Proceedings of the Symposium A of the European Materials Research, Warsaw, September 2008

Confined Metamaterial Structures Based on Coordinate Transformations

B. Vasić, R. Gajić
Institute of Physics, Pregrevica 118, P.O. Box 68, 11080 Belgrade, Serbia
G. Isić
School of Electronic and Electrical Engineering, University of Leeds
Leeds LS2 9JT, United Kingdom
and K. Hingerl
Zentrum für Oberflächen- und Nanoanalytik und Universität Linz
Altenbergerstr. 69, A-4040 Linz, Austria

We apply transformation optics to structures in which the electromagnetic field is confined by highly conducting coatings. The possibility of changing the field propagation direction without perturbation is demonstrated on the example of a waveguide bend. Using this approach it is also possible to reshape a confined structure in order to meet certain external requirements and to redistribute a field in order to obtain desired field distribution. The structure implementation implies replacing a part of given confined structure with a metamaterial designed using the technique of transformation optics. Simplification of structure realization based on using reduced set of material parameters is examined. Our theoretical considerations are confirmed by full wave finite element simulations of a waveguide bend.

PACS numbers: 03.50.De, 41.20.Jb, 42.70.−a

1. Introduction

Transformation optics [1, 2] is based on the invariance of the Maxwell equations under coordinate transformations while material parameters (permittivity and permeability) are changed accordingly [3]. The material becomes anisotropic and inhomogeneous in the general case, and it is implemented as a metamaterial. Transformation optical structures can be used for manipulation of an electromagnetic field in a unique way. The first class of the devices changes the field within a certain domain while the field is unchanged elsewhere, remaining invisible from outside. This class includes electromagnetic cloaks [1], field rotators [4], field concentrators [5], metamaterial coatings for scatterers reshaping [6], and perfect lenses [7, 8]. Several authors used transformation optics for waveguide applications, for design as well as for modeling [9–11]. The embedded coordinate transformations have been introduced [12, 13] in order to transfer changed electromagnetic fields outside a device thus extending the scope and applicability of transformation optics.

Here we give the general description of transformation optics for confined structures where an electromagnetic field is confined by metallic coatings. The control of the field propagation is achieved by replacing a part of an original structure with a metamaterial designed using transformation optics. In this way it is possible to change the field propagation direction, redistribute the field and reshape the structure. As an example, we consider a waveguide bend and its implementation with an ideal and reduced set of material parameters. The results of our analysis are confirmed by finite element simulations.

2. Coordinate transformations of confined structures

Application of transformation optics to confined structures can be explained by looking at Fig. 1. The left part, Fig. 1a, shows a straight segment of an original confined structure in Cartesian coordinates (x, y, z). It guides a wave excited on the left edge while the field values on structure boundaries are defined by perfect electric conductor boundary conditions. The plan is to change the field propagation direction by appropriate reshaping of the original structure to the transformed one in Fig. 1b leaving the field distribution in domains D_1', D_3' the same as in D_1, D_3, respectively. Consider a transformation of coordinates described by

$$x' = x'(x, y, z),\ y' = y'(x, y, z),\ z' = z'(x, y, z). \quad (1)$$

The Maxwell equations have a coordinate-independent form if the permittivity and permeability tensors are changed according to [14]:

$$\overline{\overline{\varepsilon'}} = \frac{J\overline{\overline{\varepsilon}}J^{\mathrm{T}}}{\det J}, \quad \overline{\overline{\mu'}} = \frac{J\overline{\overline{\mu}}J^{\mathrm{T}}}{\det J}, \tag{2}$$

where $\overline{\overline{\varepsilon}}$ and $\overline{\overline{\mu}}$ are the tensors of permittivity and permeability in the original structure, while $J = \partial(x', y', z')/\partial(x, y, z)$ is the Jacobi transformation matrix.

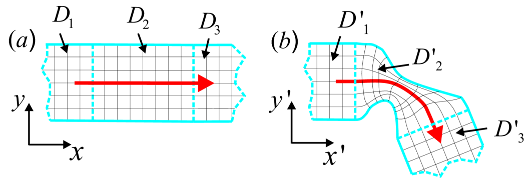

Fig. 1. The original straight structure in (a) is deformed by a coordinate transformation becoming the structure in (b). Since D_1' and D_3' are obtained from D_1 and D_3 by orthogonal transformations, only implementation of the medium within D_2' requires metamaterials. Arrows show the wave propagation direction.

