen, the waveguide dispersion increases with input power.

4. Conclusion

The variation of some important parameters in fibers, like V no., propagation constant and the waveguide dispersion with input power has been discussed. We have shown that, in middle and high-power regimes, these variations will be important and cannot be neglected in designing of such lasers.

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