Theoretical Analysis of the TE Mode Cerenkov Type Second Harmonic Generation in Ion-Implanted X-Cut Lithium Niobate Planar Waveguides

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We present a study of the Cerenkov configuration second harmonic generation in X-cut ion-implanted lithium niobate waveguides. An approximate solution of conversion efficiency is given and plotted which shows that it is very sensitive to the waveguide depth and pump wavelength. The results can be used in the design of waveguides for the efficient second harmonic generation in the Cerenkov configuration.

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1. Introduction

Lithium niobate ($LiNbO_3$) is a widely-used material in integrated nonlinear optics for its remarkable electro-optical, acousto-optical and nonlinear properties. Several processes of fabricating waveguides, such as titanium indiffusion (TI) [1], proton exchange (PE) [2], have been extensively developed. Since Destefanis et al. successfully fabricated lithium niobate waveguides with helium ion in 1978 [3, 4], ion-implantation (IP) as an effective technology has aroused extensive concern to make optical waveguide.

Ion implantation is the process of depositing a chemical species into a substrate by direct bombardment of the substrate with high-energy ions of the chemical for deposition. Over the years, ion implantation has steadily replaced thermal diffusion for doping a material in wafer fabrication because of its many advantages. The greatest advantage of ion implantation over diffusion is its more precise control for depositing atoms into the substrate. Besides, advantages of ion implantation include wide selection of masking materials, less sensitive to surface cleaning procedures as well as excellent lateral dose uniformity.

Optical second harmonic generation in the form of the Cerenkov radiation from a planar waveguide was first reported by Tien et al. in 1970 [5]. When the nonlinear polarization in a waveguide has a faster phase velocity than that of a free wave at the harmonic frequency in the material, a Cerenkov configuration second harmonic generation (SHG) will occur. By now several theoretical works on the Cerenkov radiation SHG have been done [6, 7], but there is no study of the Cerenkov SHG (CSHG) on ion-implanted waveguides.

In this paper, we will present in detail our theoretical analysis of the Cerenkov SHG from an ion-implanted X-cut lithium niobate planar waveguide. We will study the influences of waveguide parameters on the Cerenkov radiation conversion efficiency.

2. Theory

In this section, we will present a theoretical calculation of CSHG in X-cut $LiNbO_3$ planar waveguides realized by ion-implantation. Figure 1 shows the waveguide structure and the crystal orientation that we consider. The axes a, b and c are the principal axes of $LiNbO_3$. For an X-cut waveguide, the optical axis c is parallel to the axis y. The light propagates in the z-direction. In the following calculations, we use the subscripts f for the fundamental fields and h for the harmonic fields. The subscripts 1, 2, and 3 denote the waveguide, the substrate, and the cladding, respectively.

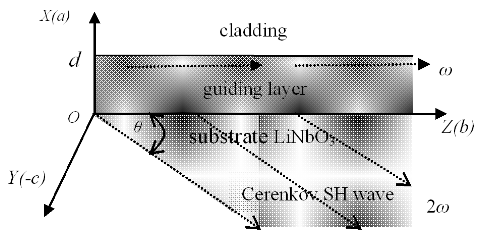

Fig. 1. Definition of crystal orientation (X-cut) and the arrangement of the Cerenkov SHG regime in an optical waveguide.

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2.1. Fundamental wave TE guided mode

The fundamental field is a guided wave, Maxwell's equations for the fields are

$$\nabla \times E_f = -\mathrm{i}\omega_f \mu_0 H_f, \tag{1.1}$$

$$\nabla \times H_f = \mathrm{i}\omega_f \varepsilon_{rf} \varepsilon_0 E_f. \tag{1.2}$$

In the X-cut IP waveguides, the TE modes, which have the components E_y, H_x and H_z, can propagate. Using Eqs. (1.1) and (1.2), we can obtain the following equation for the fundamental mode:

$$\frac{\mathrm{d}^2 E_{fy}(x)}{\mathrm{d}x^2} + \left(k_f^2 n_{fe}^2 - \beta_f^2\right) E_{fy}(x) = 0. \tag{1.3}$$

The solution of Eq. (1.3) can be obtained by using the boundary conditions and is expressed as

$$E_{fy}(x) = \begin{cases} A\left[\cos(k_{f1}d) + T_2 \sin(k_{f1}d)\right] \mathrm{e}^{-k_{f3}(x-d)}, & d < x, \\ A\left[\cos(k_{f1}x) + T_2 \sin(k_{f1}x)\right], & 0 < x < d, \\ A\mathrm{e}^{k_{f2}x}, & x < 0, \end{cases} \tag{1.4}$$

with

$$\begin{cases} k_{f1}^2 = k_f^2 n_{fe1}^2 - \beta_f^2, \\ k_{f2}^2 = \beta_f^2 - k_f^2 n_{fe2}^2, \\ k_{f3}^2 = \beta_f^2 - k_f^2, \end{cases} \quad T_2 = \frac{k_{f2}}{k_{f1}}.$$

