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Perturbative Solution of Optical Bloch Equations for Analysis of Electromagnetically Induced Absorption

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We study phenomenon of electromagnetically induced absorption in the Hanle configuration by solving time-dependent optical Bloch equations for the case of the closed multilevel $F_{\mathrm{g}} = 1 \rightarrow F_{\mathrm{e}} = 2$ transition. Our model gives optical Bloch equations as a non-homogeneous system of ordinary linear differential equations. For weak laser fields ($\Omega \ll \Gamma$ i.e. Rabi frequency small compared to spontaneous emission rate), a perturbative method to solve linear differential equations can be applied. Perturbative method is realized by solving (in the time-domain) higher-order corrections to the density matrix which in the sum converge to the exact solution of optical Bloch equations. By its form, each successive correction is also system of ordinary linear differential equations which depends on the solution of previous ones. Corrections are partitioned such that odd give corrections to optical coherences, while even give corrections to populations and Zeeman coherences. We present numerical results for the behavior of density matrix elements with successive corrections, and compare them with exact solution of optical Bloch equations. Electromagnetically induced absorption is observed as a 4th and higher (even) correction to populations, when behavior in respect to both time and magnetic field is viewed. Since in our method each correction depends on the solution of previous ones, we can analyze how (through mechanism of transfer of coherences and transfer of populations between Zeeman sublevels) electromagnetically induced absorption is formed. We also discuss qualitative differences in the behavior (with respect to time) of certain density matrix elements for magnetic fields "inside" and "outside" electromagnetically induced absorption resonance.

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1. Introduction

During past few decades, fascinating properties of coherently prepared atomic media have become subject of extensive studies. Experiments involving manipulation of quantum coherence were performed by many groups. One of most studied phenomena is phenomenon of electromagnetically induced transparency (EIT) and closely related to it, but opposite in sign and with different properties is phenomenon of electromagnetically induced absorption (EIA). Even though less studied than EIT, since its first experimental evidence [1], EIA has received considerable interest. Several groups have experimentally observed EIA in different atomic systems and under different conditions. Specific conditions have to be fulfilled so that EIA can be observed on resonant atomic transitions

$$F_{\mathrm{g}} = F \rightarrow F_{\mathrm{e}} = F + 1, \quad F > 0, \tag{1}$$

i.e. for the type of dipole transitions that gives the positive-sign resonance — the total angular momentum of the excited level F_{e} must be larger than angular momentum of ground level $F_{\mathrm{g}}, F_{\mathrm{e}} = F_{\mathrm{g}} + 1$. Ground level must be degenerate in order to allow the long-lived Zeeman coherence and transition should be closed. Although conditions (1) are widely accepted and most investigated, recent studies [2] show that EIA can also be observed even if they all are not fulfilled. Both two-photon resonances in a bichromatic light field (pump-probe spectroscopy) and magneto-optical resonances in the Hanle configuration have been explored. Theoretically, EIA is studied by solving optical Bloch equations numerically [3–5], and several realizations of analytical solving have been done. First explanation for the physical origins of EIA phenomenon was given by Taichenachev [6] using simple analytically tractable model of a four-level N system — EIA resonance is caused by spontaneous transfer of the light-induced low-frequency coherence from the excited level to the ground one.

In this paper, we use method of perturbations to solve optical Bloch equations (OBEs). This is new and elegant way which allows us to study peculiarities about this phenomenon. Simplest closed multilevel $F_{\mathrm{g}} = 1 \rightarrow F_{\mathrm{e}} = 2$ transition in the Hanle configuration is analyzed in the time domain.

2. Method of perturbations for the system of linear differential equations applied to OBEs

We apply set of approximation schemes to solve the system of linear differential equations. Here is the system of linear differential equations in matrix form

$$\dot{x}(t) = Ax(t) - y. \tag{2}$$

A is the system's matrix, $x(t)$ is the solution and y is a right-hand side of nonhomogeneous system. Main assumption is that matrix A has two parts, A_{PERT} which

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contains relatively "small" elements and A_0 with all other elements of the matrix. The idea is to consider matrix A_{PERT} a perturbation and to solve system of linear differential equations in a perturbative manner. The goal is to find solution $x(t)$ in terms of corrections

$$x(t) = x_0(t) + x_{\mathrm{R}}(t) = x_0(t) + x_1(t) + x_2(t) + x_3(t) + \ldots, \tag{3}$$

where $x_0(t)$ is the exact solution of unperturbed part and we use this solution as an approximation to the solution of residual part $x_{\mathrm{R}}(t)$. Inserting Eq. (3) into original problem (Eq. (2)) gives

$$\dot{x}(t) = (A_0 + A_{\mathrm{PERT}})\,[x_0(t) + x_{\mathrm{R}}(t)] - y. \tag{4}$$

