Modeling the Dynamic Emission of a Polarized Cr^{4+}:YAG Crystal Investigated by Double-Pulse Pumping Generated by a Nd-Laser

B. Abdul Ghani* and M. Hammadi

Atomic Energy Commission, P.O. Box 6091, Damascus, Syria

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A mathematical model describing the dynamic emission of intracavity polarized isotropic Cr^{4+}:YAG solid-state saturable absorber was developed. This model considers double pumping laser pulses to simulate the actions of Cr^{4+}:YAG as a dual Q-switched crystal (1.06 μm) and lasing medium (1.4 μm). The model describes the time evolution of interaction between the pumping laser pulse, partial polarizer and the polarized saturable absorber. The analysis of the polarization process is based on the assumption that at each moment of lasing evolution, the state of polarization represented by an eigenstate corresponds to the lowest radiation losses state. The model offers a simple mechanism for studying the kinetics of the pulsed lasers and the influence of the variations of the pumping laser power and nonlinear anisotropy parameter on the characteristics of the output laser pulses 1.06 μm and 1.4 μm. The angular rotation of the passive switch Cr^{4+}:YAG reveals that the transmission of the polarized 1.06 μm laser radiation is strongly anisotropic in the saturation regime. The suggested model estimates the transmission of pumping laser density of the polarized light 1.06 μm radiation as a function of nonlinear anisotropy parameter, temporal behavior of the relative population inversion of Nd-laser, population inversion of the different polarized levels of Cr^{4+}:YAG pulsed laser and the output laser pulse densities under impact of different values of the nonlinear anisotropy which is due to the self-induced anisotropy of its saturated absorption.

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1. Introduction

Due to its good properties, the Cr^{4+}:YAG crystal is widely implemented as a polarized saturable absorber in practical Q-switching solid-state lasers. These properties are related to thermo-mechanical properties, large absorption cross-section at wavelength 1.06 μm ($\sigma_{GS} = 5.7 \times 10^{-18}$ cm^2), long enough excited-state lifetime (3.5 μs) and low saturable fluence at 1.06 μm [1–4]. The good bleaching and thermo-optics characteristics of this crystal are due to the self-induced anisotropy of its saturated absorption.

The description of the effect of polarization anisotropy can be achieved within the framework of the model of linearly absorbing dipoles oriented along the principal crystallographic axes of the host material. The saturation absorption in Cr^{4+}:YAG is induced by three groups of resonantly absorbing linear dipoles (Cr^{4+} centers) [5–9]. The propagation of resonant radiation through a medium with a regular orientation of absorbing dipoles is accompanied by a self-induced anisotropy of the saturable absorption and by self-induced changes in the state of polarization of light. These effects should influence the laser density and polarization of the radiation generated in a laser with a cavity containing a passive crystalline switch.

The pumping polarized laser pulse generated by the Nd-laser (Nd-glass, Nd-YAG, Nd-YVO_4, etc.) and the polarizer is absorbed in Cr^{4+}:YAG crystal. The direction of the polarization of the generated giant laser pulse (Q-switched) depends on the process of the angular orientation of the Cr^{4+}:YAG crystal inside the active laser resonator. The orientations of the x and y axes are chosen so that the total cavity losses (small signal losses) are in its minimum along x axes and maximum along y axes. It means that the axis [001] (Fig. 1) can be chosen parallel to the optical axis of the laser beam. The other two axes [010] and [100] are oriented with respect to the optical axis z at the angle ϑ and $\vartheta + \pi/2$, respectively [5, 10].

The partial polarizer (PP) has the form of glass plate forming the angle β with the axis orthogonal to longitudinal axis of the laser cavity (experimental setup of Q-switched Nd-laser by polarized isotropic Cr^{4+}:YAG crystal is shown in [10]). This angle describes the linear anisotropy of the laser cavity. The linear anisotropy of the cavity arises due to specific orientation of the

* corresponding author; e-mail: scientific@aec.org.sy (B.A. Ghani)

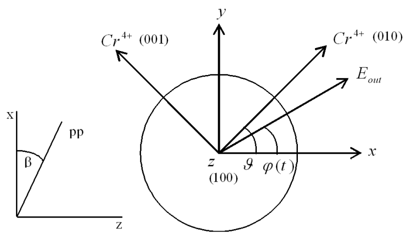

Fig. 1. Angular orientations of Cr^{4+}:YAG crystal axes $(\vartheta, \vartheta+\pi/2)$ and polarization angle of the electrical field $E_{\mathrm{out}}(\varphi(t))$ of the pumping output laser pulse.

