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# New Paradigm of Triplet Superconductivity

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Since 1980, more than 10 or so triplet superconductors have been discovered. Now we can put them into two separate classes. Type A consists of e.g. $(TMTSF)_2PF_6$, $(TMTSF)_2ClO_4$, $UPt_3$, $Sr_2RuO_4$, $PrOs_4Sb_{12}$. These triplet superconductors are characterized with the extreme smallness of the spin–orbit coupling energy $E_{\rm so}$ ($\ll \Delta$, where $\Delta$ is the superconducting gap). Also like superfluid $^3$He-A, the order parameter of these superconductors are characterized by $\hat{l}$ (the chiral vector) and $\hat{d}$ (the spin vector). In these superconductors, an Abrikosov vortex splits into a pair of half quantum vortices at low temperatures. Type $A_1$ comprises most of non-centrosymmetric triplet superconductors discovered recently, e.g. $CePt_3Si$, $CeIrSi_3$, $CeRhSi_3$, and $Li_2Pt_3B$. They are characterized by $\hat{l}$ and $\hat{d}_1 + i\hat{d}_2$ like superfluid $^3$He-$A_1$. The spin–orbit coupling energy $E_{\rm so}$ is extremely large $E_{\rm so} \approx 10^3$ K. Therefore, as noted by Frigeri et al., the Fermi surface splits for the up-spin one and the down-spin one. However, contrary to Frigeri et al., the superconductivity should occupy only the larger Fermi surface (say for spin-up). The other Fermi surface remains in the normal state. Also in type $A_1$ superconductors, an Abrikosov vortex does not split into a pair of half quantum vortices. Further all thase triplet superconductors (both type A and type $A_1$) harbor the zero mode or the Majorana fermion attached to each vortex, of which implication should be further explored.

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## 1. Introduction

In 1980 Jerome et al. [1] discovered the first organic superconductor $(TMTSF)_2PF_6$ which was the first triplet superconductor as shown later by proton NMR [2]. In the meanwhile, many triplet superconductors have been discovered and reviewed in [3]. Now we understand that these triplet superconductors are characterized by the equal spin pairing (ESP) and with extremely small spin–orbit coupling energy $E_{\rm so} \approx 10^{-3}\Delta$, where $\Delta$ is the superconducting energy gap [4]. Therefore, the superconducting order parameter is characterized by $\hat{l}$ (the chiral vector) and $\hat{d}$ (the spin vector) as in superfluid $^3$He-A except for the gap symmetry [5, 6]. The gap functions of some of the triplet superconductors are shown in Fig. 1 [7].

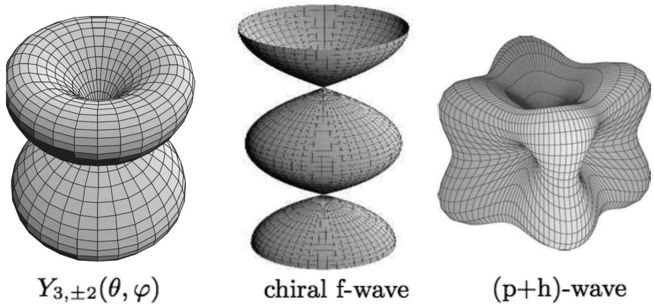


Fig. 1. Gap functions of some of the triplet superconductors e.g. $Y_{3,\pm2}(\theta,\varphi)$ for $UPt_3$ and $CePt_3Si$, chiral $f$-wave for $Sr_2RuO_4$, $(p+h)$-wave for $PrOs_4Sb_{12}$.

On the other hand, the discovery of a triplet superconductor $CePt_3Si$ without inversion symmetry by Bauer et al. in 2004 [8] was a big surprise. According to Anderson [9], such a triplet superconductor should not exist. In order to clarify the effect of the parity-breaking term (e.g. the Rashba term [10]), Frigeri et al. [11, 12] considered a model with a Rashba term. They found, first of all, that the Fermi surface is split into the one for up-spin and the other for down-spin as shown in Fig. 2. Second, they found that the triplet superconductor can exist with the strong admixture of the singlet component. In the meanwhile, a number of triplet superconductors in noncentrosymmetric crystals have been discovered in $CeIrSi_3$, $CeRhSi_3$, UIr, and $Li_2Pt_3B$ [13–16]. Also the spin–orbit coupling energy $E_{\rm so}$ or the Rashba term is found invariably extremely large [13, 16] i.e. $E_{\rm so} \approx 10^3$ K in these systems. This makes the triplet superconductor with the admixture of the singlet superconductor (as proposed by Frigeri et al. [11, 12]) very unlikely. Recently in [17] we have proposed that the triplet superconductors in non-centrosymmetric crystals should be like the one in $^3$He-A$_1$. The superconductivity occupies the Fermi surface associated with one spin component (say spin up) while the other Fermi surface remains in the normal state. With this assumption, we can describe many unusual experimental results of $CePt_3Si$.

