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The Analytical Solution of Rate Equations in End-Pumped Fiber Lasers with Minimum Approximation and Temperature Distribution during the Laser Operation

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In this paper, as the first the rate equations in end-pumped fiber laser have been solved analytically with minimum approximation and the output power versus input power has been derived. Then, the heat conduction equation in the fiber which is in operation has been solved and the temperature distribution has been derived. The results were applied for a Yb-doped fiber laser and discussed.

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1. Introduction

High-power fiber lasers have many applications in medicine, military, industry, etc. due to their advantages. They have high beam quality and high efficiency with respect to the conventional solid-state lasers [1–8]. In high-power regime, thermal effects influence the laser operation and cannot be neglected [9]. The thermal effects will produce the poor beam quality and knowledge of them will be important in laser design.

In this paper, as the first, we analytically solved the rate equations in single end-pumped Yb-doped fiber laser. To solve the rate equations, we considered the minimum approximation, this means taking into account the upper population density of fiber and absorption cross-section, which have been ignored in the literature [10]. So, we improved the analytical rate equation solutions and found the output laser power versus input parameters. By using the output power of the laser, we solved the heat conduction equation in the laser during operation. In this way, we considered heat deposited in the fiber due to pump and laser powers and solved the heat equation. The temperature distribution during the operation has been obtained and compared with temperature distribution in turn-off. The results were applied for Yb-doped fiber lasers and the plots have been discussed.

2. Rate equations

The steady-state rate equations in an end-pumped fiber, illustrated in Fig. 1, are as follows [10]:

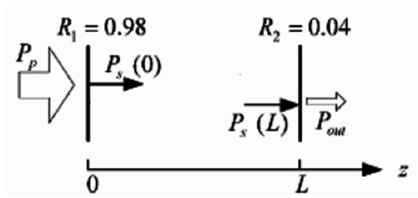

Fig. 1. Schematic illustration of Yb-doped end-pumped fiber laser.

$$\frac{N_2(z)}{N} = \frac{\frac{P_p(z)\Gamma_p\sigma_{ap}}{h\nu_p A} + \frac{P_s(z)\Gamma_s\sigma_{as}}{h\nu_s A}}{\frac{P_p(z)\Gamma_p(\sigma_{ap}+\sigma_{ep})}{h\nu_p A} + \frac{1}{\tau} + \frac{P_s(z)\Gamma_s(\sigma_{as}+\sigma_{es})}{h\nu_s A}},$$
$$\frac{dP_p(z)}{dz} = \Gamma_p\big[(\sigma_{ap}+\sigma_{ep})N_2(z) - N\sigma_{ap}\big]P_p(z) - \alpha_p P_p(z),$$
$$\frac{dP_s(z)}{dz} = \Gamma_s(z)\big[(\sigma_{as}+\sigma_{es})N_2(z) - N\sigma_{as}\big]P_s(z) - \alpha_s P_s(z), \quad (1)$$

where $P_p(z)$ and $P_s(z)$ are the pump power and the laser (signal) power, respectively. N is Yb^{3+} dopant concentration in the core with a cross-section area A. Γ_p and Γ_s are the power filling factors. σ_{as} and σ_{es} are the absorption and emission cross-sections of laser, whereas σ_{ap} and σ_{ep} are absorption and emission cross-sections of the pump, respectively. υ_p and υ_s are the pump and the laser frequency, respectively, α_p and α_s are the fiber scattering loss for pump and laser, respectively, and τ is the spontaneous lifetime.

We consider K as a constant which is the average of $N_2(z)$:

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$$K = \frac{1}{L}\int_0^L N_2(z)\,\mathrm{d}z. \tag{2}$$

The gain of the fiber can be derived from (1) as follows:

$$G_\mathrm{s}(L) = \int_0^L \frac{\mathrm{d}P_\mathrm{s}(z)}{P_\mathrm{s}(z)} = \int_0^L \Gamma_\mathrm{s}(\sigma_\mathrm{as}+\sigma_\mathrm{es})N_2(z)\,\mathrm{d}z - \int_0^L (\Gamma_\mathrm{s}\sigma_\mathrm{as}N+\alpha_\mathrm{s})\,\mathrm{d}z, \tag{3}$$

where $G_\mathrm{s}(L)$ can be obtained from the threshold condition

$$R_1R_2\exp(2G_\mathrm{s}(L)) = 1. \tag{4}$$

Here R_1 and R_2 are the reflection coefficients of the mirrors. So, by using (2)–(4) we find

$$K = \frac{\frac{1}{L}\ln\left(\frac{1}{\sqrt{R_1R_2}}\right)+(\Gamma_\mathrm{s}\sigma_\mathrm{as}N+\alpha_\mathrm{s})}{\Gamma_\mathrm{s}(\sigma_\mathrm{as}+\sigma_\mathrm{es})}. \tag{5}$$