Cr^{4+}:YAG crystal axes, and the nonlinear anisotropy due to the self-induced anisotropy determined by angle ϑ.

In this work, the ground-state is assumed to be completely polarized along one axis and moreover, the relative positions of resonant linear dipole moments orientations (polarization angle $\varphi(t)$) are considered. The realization of the double pulse pumping process is reported in details in [11, 12]. The realization of the double pulse pumping process is reported in detail in [11, 12].

It has been reported in [3, 5, 11, 12] that the models of similar problems describing the absorption and oscillation processes by single and double pumping pulse have been achieved by different considerations for the pumping process. In fact, these models did not consider the influence of the variation of the source's pumping polarized laser power from one operation to another. Besides, previous works did not study the formed nonlinear anisotropy in the switch in the case of double pulse pumping process. Moreover, they did not consider the influence of the total cavity losses (the Fresnel losses of the PP).

This work investigates the influence of the variation of the pumping source power and mainly focuses on studying the nonlinear anisotropy effect of Cr^{4+}:YAG saturable absorber in passive switch. Moreover, this work determines the influence of the polarized Cr^{4+}:YAG crystal angular position in the cavity of a Nd-laser on the laser density and polarization characteristics.

2. Mathematical model

The suggested mathematical model, presented in this work, is built on some basic ideas from different models reported in [10–13]. This developed model describes the Nd-polarized laser pumping source, the partial polarizer and the isotropic Cr^{4+}:YAG polarized solid-state saturable absorber.

2.1. Polarized pumping source medium

The time variation of the relative population inversion for Nd-laser is given as follows:

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = G - DY - 2B_{32}(\nu)U_Q Y, \quad (1)$$

where the physical constants and parameters are described in detail in [11].

Generally, two orthogonal linear polarized states can form elliptical polarization. These polarized states can be realized practically using two methods. Electronically, by reversing the direction of the applied electrical field containing the isotropic polarized Cr^{4+}:YAG crystal (or across the Pockels cell) without disturbing other optical elements. Mechanically, by rotating the intracavity quarter wave plate with an angle value of 90° [10, 14].

In order to qualitatively understand the above mentioned polarization anisotropic behavior after the onset of the saturation absorption, it should be assumed that the dipole moments of the transitions from the ground--state (lower laser level) to the first excited-state (upper laser level) must be aligned to one of the crystallographic axes [100], [010], or [001] [5, 10].

The time evolution of density of the field intensity inside the active medium, considering the interaction between the pumping laser pulse, partial polarizer and the polarized absorber Cr^{4+}:YAG and the Fresnel diffraction losses of tilted glass plate can be given by the following equation [11]:

$$\frac{\mathrm{d}U_Q}{\mathrm{d}t} = \Big\{ v\mu(\chi Y - K_{\mathrm{loss}}) - v^*\mu^*\left[\sigma_{GS}N_1^s\cos^2(\vartheta-\varphi) + \sigma_{ES}N_2^s\sin^2(\vartheta-\varphi)\right] - \frac{v\mu}{L}(\alpha_x\cos^2\varphi + \alpha_y\sin^2\varphi)\Big\}U_Q, \quad (2)$$

where α_x and α_y are the Fresnel losses according to x and y axes, respectively, N_i^s $(i = 1, 2, 3)$ are the population densities of the lower and upper laser levels and higher excited level respectively, ϑ is the angle between the group Cr^{4+} of active centers of [010] orientation and x axis giving the direction of the minimum non-saturated losses α_x of the laser cavity. The other physical and geometrical parameters are reported in detail in [11].

The polarization state of a Nd-laser passively Q-switched with Cr^{4+}:YAG is governed by the relative orientations of the Cr^{4+}:YAG crystal and the intracavity partial polarizer as well as by the density of a giant laser pulse in the switch [9]. The dynamical interaction, during the formation stage of the giant laser pulse, between the initial linear anisotropy of the passive switch, the linear anisotropy of the patial polarizer and the self-induced nonlinear anisotropy of the passive switch leads to different scenarios in the build up of polarization state. The complicated behavior of the polarization state refers to the interact between partial polarizer and the isotropic polarized Cr^{4+}:YAG crystal.