If the electric and magnetic field in the original structure are $\boldsymbol{E}$ and $\boldsymbol{H}$, the fields in the transformed structure are given by

$$\boldsymbol{E}' = \left(J^{\mathrm{T}}\right)^{-1}\boldsymbol{E}, \quad \boldsymbol{H}' = \left(J^{\mathrm{T}}\right)^{-1}\boldsymbol{H}. \tag{3}$$

Assuming that the transformations of domains D_1 and D_3 to D_1' and D_3', respectively, comprises of translations and rotations (hence $JJ^{\mathrm{T}} = 1$), $\overline{\overline{\varepsilon'}}$ and $\overline{\overline{\mu'}}$ for D_1' and D_3' are the same as the corresponding $\overline{\overline{\varepsilon}}$ and $\overline{\overline{\mu}}$ for D_1 and D_3 except that they may be rotated (together with D_1 and/or D_3). Thus, the metamaterial with parameters given by (2) implementing the described transformation is restricted to D_2'. As a result, the wave becomes rotated and displaced after passing through the metamaterial. This is schematically depicted by the curved arrow in Fig. 1b corresponding to the straight arrow in Fig. 1a. In [12] and [13], a reflectionless transfer of fields through a transformation optical metamaterial is explained by heuristic means while in the present case a continuous transformation ensures that the wave is rotated without any perturbation.

The distortion of domain D_2 to D_2' can be used in order to obtain a desired outline satisfying particular external requirements, e.g. when by-passing an obstacle is needed. Reshaping the domain D_2 is accompanied by fields rescaling. According to (3), in parts of D_2' that are compressed, the intensity of field components is proportionally increased, which can be employed for power flow concentration, e.g. in various sensor applications.

3. Waveguide bend

In this section we illustrate the proposed design method. Figure 2 shows the coordinate transformation implementing a waveguide bend [12, 10]. The transformation of D_2 to D_2' reads

$$x' = x\cos(\alpha y), \quad y' = x\sin(\alpha y), \quad z' = z, \tag{4}$$

where $\alpha = \theta/L$, θ is the bend angle (here $\theta = \pi/2$), while $L = |AD|$. If the medium in D_2 is vacuum, L equals the optical length of D_2'. According to (2), the relative permittivity and permeability tensors in D_2' are given by

$$\overline{\overline{\varepsilon'}} = \overline{\overline{\mu'}} = \operatorname{diag}\left((\alpha r')^{-1}, \alpha r', (\alpha r')^{-1}\right),$$
$$r' = \sqrt{x'^2 + y'^2}, \tag{5}$$

expressed in cylindrical coordinates.

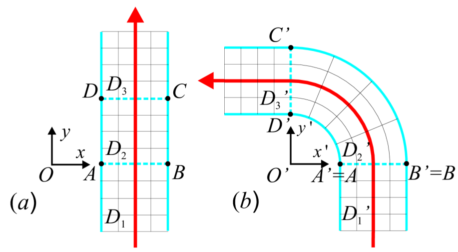

Fig. 2. The coordinate transformation employed in the waveguide bend, Eq. (4): (a) straight waveguide, (b) bent waveguide. The transformation optical metamaterial for D_2' described by Eq. (5) implements the curvature of the grid in D_2'.

As can be seen from (5), the material parameters depend only on the radial coordinate and the degree of fabrication difficulty is determined solely by the ratio between the inner and outer radius of the bend. Following the procedure used in [15] and [16], a reduced set of non--magnetic material parameters for TM mode of field polarization is found as

$$\varepsilon'^{rr} = (\alpha r)^{-2}, \quad \varepsilon'^{\phi\phi} = 1, \quad \mu'^{zz} = 1. \tag{6}$$

Magnetism is removed from this material while its dispersion (which determines a trajectory of a field propagation) stays the same. In this way, reduced parameters would significantly improve the feasibility, but at the expense of some reflection due to impedance mismatch [17].

In order to confirm the previous analysis, finite element simulations of field propagation through the waveguide bends have been performed using the COMSOL Multiphysics software. The second transverse magnetic mode ($\boldsymbol{H} \parallel \hat{\boldsymbol{z}}$) was excited at the left edge whereas the side boundaries correspond to perfect electric conductors. The following cases have been considered: (a) empty bent waveguide, (b) bent waveguide with the ideal parameters given by (5) and (c) bent waveguide with the reduced parameters given by (6). The z-components of magnetic field for the bends are shown in Fig. 3 whereas the reflection and transmission coefficients as a function of wavelength are shown in Fig. 4. Coefficient $r(t)_{mn}$ corresponds to reflection (transmission) of the incident mode $n = 2$ to mode m. As can be seen, reflection in the empty bend is very small (Figs. 4a1, a2, a3) but it causes intensive modal mixing after the bend (all transmission coefficients have significant values, Figs. 4a4, a5, a6) which results in a total field perturbation, Fig. 3a. On the other hand, the field passes through the bend with ideal parameters unperturbed and without any reflection, Fig. 3b (coefficients for this case are not shown since this bend is an ideal structure, perfectly matched to the surrounding, $t_{22} = 1$ and all other coefficients equal zero). In a case of the bend with reduced parameters, Fig. 3c, small reflection occurs, Figs. 4b1, b2, b3. However, modal mixing is rather small, the second mode dominates in the transmitted field, Figs. 4b4, b5, b6, while the bending effect works fine. In the case of a bend with ideal parameters, the parameter α only determines phase shift of field propagation through a bend. However, in the case of the bend with reduced parameters, the choice of α is important since we found the smallest reflection for $\alpha = 2(|O'A'| + |O'B'|)^{-1}$, that is, when the reduced parameters are impedance matched to the vacuum along the central line of a bend.