Using the boundary conditions, we get the eigenvalue equation

$$k_{f1} d = m\pi + \arctan T_2 + \arctan T_3 \quad (m = 0, 1, 2, \ldots) \tag{1.5}$$

with

$$T_3 = \frac{k_{f3}}{k_{f1}}.$$

2.2. Second harmonic TE radiation mode

Maxwell's equations for the harmonic fields are

$$\nabla \times E_h = -\mathrm{i}\omega_h \mu_0 H_h, \tag{2.1}$$

$$\nabla \times H_h = \mathrm{i}\omega_h \left(\varepsilon_h E_h + P\right), \tag{2.2}$$

where P is the nonlinear polarization generated by fundamental field, and for X-cut $LiNbO_3$:

$$P_y = \varepsilon_0 d_{33} E_{fy}^2. \tag{2.3}$$

For the TE modes, from Eqs. (2.1) and (2.2), we can derive the following equations for the harmonic fields:

$$\nabla^2 E_{hy} + k_h^2 n_{he}^2 E_{hy} = -\mu_0 \omega_h^2 P_y, \tag{2.4}$$

$$H_{hx} = \frac{1}{\mathrm{i}\mu_0 \omega_h} \frac{\partial E_{hy}}{\partial z}, \tag{2.5}$$

$$H_{hz} = -\frac{1}{\mathrm{i}\mu_0 \omega_h} \frac{\partial E_{hy}}{\partial x}. \tag{2.6}$$

To solve Eq. (2.4), we look for a solution for the harmonic field of the form

$$E_{hy}(x, z) = E_{hy}(x) \mathrm{e}^{-\mathrm{i}\beta_h z}. \tag{2.7}$$

Putting this equation into Eq. (2.4), we can obtain

$$\frac{\mathrm{d}^2 E_{hy}(x)}{\mathrm{d}x^2} + (k_h^2 n_{he}^2 - \beta_h^2) E_{hy}(x) = -\mu_0 \omega_h^2 P_y(x) \tag{2.8}$$

with $k_h = 2k_f$, $\beta_h = 2\beta_f$.

The solution of (2.8) is the sum of the general solution $G(x)$ of the equation without the source term and a particular solution $Q(x)$ as the forced field which is generated directly by the nonlinear polarization P_y. Therefore, the solution can be written as

$$E_{hy}(x) = G(x) + Q(x). \tag{2.9}$$

The expression for $G(x)$ can be written as

$$G(x) = \begin{cases} D\mathrm{e}^{-k_{h3}(x-d)}, & d < x, \\ A\mathrm{e}^{-\mathrm{i}k_{h1}x} + B\mathrm{e}^{\mathrm{i}k_{h1}x}, & 0 < x < d, \\ C\mathrm{e}^{\mathrm{i}k_{h2}x}, & x < 0, \end{cases} \tag{2.10}$$

with $k_{h1}^2 = K_h^2 n_{he1}^2 - \beta_h^2$, $k_{h2}^2 = K_h^2 n_{he2}^2 - \beta_h^2$, $k_{h3}^2 = \beta_h^2 - K_h^2$.

For $Q(x)$ we define

$$Q(x) = \begin{cases} Q_3(x), & d < x, \\ Q_1(x), & 0 < x < d, \\ Q_2(x), & x < 0. \end{cases} \tag{2.11}$$

Then we can write the harmonic field as

$$E_{hy}(x, z) = \begin{cases} \left[D\mathrm{e}^{-k_{h3}(x-d)} + Q_3(x)\right] \mathrm{e}^{-\mathrm{i}\beta_h z}, & d < x, \\ \left[A\mathrm{e}^{-\mathrm{i}k_{h1}x} + B\mathrm{e}^{\mathrm{i}k_{h1}x} + Q_1(x)\right] \mathrm{e}^{-\mathrm{i}\beta_h z}, & 0 < x < d, \\ \left[C\mathrm{e}^{\mathrm{i}k_{h2}x} + Q_2(x)\right] \mathrm{e}^{-\mathrm{i}\beta_h z}, & x < 0. \end{cases} \tag{2.12}$$

The constants A, B, C, and D in Eq. (2.12) can be determined by using the boundary conditions. The particular solution $Q(x)$ represents the forced field which is generated by the nonlinear polarization $P_y(x)$. $P_y(x)$ is determined by the fundamental field $E_{fy}(x)$.