By analytically integrating the second equation in (1) and by using (5), the pump power will be

$$P_\mathrm{p}(z) = P_\mathrm{p}(0)\exp\big((\Gamma_\mathrm{p}(\sigma_\mathrm{ap}+\sigma_\mathrm{ep})K - (\Gamma_\mathrm{p}N\sigma_\mathrm{ap}+\alpha_\mathrm{p}))z\big). \tag{6}$$

In this equation, the first term is due to upper level population density which we considered in this literature. The third equation of (1) can be rewritten as

$$\left(\frac{\mathrm{d}P_\mathrm{s}(z)}{\mathrm{d}z}+\alpha_\mathrm{s}P_\mathrm{s}(z)\right)+\left(\frac{\nu_\mathrm{s}}{\nu_\mathrm{p}}\right)\left(\frac{\mathrm{d}P_\mathrm{p}(z)}{\mathrm{d}z}+\alpha_\mathrm{p}P_\mathrm{p}(z)\right)+\frac{h\nu_\mathrm{s}AK}{\tau} = 0. \tag{7}$$

Finally, by integrating the above equation and using (6) we obtain

$$P_\mathrm{s}(z) = -\frac{\left(\frac{\nu_\mathrm{s}}{\nu_\mathrm{p}}\right)P_\mathrm{p}(0)\left[\Gamma_\mathrm{p}\left((\sigma_\mathrm{ap}+\sigma_\mathrm{ep})K-N\sigma_\mathrm{ap}\right)\right]}{\Gamma_\mathrm{p}\left((\sigma_\mathrm{ap}+\sigma_\mathrm{ep})K-N\sigma_\mathrm{ap}\right)+(\alpha_\mathrm{s}-\alpha_\mathrm{p})} \times\left[\exp\left((\Gamma_\mathrm{p}\left((\sigma_\mathrm{ap}+\sigma_\mathrm{ep})K-N\sigma_\mathrm{ap}\right) - \alpha_\mathrm{p})z\right)-\exp(-\alpha_\mathrm{s}z)\right] + \frac{h\nu_\mathrm{s}AK}{\tau\alpha_\mathrm{s}}\left(\exp(-\alpha_\mathrm{s}z)-1\right) + (1-R_1)P_\mathrm{p}(0)\exp(-\alpha_\mathrm{s}z). \tag{8}$$

That in the above equation, the following boundary condition has been used

$$P_\mathrm{s}(0) = (1-R_1)P_\mathrm{p}(0), \tag{9}$$

where $P_\mathrm{s}(0)$ and $P_\mathrm{p}(0)$ are the laser power and the pump power in the beginning of the first mirror, respectively.

We have used the following parameters in the calculations [11]: $\lambda_\mathrm{p} = 920$ nm, $\lambda_l = 1090$ nm, $\tau = 1$ ms, $\sigma_\mathrm{ap} = 6\times10^{-21}$ cm^2, $\sigma_\mathrm{ep} = 2.5\times10^{-22}$ cm^2, $\sigma_\mathrm{es} = 2\times10^{-21}$ cm^2, $\sigma_\mathrm{as} = 1.4\times10^{-23}$ cm^2, $A = 5\times10^{-7}$ cm^2, $L = 50$ m, $P_\mathrm{p}(0) = 20$ W.

Figure 2 shows the variation of input pump and laser powers within the fiber. As seen from this figure, the input pump power reaches about 4 W at the end of fiber while the output power is about 10 W. Figure 3 shows the variation of laser power within the fiber for cases of scattering loss present and absent.

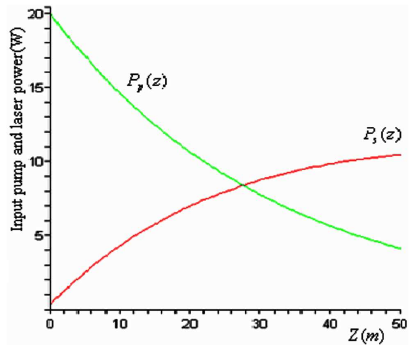

Fig. 2. Input and laser power versus z in Yb fiber laser.

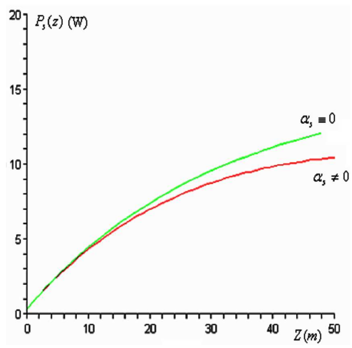

Fig. 3. Laser power versus z for cases of scattering loss present and absent.