In fact, the evolution of the lasing polarization state can be represented at each moment by an eigenstate corresponding to the lowest radiation losses. The bleaching of the switch can be more effective when the plane of minimal polarization losses in the cavity coincided with the orientation of one of the two groups of the Cr^{4+} active centers [5]. Therefore, the changed state of the polarization angle of the giant laser pulse can be expressed by the following evolution:

$$\varphi(t)=\frac{1}{2}\arctan\left(\frac{\sin 2\vartheta}{\cos 2\vartheta-\frac{\alpha_y-\alpha_x}{2L^*[\sigma_{GS}N_1^s-\sigma_{ES}(N_2^s+N_3^s)]}}\right),$$

where $\alpha_y-\alpha_x$ is the PP partial losses difference, being determined from the Fresnel formulae for tilted glass plate (at the angle β).

In any laser resonator, a part of the laser pulse will be lost either by spillover at the mirrors or by the limiting aperture. These losses depend on the diameter of the laser beam in the plane of the aperture and aperture radius r_{a}. In fact, the losses depend on the combination of the parameters $L_{\mathrm{r}}, \lambda_{\mathrm{p}}, r_{\mathrm{a}}$. This combination is called "Fresnel" number $N=r_{\mathrm{a}}^2/\lambda_{\mathrm{p}}L_{\mathrm{r}}$. For the plane parallel resonator with circular aperture and for a relatively large Fresnel number $N\neq 1$, the Fresnel losses of the tilted glass plate are given by the following relation [15]:

$$\alpha_y=8\delta(M+\delta)K_{pl}^2/[(M+\delta)^2+\delta^2],\quad \delta=0.824,$$

$$M=(8\pi N)^{1/2},$$

where K_{pl}^2 is the $(p+l)$-th zero of the Macdonald or Bessel function of order l, $K_{pl}^2=\left(\frac{\mathrm{i}\pi}{2}\right)^2\exp\left((p+l)\frac{\mathrm{i}\pi}{2}\right)H_{\mathrm{p}}^{(1)}(x)H_l^{(1)}(x)$ and $H_p^{(1)}(x)$ is the Hankel function [16].

2.2. Polarized solid-state saturable absorber medium

The energy levels diagram for Nd:laser Q-switched by polarized isotropic Cr^{4+}:YAG crystal is shown in Fig. 2. The angular orientations of Cr^{4+}:YAG crystal axes $(\vartheta, \vartheta+\pi/2)$ and polarization angle of the electrical field $E_{\mathrm{out}}(\varphi(t))$ of the pumping output laser pulse is shown in Fig. 1.

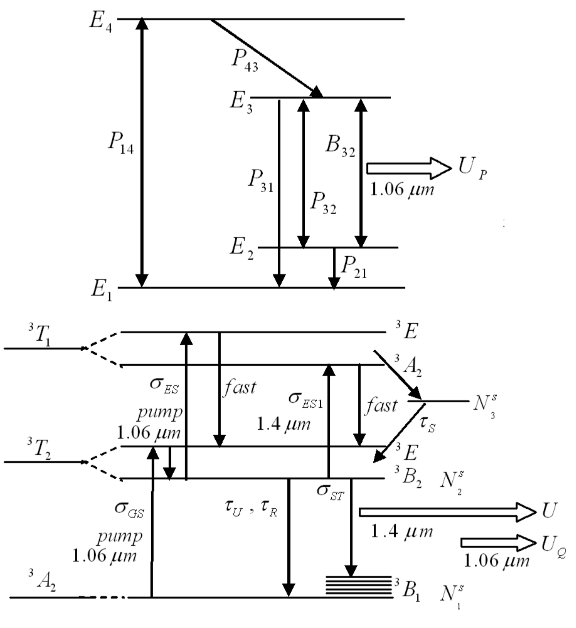

Fig. 2. Energy levels diagram for Nd-laser Q-switched by polarized isotropic Cr^{4+}:YAG crystal [4].

The double pulse pumping is expressed mathematically through the existence of the assumed intermediate level [11]. Suppose that N_1^s of the Cr^{4+} active centers exists in the lower laser level. The $N_2^{*s}=N_2^s+N_3^s$ of the Cr^{4+} active centers transited to the upper laser level and to the assumed intermediate level due to the absorption processes (Fig. 2) [11, 12].