We start with a simplified form of the original problem which can be solved exactly, i.e. the 0th order correction $x_0(t)$ is simply the solution of the unperturbed A_0. To obtain ith-order correction successively we solve Eq. (4) for $x_{\mathrm{R}}(t) = x_1(t) + \ldots + x_i(t)$. That way we get the equation that contains addends of the equation of the previous orders and they cancel each other. Approximation is in considering term $A_{\mathrm{PERT}}x_i(t)$ negligible compared to the rest of terms in the equation. This process gives each successive correction as follows:

$$\dot{x}_{n+1}(t) = A_0 x_{n+1}(t) + A_{\mathrm{PERT}} x_n(t). \tag{5}$$

Using previous formula, we solve time-dependent optical Bloch equations

$$\frac{\partial\hat{\rho}}{\partial t} = -\frac{\mathrm{i}}{\hbar}[\hat{H}_0, \hat{\rho}] - \frac{\mathrm{i}}{\hbar}[\hat{H}_{\mathrm{I}}, \hat{\rho}] - \widehat{SE} - \gamma\hat{\rho} + \gamma\hat{\rho}_0, \tag{6}$$

where $\hat{H}_0$ and $\hat{H}_{\mathrm{I}}$ are Hamiltonian parts describing interaction with the magnetic field B (Zeeman splitting) and the laser light field (characterized by the Rabi frequency Ω), $\widehat{SE}$ is abbreviated spontaneous emission, and γ is part that describes relaxation that is not from spontaneous emission. When applying perturbation method to OBEs, the laser light field is considered a perturbation and all terms containing Rabi frequency are part of A_{PERT}, $\gamma\hat{\rho}_0$ is y and rest are parts of A_0. Here is the list of values we have used in our calculations: $\Gamma = 2\pi 6$ MHz, $\Omega = 0.02\Gamma$, $\gamma = 0.001\Gamma, l_{F\mathrm{g}} = 1, l_{F\mathrm{e}} = 1$. We take linearly polarized light and solve OBEs for $F_{\mathrm{g}} = 1 \to F_{\mathrm{e}} = 2$ as a simplest transition for which conditions from Eq. (1) are fulfilled. It is obvious from the list of parameters that the condition $\Omega \ll \Gamma$ is satisfied.

3. Results and discussion

For each density matrix element either even or odd corrections are non-zero. Furthermore, ascending sequenced even/odd non-zero corrections are in general nearly by order of magnitude smaller and their sum converges to the exact solution (see Fig. 1).

Figure 2a schematically shows (by the column of symbols in the middle of the figure) order of appearance of non-zero corrections. Our results show that odd corrections give corrections to optical coherences and even give corrections to populations and Zeeman coherences. The 0th correction is simply the initial condition for isotropic

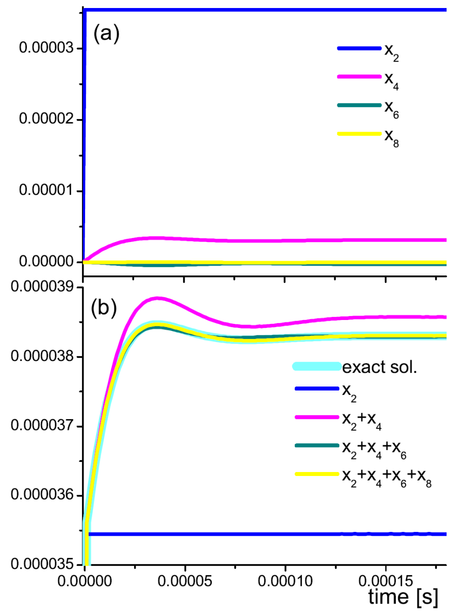

Fig. 1. Successive corrections (a) and sum of corrections compared with exact solution (b) for density matrix element $\rho_{\mathrm{e0,e0}}$.

ground state population (for each ground-state sublevel). First correction is non-zero for optical coherences, but not all of them — only the closest neighbors i.e. coherences between Zeeman sublevels for which the selection rule $\Delta m = \pm 1$ holds. The 2nd correction changes all populations and only Zeeman coherences of the sublevels for which $\Delta m_{g,e} = \pm 2$ holds. The matrix elements $\rho_{\mathrm{e-2,e+2}}$ and $\rho_{\mathrm{e+2,e-2}}$ are not perturbed by this correction. The 3rd correction is non-zero for all optical coherences, and 4th for all populations and all Zeeman coherences. Here and in the previous text, by all we mean coherences that are radiatively coupled and have physical sense.