Therefore, the nonlinear polarization can be written as

$$p_y(x) = \varepsilon_0 d_{33} E_{fy}^2(x) = \begin{cases} 0, & d < x \\ \varepsilon_0 d_{33}^{(1)} A^2 \left[\cos(k_{f1}x) + T_2 \sin(k_{f1}x)\right]^2, & 0 < x < d, \\ \varepsilon_0 d_{33}^{(2)} A^2 e^{2k_{f2}x}, & x < 0. \end{cases} \tag{2.13}$$

We can find the particular solution $Q(x)$:

$$Q(x) = \mu_0 \varepsilon_0 \omega_h^2 A^2 f(x) \tag{2.14}$$

with

$$f(x) = \begin{cases} 0, & d < x, \\ -d_{33}^{(1)} \left[\frac{1+T_2^2}{2k_{h1}^2} + \frac{(1-T_2^2)\cos 2k_{f1}x + 2T_2 \sin 2k_{f1}x}{2(k_{h1}^2 - 4k_{f1}^2)}\right], & 0 < x < d, \\ -\frac{d_{33}^{(2)}}{k_{h2}^2 + 4k_{f2}^2} e^{2k_{f2}x}, & x < 0. \end{cases} \tag{2.15}$$

2.3. Efficiency of SHG

In the case of a planar waveguide uniformly excited over an interaction width W, the harmonic power at the exit face of the structure is calculated by the formula

$$p_h = \frac{W}{2} \mathrm{Re} \int_{-\infty}^{+\infty} (E_h \times H_h^*) z_0 \mathrm{d}x, \tag{3.1}$$

which can be calculated into

$$P_h = \frac{1}{2} W \mu_0 \varepsilon_0^2 \beta_h \omega_h^3 A^4 L \tan\theta \, |c|^2, \tag{3.2}$$

where L and W are the length and interaction width of the waveguide respectively, θ is the Cerenkov angle, and the fundamental power is given by

$$P_f = \frac{\beta_f W d_{\mathrm{eff}} A^2 (1 + T_2^2)}{4\omega_f \mu_0}. \tag{3.3}$$

The conversion efficiency is given by

$$\eta = \frac{P_h}{P_f} = \frac{8\mu_0^3 \varepsilon_0^2 \omega_h^5}{\beta_h} \frac{L P_f}{W} \frac{\tan\theta \, |c|^2}{d_{\mathrm{eff}}^2} \frac{1}{(1+T_2^2)^2}, \tag{3.4}$$

where d_{eff} is the effective depth of the fundamental TE mode

$$d_{\mathrm{eff}} = \frac{1}{k_{f2}} + d + \frac{1}{k_{f3}}, \tag{3.5}$$

c is a parameter that depends on the forced field

$$|c|^2 = \frac{|k_{h1}(k_{h3} f_{13} + f'_{13}) + k_{h1}(k_{h1} \sin k_{h1}d - k_{h3} \cos k_{h1}d) f_{12} - (k_{h1} \cos k_{h1}d + k_{h3} \sin k_{h1}d) f'_{12}|^2}{k_{h2}^2 (k_{h1} \cos k_{h1}d + k_{h3} \sin k_{h1}d)^2 + k_{h1}^2 (k_{h3} \cos k_{h1}d - k_{h1} \sin k_{h1}d)^2}, \tag{3.6}$$

with

$$f_{12} = -\frac{d_{33}^{(2)}}{k_{h2}^2 + 4k_{f2}^2} + d_{33}^{(1)} \left[\frac{1+T_2^2}{2k_{h1}^2} + \frac{1-T_2^2}{2(k_{h1}^2 - 4k_{f1}^2)}\right],$$

$$f_{13} = d_{33}^{(1)} \times \left[\frac{1+T_2^2}{2k_{h1}^2} + \frac{(1-T_2^2)\cos 2k_{f1}d + 2T_2 \sin 2k_{f1}d}{2(k_{h1}^2 - 4k_{f1}^2)}\right],$$

$$f'_{12} = -\frac{2d_{33}^{(2)} k_{f2}}{k_{h2}^2 + 4k_{f2}^2} + \frac{2d_{33}^{(1)} T_2 k_{f1}}{k_{h1}^2 - 4k_{f1}^2},$$

$$f'_{13} = d_{33}^{(1)} \times \frac{2(T_2^2 - 1) k_{f1} \sin 2k_{f1}d + 4T_2 k_{f1} \cos 2k_{f1}d}{2(k_{h1}^2 - 4k_{f1}^2)}.$$

3. Discussion

Formula (3.4) shows that the conversion efficiency in CSHG increases linearly with the interaction length L, this is consistent with the conclusions of Refs. [8–10] for large Cerenkov angles. That is to say, formula (3.4) is suitable under this condition.