The possibility to change an initial relationship between the ground-state populations of the Cr^{4+} active centers groups in a passive switch Cr^{4+}:YAG and the polarization plane coinciding with orientation of one of the Cr^{4+} active centers group can be achieved by the radiation, whose wavelength falls into the Cr^{4+}:YAG resonant absorption band. In the absence of radiation there are the same numbers of [010] and [100] groups. When the radiation is switched on, then some part of Cr^{4+} active centers is transited from the ground to excited state causing a remove of ground-state population equilibrium between the groups of [010] and [100] [8].

The time evolution of population density of the lower laser level of isotropic Cr^{4+}:YAG polarized crystal of the polarization ellipse is given by the following relation [11–13]:

$$\frac{\mathrm{d}N_1^s}{\mathrm{d}t}=-\frac{\sigma_{GS}v^*U_Q}{W_p}N_1^s\cos^2(\vartheta-\varphi)+\frac{N_2^s}{\tau_U}+\frac{N_2^s}{\tau_R}+\frac{(\sigma_{ST}-\sigma_{ES1})v^*U}{W_L}N_2^s. \tag{3}$$

The time evolution of the population density of the first excited-state of polarized solid-state saturable absorber (Cr^{4+}:YAG) is given by the following relation (taking into account that the orientation of the dipole moments of Cr^{4+} active centers in this state at each moment is perpendicular on the dipole moments orientation in the ground state) [11, 12]:

$$\frac{\mathrm{d}N_2^s}{\mathrm{d}t}=\frac{\sigma_{GS}v^*U_Q}{W_{\mathrm{p}}}N_1^s\cos^2(\vartheta-\varphi)-\gamma\frac{\sigma_{ES}v^*U_Q}{W_{\mathrm{p}}}N_2^s\sin^2(\vartheta-\varphi)+\frac{N_3^s}{\tau_S}-\frac{N_2^s}{\tau_U}-\frac{(\sigma_{ST}-\sigma_{ES1})v^*U}{W_L}N_2^s-\frac{N_2^s}{\tau_R}. \tag{4}$$

The time evolution of the population density of the polarized intermediate level of the Cr^{4+}:YAG is given as follows [11, 12]:

$$\frac{\mathrm{d}N_3^s}{\mathrm{d}t}=\gamma\frac{\sigma_{ES}v^*U_Q}{W_{\mathrm{p}}}N_2^s\sin^2(\vartheta-\varphi)-\frac{N_3^s}{\tau_S}. \tag{5}$$

The time evolution of density of the field intensity of pulsed Cr^{4+}:YAG laser is given as [11, 12]:

$$\frac{\mathrm{d}U}{\mathrm{d}t}=c(\sigma_{ST}-\sigma_{ES1})\left(N_2^s-\frac{g_2}{g_1}N_1^s\right)U+\frac{\Omega W_L}{4\pi\tau_R}N_2^s, \tag{6}$$

where the physical constants and parameters are described in detail in [11].

3. Numerical solution of rate equations

The rate equations (1)–(6) represent a system of stiff ordinary coupled nonlinear differential equations. These equations describe the dynamic emission in the Nd-laser rod and Cr^{4+}:YAG polarized pulsed laser. The isotropic Cr^{4+}:YAG polarized crystal acts as a passive Q-switch solid-state absorber at the wavelength 1.06 μm and as a pulsed laser at the wavelength 1.4 μm. A computer program, based on the Runge–Kutta method, was written to solve these equations. This program allows the investigation of the effect of the input pumping polarized laser pulse parameters such as pumping power (maximum amplification coefficient, loss coefficient and pumping rate) and especially the effect of nonlinear anisotropy factor or polarization anisotropy on the output laser pulse characteristics of the Nd and Cr^{4+}:YAG lasers.

The physical constants of Eqs. (1)–(6) and the geometrical dimensions of the laser cavity are given in [11]. The results are reduced for Nd-glass laser as a pumping source of the laser system. The initial values of the rate equations were chosen exactly as in [11].

4. Results and discussion

The polarization characteristics can be formed by the Cr^{4+}:YAG switch at the stage of growth of a giant laser pulse when the absorption saturation in Cr^{4+}:YAG becomes significant. The self-induced polarization anisotropy in the cavity also appears as a result of a self-induced anisotropy of the absorption saturation and of self-induced changes in polarization within the cavity.

