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Anomalous Hall Effect in IV–VI Semiconductors

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We consider theoretically the topological contribution to the anomalous Hall effect in narrow-gap IV–VI magnetic semiconductors in which the relativistic terms are relatively large and determine both the non-parabolicity of the energy spectrum and strong spin–orbit interaction. We use the relativistic Dirac model and linear response theory to calculate this contribution. Experimental data on the anomalous Hall effect in these compounds are also presented and discussed.

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1. Introduction

The off-diagonal conductivity of ferromagnetic metals and magnetic semiconductors includes two terms: one is proportional to external magnetic field and is related to the ordinary Hall effect, while the second term is proportional to magnetization of the sample and does not disappear in zero magnetic field. The latter term is known as the anomalous Hall effect (AHE). There is currently a lot of interest in AHE in magnetic macroscopic systems and low-dimensional structures [1–6], which is stimulated by application possibilities of the effect. The main origin of the AHE is the spin–orbit interaction in the presence of spin polarization. There are two groups of mechanisms responsible for AHE — extrinsic mechanisms (skew scattering and side jump) and intrinsic mechanism. The intrinsic mechanism of the AHE effect is related to the topology of electron bands, and therefore it is often referred to as topological mechanism. The relation to the topology can be understood if the Hall conductivity is presented in terms of the Berry phase along a contour at the Fermi surface [5].

The IV–VI magnetic semiconductors attract currently much attention from the experimental and application points of view [7]. The AHE has been measured for instance in PbSnMnTe, SnMnTe and SnMnEuTe mixed crystals containing Mn ions up to 16 at.% [8, 9]. Analysis of the corresponding experimental results should take into account peculiarity of the energy spectrum of IV–VI semiconductors. However, no theoretical considerations of the AHE in IV–VI crystals have been presented so far. In this paper we consider narrow-gap IV–VI magnetic semiconductors, where the relativistic terms are not small and determine both the non-parabolicity of energy spectrum and a strong spin–orbit interaction. We use the relativistic Dirac model to describe electronic structure of the system, and the linear response theory to calculate the topological contribution to the anomalous Hall conductivity. We also present some experimental data. From a brief comparison of numerical and experimental results we conclude that the extrinsic contribution to the AHE conductivity in the system studied experimentally is dominant.

2. AHE in a relativistic Dirac model

We consider the relativistic Dirac model for electrons and holes in IV–VI semiconductors with a non-zero magnetization due to magnetic impurities. Hamiltonian of the system may be written as

$$H = \begin{pmatrix} \Delta - g_{\mathrm{c}} M \sigma_z & -\mathrm{i} v_0 \boldsymbol{\sigma} \cdot \frac{\partial}{\partial \boldsymbol{r}} \\ -\mathrm{i} v_0 \boldsymbol{\sigma} \cdot \frac{\partial}{\partial \boldsymbol{r}} & -\Delta - g_{\mathrm{v}} M \sigma_z \end{pmatrix}, \quad (1)$$

where Δ is the half-width of the energy gap, v_0 is the band interaction parameter, M is the magnetization, while g_{c} and g_{v} are constants which describe magnetic splitting in the conduction and valence bands, respectively. The Pauli matrices $\sigma_x, \sigma_y, \sigma_z$ operate in the spin space, with the axis z along the magnetization.

To calculate the topological contribution we start from the general Kubo formula for dynamical ($\omega \neq 0$) conductivity,

$$\sigma_{xy}(\omega) = \frac{e^2 \hbar}{\omega} \mathrm{Tr} \int \frac{\mathrm{d}\varepsilon}{2\pi} \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} v_x G_{\boldsymbol{k}}(\varepsilon + \omega) v_y \, G_{\boldsymbol{k}}(\varepsilon), \quad (2)$$

where the velocity components in our model have the following form:

$$v_x = \frac{v_0}{\hbar} \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}, \quad v_y = \frac{v_0}{\hbar} \begin{pmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{pmatrix},$$

and $G_{\boldsymbol{k}}(\varepsilon)$ is the Green function corresponding to the Hamiltonian (1).

After calculating the trace in Eq. (2), taking the limit $\omega \to 0$, and integrating over ε, the leading term of σ_{xy} (linear in M) for the Fermi level in the valence band takes the form

$$\sigma_{xy} = -\frac{2e^2 v_0^2 g M}{3\hbar} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} f(E_{0\boldsymbol{k}}) \left[\frac{P_0'''(E_{0\boldsymbol{k}})}{(2E_{0\boldsymbol{k}})^4} - \frac{12 P_0''(E_{0\boldsymbol{k}})}{(2E_{0\boldsymbol{k}})^5} + \frac{60 P_0'(E_{0\boldsymbol{k}})}{(2E_{0\boldsymbol{k}})^6} - \frac{120 P_0(E_{0\boldsymbol{k}})}{(2E_{0\boldsymbol{k}})^7} \right], \quad (3)$$

where $g_{\mathrm{c}} = g_{\mathrm{v}} = g$, $P_0(\varepsilon) = \mathrm{d}P(\varepsilon_1, \varepsilon_2)/\mathrm{d}\varepsilon_2|_{\varepsilon_1, \varepsilon_2 = \varepsilon}$, and $E_{0\boldsymbol{k}} = -\left(\Delta^2 + v_0^2 k^2\right)^{1/2}$ is the energy spectrum of valence band in the limit of $M = 0$. Here, $P(\varepsilon_1, \varepsilon_2)$ is the trace of the numerator in Eq. (2) (details will be presented elsewhere). Apart from this, P_0', P_0'', and P_0''' are the first, second, and third order derivatives of P_0. Similar formula also can be derived for the contribution from the conduction band.