In addition to the dependence of interaction length, the efficiency depends drastically on waveguide parameters. We have calculated the conversion efficiency for IP waveguides on X-cut $LiNbO_3$ by changing the waveguides depth and the pump wavelength to show how the Cerenkov radiation varies with these parameters, with the length and interaction width of 10 mm and 10 μm, respectively. The dependence of the refractive indices in the substrate on the wavelength has been obtained by the Chebyshev method [11] and the refractive indices in the guiding layer come from Ref. [12]. For most of the waveguides realized by ion-implantation, the waveguide depth is between 1.5 μm and 3.0 μm. Therefore our calculations are done in this extent.

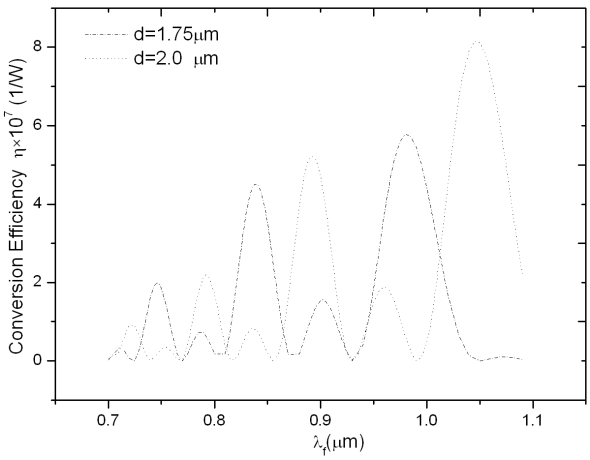

Fig. 2. Efficiency with different waveguide depth as a function of pump wavelength.

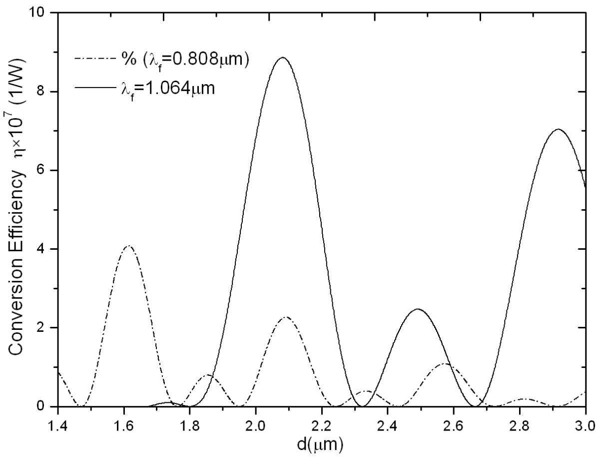

Fig. 3. Cerenkov SHG efficiency with different pump wavelength as a function of waveguide depth.

Figures 2 and 3 are obtained under the condition of $m = 0$. In the range of pump wavelength $\lambda_f = 0.7$–1.1 μm, the minimum waveguide depth is beyond 3.0 μm when the conversion efficiency is not equal to 0 at $m = 1, 2, 3\ldots$ We can see from Figs. 2 and 3 that for a given waveguide depth the efficiency is very sensitive to the pump wavelength, and for a given pump wavelength the efficiency is also very sensitive to the waveguide depth. Figure 2 shows that for a given waveguide depth, the relatively maximum conversion efficiency can be obtained at a certain pump wavelength, and the relatively maximum conversion efficiency of $d = 2.0$ μm is higher than that of $d = 1.75$ μm. For a given pump wavelength, the relatively maximum conversion efficiency can be obtained at a certain waveguide depth, and the relatively maximum conversion efficiency of $\lambda_f = 1.064$ μm is higher than that of $\lambda_f = 0.808$ μm, as can be seen from Fig. 3. Figure 3 also indicates that the minimum waveguide depth increases with the increase in pump wavelength when the conversion efficiency is not equal to 0.

4. Conclusion

We have proposed a model analysis for calculating the conversion efficiency of the Cerenkov type second harmonic generation in ion-implanted X-cut lithium niobate planar waveguides. We have studied the influences of the different parameters of waveguides, depth and pump wavelength, on the CSHG. Theoretical results indicate that the relatively maximum conversion efficiency increases with the increase in the waveguide depth and the pump wavelength. The model allows us to determine the optimum parameters for an integrated optical visible CSHG source pumped by semiconductor lasers.

However, we have not considered the SHG efficiency at small Cerenkov angles. For large Cerenkov angles and interaction lengths it yields the expected L dependence (as we have discussed), while in the limit of small Cerenkov angles the dependence is found to have the form of $L^{3/2}$ [8–10]. We would intend to study this area in our future work.

Acknowledgments

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