The appearance of the polarization ellipticity β during the growth of a giant pulse may be related to the self--induced birefrigence in the Cr^{4+}:YAG crystal during the absorption saturation stage [17].

Figure 3 shows the maximum transmission of pumping laser density of the polarized 1.06 μm light radiation as a function of rotation angle for different values of maximum amplification coefficient. It can be seen from this figure that at low laser density (which corresponds to low value of maximum amplification coefficient) the transmission laser densities are closely isotropic. As the pumping laser density increases, the optical Cr^{4+} active centers become steadily saturated and the transmission of 1.06 μm light increases, so that the anisotropic behavior is clearly appeared. The amplitude of the anisotropic transmission modulation apparently reaches a maximum for intermediate values of laser density. The density of radiation and the state of polarization are affected radically by the orientation and position of the Cr^{4+}:YAG and also by the nature of the intracavity polarizer (partial polarizer, total polarizer).

If the electric vector of the laser radiation is not exactly parallel with any particular dipole moment, then partial overlap with another dipole moment can occur. From the other hand, if the laser radiation propagates along [001]

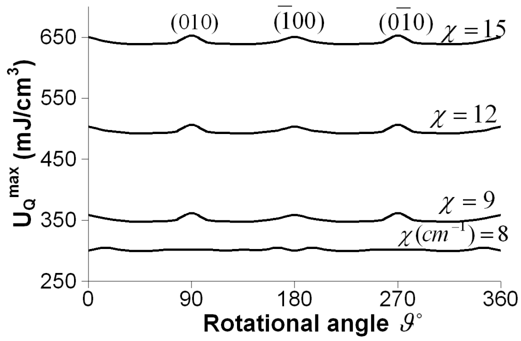

Fig. 3. Transmission of pumping laser density of the polarized light as a function of rotation angle ϑ for several values of input density.

optical axis, then the required intensity for saturating the transition is at its minimum and the transmission light at its maximum. This means that the electric vector is parallel to [010] or [100] transition moments for $\vartheta = 0°$ and 90°, respectively. Then only one subset or one-third of the total active centers is sampled (participates in the laser action) and for $\vartheta = 45°, 135°$ two subsets or two--third of the total active centers with [010] and [100] moments are simultaneously participating in the laser action (sampled) and the intensity needed for saturation is subsequently larger. This qualitatively interprets the behavior of the four peaks and troughs.

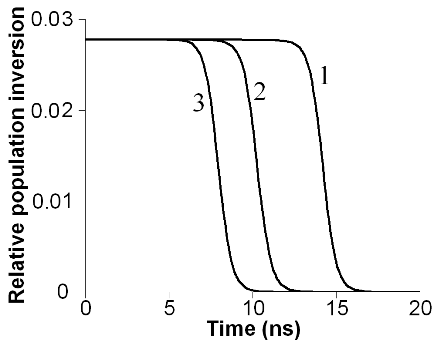

Fig. 4. Temporal behavior of the relative population inversion for $\vartheta = 0°$ (1), $\vartheta = 45°$ (2), $\vartheta = 90°$ (3).

Figures 4 and 5 show the temporal behavior of the relative population inversion and the Q-switched Nd-laser pulse density for different values of nonlinear anisotropy angles ϑ and for maximum value of amplification coefficient $\chi = 15$ cm^{-1}.

It can be noticed that by increasing the rotation angle ϑ of the polarized isotropic Cr^{4+}:YAG crystal, the delay time for both temporal relations of the relative population inversion and laser pulse density decreases. It can also be seen from Fig. 5 that the pulse duration is very short (approximately 75 ns).

Figure 6 shows the temporal behavior of the ground population density of the polarized isotropic Cr^{4+}:YAG

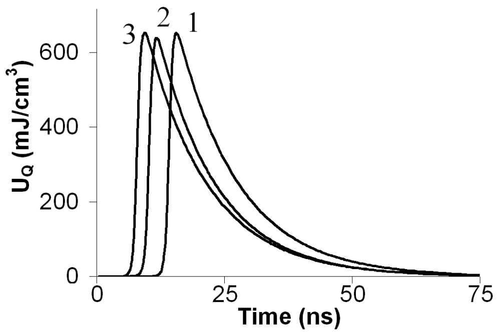

Fig. 5. Temporal behavior of laser pulse density of the Q-switched Nd- laser for $\vartheta = 0°$ (1), $\vartheta = 45°$ (2), $\vartheta = 90°$ (3).