3. Temperature distribution

Considering the calculated powers, we now calculate the temperature distribution in the fiber by solving the following heat conducting equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial I}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2} = -\frac{Q_\mathrm{th}}{K_\mathrm{c}}, \tag{10}$$

where $Q_\mathrm{th} = Q_\mathrm{p}+Q_\mathrm{s}$ is the total heat power density and is related to the input and output powers as

$$Q_\mathrm{p} = \frac{1}{\pi a^2}\frac{\mathrm{d}P_\mathrm{p}(z)}{\mathrm{d}z}, \quad Q_\mathrm{s} = \frac{1}{\pi a^2}\frac{\mathrm{d}P_\mathrm{s}(z)}{\mathrm{d}z}. \tag{11}$$

The temperatures in the core and clad will be [12]:

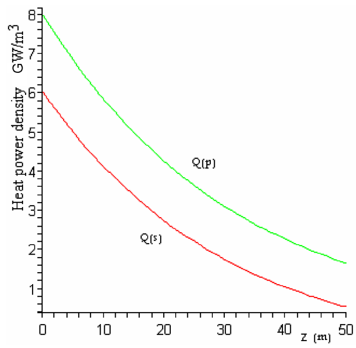

Fig. 4. Comparison of the heat power density.

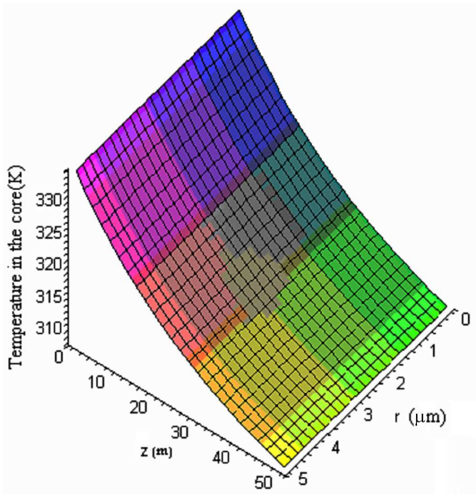

Fig. 5. Temperature distribution, operation off.

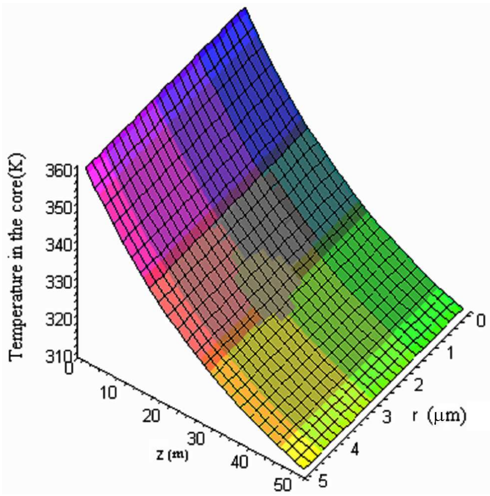

Fig. 6. Temperature distribution, operation on.

$T_{\mathrm{core}}(r,z) = T_{\mathrm{c}} + \frac{Q_{\mathrm{th}}}{2K_{\mathrm{c}}}a^2\left(\ln\left(\frac{b}{a}\right) + \frac{K_{\mathrm{c}}}{bH} + \frac{1}{2} - \frac{r^2}{2a^2}\right)$, for $r \le a$, and $T_{\mathrm{clad}}(r,z) = T_{\mathrm{c}} + \frac{Q_{\mathrm{th}}}{2K_{\mathrm{c}}}a^2\left(\ln\frac{b}{r} + \frac{K_{\mathrm{c}}}{bH}\right)$, for $r > a$, where b is the fiber radius, K_{c} is the heat conductor, H is the convention coefficient and T_{c} is the coolant temperature, $T_{\mathrm{c}} = 300$ K. Figure 4 shows the heat power densities of pump, Q_p, and laser, Q_{s}, versus z.

Figures 5 and 6 show the temperature distribution in the core versus r and z by considering the heat generation due to pump and due to pump and laser, respectively. As seen from these figures, during laser operation, the temperature distribution shows the increase of about 10%.

4. Conclusion

In this paper, we considered a diode end-pumped fiber laser and solved the rate equation by considering the upper population density which has been ignored in the literature. After finding the relation for output power, the effect of laser heat generation has been studied and the temperature distribution in fiber laser during operation has been studied.

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