3. Numerical results

From Eq. (3) follows that the intrinsic contribution is determined by the integral over all occupied states. In the Dirac model assumed here this means that we need to integrate over energy from $-\infty$ to the Fermi level. The Fermi level is measured from the middle of the energy gap. Assuming the Fermi factor $f = 1$, from the symmetry follows that the integral from the conduction band bottom to the Fermi level μ in the conduction band is equal to minus integral from the valence band top to the Fermi level $-\mu$ in the valence band. This allows us to write the total Hall conductivity as $\sigma_{xy} = \sigma_{xy}^0 - \Delta\sigma_{xy}$, where σ_{xy}^0 is the contribution from the whole valence band while $\Delta\sigma_{xy}$ takes into account the empty (occupied) part of the valence (conduction) band.

In Fig. 1 we show $\Delta\sigma_{xy}$, calculated as a function of the energy gap (parameter Δ) for indicated impurity concentration N (holes or electrons) (Fig. 1a), and as a function of impurity concentration (Fig. 1b) for indicated values of Δ.

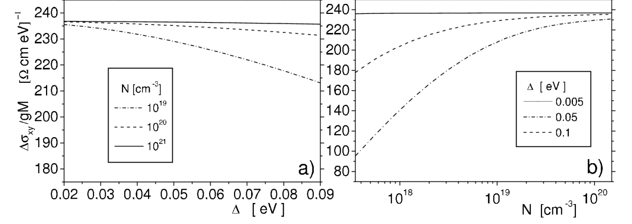

Fig. 1. Contribution $\Delta\sigma_{xy}$ to the Hall conductivity, calculated as a function of the parameter Δ (a) and impurity concentration N (b). The other parameters are: $v_0 = 5 \times 10^{-8}$ eV cm, and $T = 0$.

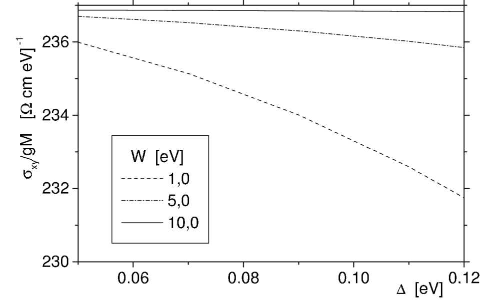

Fig. 2. Contribution σ_{xy}^0 to the Hall conductivity, calculated as a function of the parameter Δ for indicated values of the valence band width W. The other parameters are the same as in Fig. 1.

Since the valence band in the Dirac model assumed here is not bounded from the bottom, we have to impose some cut-off band edge to calculate σ_{xy}^0. The corresponding contribution from the whole valence band depends on the assumed band width W. It also depends on the parameter Δ for a particular value of W, as shown in Fig. 2. It should be noted that the constant part of the off-diagonal conductivity σ_{xy}^0 is due to the topology of fully occupied bands. Therefore, if calculated properly, it does not depend on any numerical parameters. As we see from Fig. 2, σ_{xy}^0 is independent of Δ for $W = 10$ eV.

4. Experimental results

In Fig. 3 we present the temperature dependence of the anomalous Hall constant in two sets of ferromagnetic IV–VI compounds (see also Ref. [9] and Ref. [10]).

The first set was composed of SnTe based compounds with the Curie temperature in the range 10–20 K. The

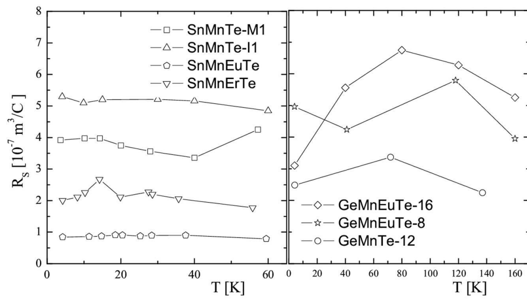

Fig. 3. The temperature dependence of the anomalous Hall coefficient for semimagnetic semiconductors based on SnTe and GeTe.

content of magnetic constituent (Mn, Er and Eu) does not exceed 14 at.%. The second set contains GeTe based materials with substantially higher Curie temperature i.e. contained between 80 and 160 K. In that case the content of magnetic constituent was higher, and reached 38 at.%. The transport and magnetic measurements (up to 13 and 9 T, respectively) were performed for the same set of temperatures. The anomalous Hall coefficients were determined from the total transverse resistivity and magnetization data by the least squares root fit to the equation $\rho_{xy} = R_0 B + \mu_0 R_S M$, where B is the magnetic field, R_0 and R_S are the normal and anomalous Hall coefficients, respectively, and μ_0 is the permeability constant.

According to presented results, one can see that within the experimental error the anomalous Hall coefficient does not depend on temperature. However, it is worth to note that in the case of the two samples (not shown in the figure), the anomalous Hall coefficient showed drastically different behavior, strong temperature dependence. The origin of this behavior has not been understood yet, but it may be connected with the presence of the inclusions of the second crystallographic or chemical phase [9].

5. Discussion

Our considerations do not include impurities, which however can play an important role. We take into account only the topological contribution. The obtained results show that the topological contribution is nonzero in IV–VI compounds, and depends on such parameters as the energy gap Δ and the location of the Fermi level. If we assume that the topological contribution to the AHE is dominant, we can present our results as a dependence of the Hall conductivity on the electron (hole) density and on temperature. The latter dependence is mostly related to the temperature dependence of magnetization M. Unfortunately, it was rather difficult to compare theoretical results with experiment, because the experimental curves are measured on samples with different contents of Mn. However, by extracting the dependence of the AHE prefactor on the carrier density, we found that these dependences are qualitatively similar. From a qualitative comparison of the experimental and theoretical results we come to the conclusion that the topological contribution to the AHE is not negligible, although not sufficient to account for the experimental observations.

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