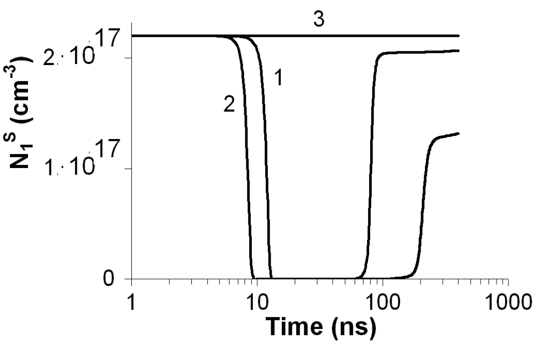

Fig. 6. Temporal behavior of population density of lower laser level of the polarized Cr^{4+}:YAG isotropic crystal for $\vartheta = 0°$ (1), $\vartheta = 45°$ (2), $\vartheta = 90°$ (3).

crystal for different values of the nonlinear anisotropy rotational angle ϑ and for maximum value of amplification coefficient $\chi = 15$ cm^{-1}. From this figure it can be noticed that the population density remains constant for a limited time. The delay time decreases by increasing the nonlinear anisotropy angle: $\vartheta = 0°$ (polarization anisotropy is near its minimum): $\vartheta = 45°$ (polarization anisotropy is very strong). For rotation angle $\vartheta = 90°$, the delay time becomes infinite and the absorption process in the ground state cannot occur in this case.

Figure 7 shows the temporal behavior of the population density of the upper laser level of the polarized isotropic Cr^{4+}:YAG crystal (pulsed laser) for different values of the nonlinear anisotropy rotational angle ϑ and for maximum value of amplification coefficient $\chi = 15$ cm^{-1}. The population density of the upper laser level reaches for $\vartheta = 0°, 45°$ its peak maximum due to the influence of the first pumping polarized laser pulse at 1.06 μm wavelength.

By increasing the nonlinear anisotropy angle (cases $\vartheta = 0°, 45°$), the delay time decreases. When $\vartheta = 90°$ the delay time becomes infinite, the upper laser level cannot be occupied. The above mentioned maxima for $\vartheta = 0°, 45°$ correspond to the minimum of the ground state shown in Fig. 6. The occupied upper laser level

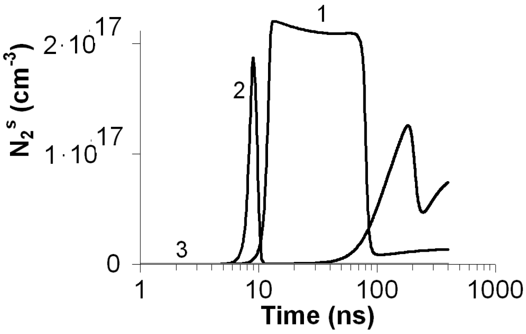

Fig. 7. Temporal behavior of population density of upper laser level of the polarized Cr^{4+}:YAG crystal for $\vartheta = 0°$ (1), $\vartheta = 45°$ (2), $\vartheta = 90°$ (3).

rapidly depletes to the ground state and to the higher excited state during the influence of the first pumping polarized laser pulse, due to the absorption, spontaneous and stimulated emission of radiations. The upper laser level starts new build up due to the effect of the second pumping polarized laser pulse and apparently second peak for nonlinear anisotropy angle $\vartheta = 45°$ can be formed [11].

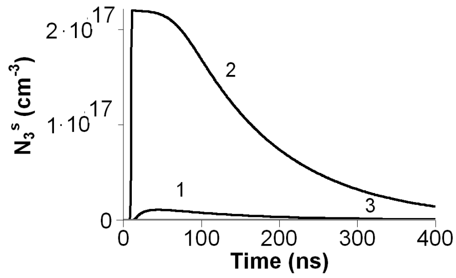

Fig. 8. Temporal behavior of the intermediate state for the nonlinear anisotropy angle for $\vartheta = 0°$ (1), $\vartheta = 45°$ (2), $\vartheta = 90°$ (3).