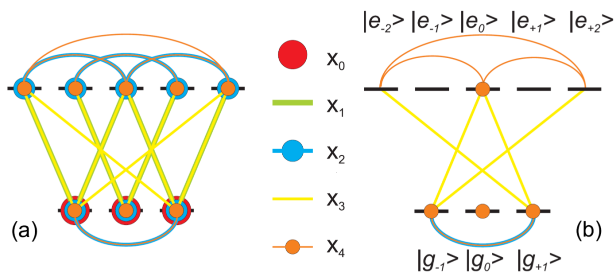

Fig. 2. Schematic order of appearance of non-zero corrections (a) and order of appearance of corrections where "critical behavior" shows (b).

Our results show that the first critical behavior (regarding dependence on the magnetic field B) is noticed for the 2nd correction of the ground-state coherence (see Fig. 2b). The critical behavior means that dependence on magnetic field besides one-photon resonance profile has a narrow (complex Lorentzian-like shape) peak resembling

EIA. Since each correction depends on the solution of previous ones, one can analyze how (through mechanism of transfer of coherences and transfer of populations between Zeeman sublevels) EIA is formed. The 4th correction brings critical behavior to the populations and the EIA is observed. It can be shown by steady-state analysis (results which we do not present here) that many properties of the EIA (sum of excited-state populations — either for the 4th correction or overall result since numerically it is negligible difference) are governed by the behavior of the second correction of the ground state coherence. Therefore, we identify the region inside narrow peak (in the behavior of the 2nd correction of the ground-state coherence) with EIA. In the further text we refer to this region as "inside" of EIA resonance, while other is "outside".

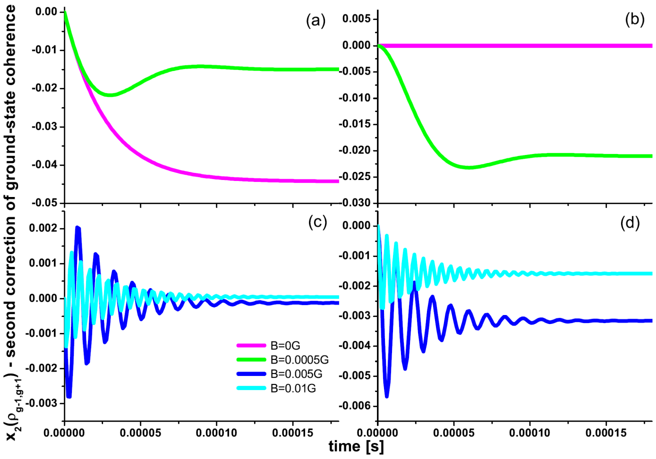

Fig. 3. Real (left) and imaginary (right) parts of time-dependence for second correction of ground-state coherence $x_2[\hat{\rho}_{g-1,g+1}]$ for four magnetic fields.

Figure 3 shows real (left) and imaginary (right) parts of time dependence for the second correction of the ground-state coherence $x_2[\hat{\rho}_{g-1,g+1}]$ for four magnetic fields. Two ((a) and (b)) of them are "inside" and two "outside" ((c) and (d)) of the EIA. What is interesting are qualitative differences in the behavior for these two regions. While "outside" EIA $x_2[\hat{\rho}g-1, g+1]$ performs oscillations, "inside" is not so. Quantitatively, period of these oscillations in time depends on the magnetic field. It can be shown (analysis that we do not present here) that by its form, equation for the second-order correction of the ground-state coherence is differential equation of forced harmonic oscillator. So, the origin of oscillatory and non-oscillatory behavior lies in the relation between right-hand side (force) and frequency of the oscillator. For some magnetic fields, relation between these two quantities is favorable for oscillatory behavior and for some not. Therefore, we see qualitative differences "inside" and "outside" EIA and in this relationship we seek deeper understanding of the EIA phenomenon.

Acknowledgments

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References

[1] A.M. Akulshin, S. Barreiro, A. Lezama, *Phys. Rev. A* **57**, 2996 (1998).

[2] R. Meshulam, T. Zigdon, A.D. Wilson-Gordon, H. Friedmann, *Opt. Lett.* **32**, 2318 (2007).

[3] J. Dimitrijević, D. Arsenović, B.M. Jelenković, *Phys. Rev. A* **76**, 013836 (2007).

[4] J. Dimitrijević, A. Krmpot, M. Mijailović, D. Arsenović, B. Panić, Z. Grujić, B.M. Jelenković, *Phys. Rev. A* **77**, 013814 (2008).

[5] J. Dimitrijević, Z. Grujić, M. Mijailović, D. Arsenović, B. Panić, B.M. Jelenković, *Opt. Express* **16**, 1343 (2008).

[6] A.V. Taichenachev, A.M. Tumaikin, V.I. Yudin, *Phys. Rev. A* **61**, 011802 (2000).