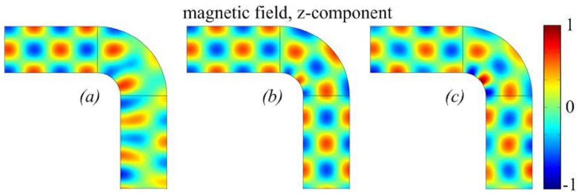

Fig. 3. Simulation results for the z-component of 0.15 m TM_2 mode propagating through waveguide bend: (a) the bend without metamaterial, (b) the bend with ideal parameters and (c) the bend with reduced parameters. Cut-off wavelength for the TM_2 mode is 0.2 m, $|O'A'| = 0.1$ m, $|O'B'| = 0.3$ m and $\alpha = 1/0.2$ m.

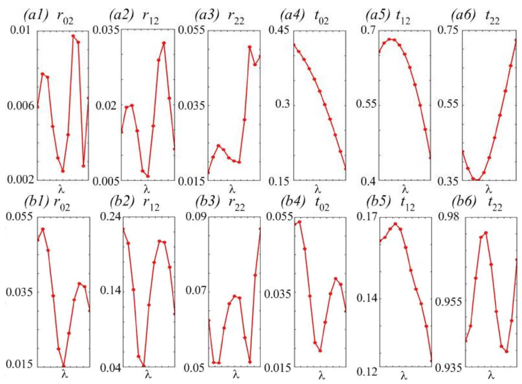

Fig. 4. The reflection and transmission coefficients for (a) the empty waveguide bend and (b) the bent waveguide with the reduced parameters. Range of wavelengths is from 0.14 m to 0.18 m. Coefficient $r(t)_{mn}$ corresponds to the amplitude ratio of reflected (transmitted) mode m to the incident one $n = 2$.

4. Conclusion

In this paper we have investigated the application of transformation optics to confined structures. On the example of a waveguide bend, we have shown how to manipulate the propagation direction of confined electromagnetic fields. In this way it is possible to obtain a transformed field outside the metamaterial domain where a field transformation is performed. In order to facilitate implementation of a waveguide bend, we considered its realization with reduced set of material parameters. Finite element simulation show that the bend works fine but small reflection arises due to impedance mismatch.

Acknowledgments

This work is supported by the Serbian Ministry of Science under project No. 141047. G.I. acknowledges support from ORSAS in the U.K. and the University of Leeds. R.G. acknowledges support from EU FP7 project Nanocharm. K.H. is grateful to the Austrian NIL-meta--NILAustria project from FFG for partial support. We thank Johann Messner from the Linz Supercomputer Center for technical support.

References

[1] J.B. Pendry, D. Schurig, D.R. Smith, *Science* **312**, 1780 (2006).

[2] U. Leonhardt, T.G. Philbin, `arXiv:0805.4778v2 [physics.optics]`, 2008.

[3] E.J. Post, *Formal Structure of Electromagnetics: General Covariance and Electromagnetics*, North--Holland, Amsterdam 1962.

[4] Huanyang Chen, C.T. Chan, *Appl. Phys. Lett.* **90**, 241105 (2007).

[5] M. Rahm, D. Schurig, D.A. Roberts, S.A. Cummer, D.R. Smith, J.B. Pendry, *Photonics Nanostruct. Fundam. Appl.* **6**, 87 (2008).

[6] O. Ozgun, M. Kuzuoglu, *Microwave Opt. Technol. Lett.* **49**, 2386 (2007).

[7] J.B. Pendry, S.A. Ramakrishna, *J. Phys., Condens. Matter* **15**, 6345 (2003).

[8] U. Leonhardt, T.G. Philbin, *New J. Phys.* **8**, 247 (2006).

[9] O. Ozgun, M. Kuzuoglu, *IEEE Microwave Compon. Lett.* **17**, 754 (2007).

[10] B. Donderici, F.L. Teixeira, *IEEE Microwave Compon. Lett.* **18**, 233 (2008).

[11] D.M. Shyroki, *IEEE Trans. Microwave Theory Tech.* **56**, 414 (2008).

[12] M. Rahm, S.A. Cummer, D. Schurig, J.B. Pendry, D.R. Smith, *Phys. Rev. Lett.* **100**, 063903 (2008).

[13] M. Rahm, D.A. Roberts, J.B. Pendry, D.R. Smith, *Opt. Express* **16**, 11555 (2008).

[14] G.W. Milton, M. Briane, J.R. Willis, *New J. Phys.* **8**, 248 (2006).

[15] D. Schurig, J.J. Mock, B.J. Justice, S.A. Cummer, J.B. Pendry, A.F. Starr, D.R. Smith, *Science* **314**, 977 (2006).

[16] W. Cai, U.K. Chettiar, A.V. Kildishev, V.M. Shalaev, *Nat. Photonics* **1**, 224 (2007).

[17] Min Yan, Zhichao Ruan, Min Qiu, *Phys. Rev. Lett.* **99**, 233901 (2007).