Figure 8 shows the temporal behavior of the intermediate state for different values of the nonlinear anisotropy angle ϑ and for maximum value of amplification coefficient $\chi = 15$ cm^{-1}. This state occupied during the effect of the first and second pumping polarized laser pulses and depleted through the upper laser level. This intermediate state closely represents the escaped Cr^{4+} active centers with oriented electric dipole moments which cannot contribute to the laser actions [11].

The temporal behavior of photons laser density emitted from pulsed Cr^{4+}:YAG laser for different values of the nonlinear anisotropy angle ϑ and for maximum value of amplification coefficient $\chi = 15$ cm^{-1} is shown in Fig. 9. The increment of the delay time and pulse width and decrement in peaks maxima of the output laser pulse density are due to the increment of the nonlinear anisotropy angle $\vartheta = 0°, 45°$. It can be seen that the laser action of Cr^{4+} active centers with oriented electric dipole moments

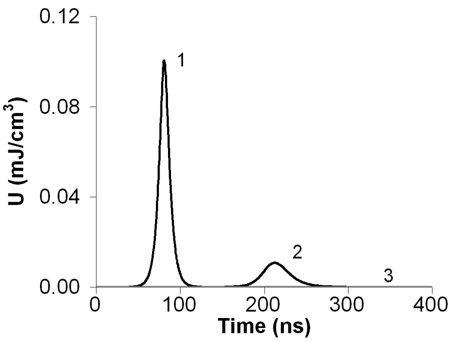

Fig. 9. Temporal behavior of photons laser density of pulsed Cr^{4+}:YAG laser for the nonlinear anisotropy angle $\vartheta = 0°$ (1), $\vartheta = 45°$ (2), $\vartheta = 90°$ (3).

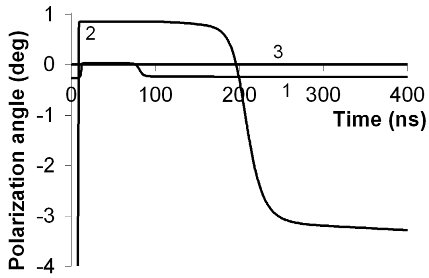

Fig. 10. Temporal behavior of polarization angle φ: $\vartheta = 0°$ (1), $\vartheta = 45°$ (2), $\vartheta = 90°$ (3).

disappears for $\vartheta = 90°$ because the absorption process in the ground state of the isotropic polarized Cr^{4+}:YAG cannot occur.

The temporal behavior of polarization angle for different values of the nonlinear anisotropy angle ϑ is shown in Fig. 10. It can be seen that for $\vartheta = 0°$, the electric field oscillates between $-0.27°$ and $0.03°$. For $\vartheta = 45°$, the electric field oscillates between $-5.99°$ and $0.85°$.

The laser characteristics are determined by self--induced anisotropy of the saturable absorber and by self-induced changes in the polarization state during the growth of the giant laser pulse when the saturation absorption starts in the crystal.

Finally, it should be mentioned that the self-induced anisotropy of resonant absorption in crystalline passive switches is reflected in the form of Eqs. (2)–(5). These equations describe the populations in the switch for each of the orientations. The numerical calculations of Eqs. (1)–(6) show strong time dependence effect of the nonlinear anisotropy angle of the used crystal.

5. Conclusion

A mathematical model describing the dynamic emission of intracavity polarized isotropic Cr^{4+}:YAG solid--state saturable absorber as a tool of dual Q-switching and lasing processes 1.06 μm and 1.4 μm by double pumping polarized pulse has been developed to describe the time evolution of laser–absorber system. The orientational dependent interaction between the polarized laser pulse's photons and polarized saturable absorber has been taken into account. The variation impact of the pumping polarized laser pulse density and rotational angle of polarized isotropic Cr^{4+}:YAG crystal are discussed in detail. The self-induced anisotropy of nonlinear absorption in crystalline passive switches containing active centers leads to a rise in angular dependence of the output laser parameters.

The study of the Cr^{4+}:YAG laser performance shows a strong dependence with crystal orientations. The selection of the polarization in the cavity is achieved by a polarizer, so that the lasing radiation evolution is slightly elliptically polarized at all stages. The selective generation of orthogonally polarized radiation can be applied in the studies of nonlinear phenomena, laser plasma interaction, and pulsed laser deposition and by birefringence measurement studies.

The adapted model in this work can be applied in pulsed Nd-lasers with passive switches made of Cr^{4+}:YAG or LiF:F_2^- crystals.

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