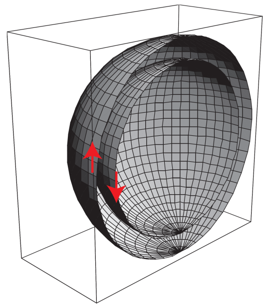

Fig. 2. The Fermi surface is split into the one for up--spin and the other for down-spin due to the parity--breaking term.

## 2. Bogoliubov–de Gennes equations

In this section, we shall explore the bound-state spectrum around an Abrikosov vortex in triplet superconductors. First of all, we note that $\hat{l}$ (the chiral vector) of many triplet superconductors is fixed parallel to the crystal $\hat{c}$-axis. Second, in the absence of a magnetic field, $\hat{d} \parallel \hat{l}$ due to the spin–orbit coupling. However, when the magnetic field $\boldsymbol{H}$ (parallel to $\hat{l}$) is applied in the vicinity of $H_{c2}(T)$ (the upper critical field), $\hat{d} \perp \hat{l}$ is realized. In this particular situation, Ivanov [18, 19] has shown that the Bogoliubov–de Gennes (BdG) equations for the triplet superconductor decouple into the one for the up--spin component and the other for the down-spin component. Then these BdG equations have the same structure as the one for $s$-wave superconductor first written down in [20, 21]. Therefore first let us consider a single vortex.

Following Caroli, de Gennes and Matricon [20, 21], the bound state energy formula is given by [22–24]:

$$\epsilon_n = n\omega_0 \quad \text{with} \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots \tag{1}$$

and $\omega_0 = I_1/I_0$, where

$$I_0 = p_{\mathrm{F}} \langle \int_0^\infty \mathrm{d}r \exp(-2K(\boldsymbol{r}, \hat{k})) \rangle \,, \tag{2}$$

$$I_1 = \langle |f| \int_0^\infty \mathrm{d}r \frac{\Delta(r)}{r} \exp(-2K(\boldsymbol{r}, \hat{k})) \rangle \,. \tag{3}$$

Now let us assume for $\Delta(r)$ as $\Delta(r) = \Delta \tanh(r/\xi)$, where $r$ is the radial distance from a vortex line. Then we find $K(\boldsymbol{r}, \hat{k}) = v_{\mathrm{F}}^{-1}\xi|f|\ln(\cosh(r/\xi))$, $I_0 = \frac{1}{2}p_{\mathrm{F}}\xi, \langle B(1/2, C|f|)\rangle$, $I_1 = \frac{1}{2}\langle |f|\Delta \int_0^1 \mathrm{d}s \times (1-s)^{c|f|-1}[\tanh^{-1}(\sqrt{s})]^{-1}\rangle$, where $B(a,b)$ is the beta-function, $C = \Delta\xi v_{\mathrm{F}}^{-1} \approx 1$, $f$ encodes the gap symmetry and $\langle \ldots \rangle$ means average over the Fermi surface.

Let us note that the bound-state spectrum is very different from the one for the singlet superconductors ($s$-wave, $d$-wave etc.). For the singlet superconductors, $\epsilon_n = (n + \frac{1}{2})\omega_0$ with $n = 0, \pm 1, \pm 2, \pm 3, \ldots$ and there is a mini-gap $\omega_0/2 \sim \Delta^2/E_{\mathrm{F}}$. On the other hand, in triplet superconductors, there is the zero mode or the Majorana fermion [25, 6, 26] with zero energy (right on the Fermi surface). The Majorana fermion wave function associated with the zero mode is given by

$$\phi_0(r, \hat{k}) \approx [\mathrm{sech}(r/\xi)]^{C|f|} \,, \tag{4}$$

and $C = \Delta\xi v^{-1} \approx 1$. These wave functions should be accessible at low temperatures ($T < 100$ mK).

## 3. Stability region of half quantum vortices

As we have discussed earlier [4, 27], the $\hat{l}$-soliton or the $\hat{l}$-domain wall as discussed by Sigrist and Agterberg [28] is too costly to be realistic. Instead we consider that the $\hat{d}$-soliton or the $\hat{d}$-domain wall is most readily created. As a consequence, half quantum vortices (HQV) are generated in type-A triplet superconductors [22–24]. Since $\hat{l}$-vector is fixed parallel to the crystaline $\hat{c}$-axis, we limit ourselves to the texture free energy associated with the $\hat{d}$-vector. Especially for $UPt_3$, $Sr_2RuO_4$, and $PrOs_4Sb_{12}$ in a magnetic field parallel to $\hat{l}$, the texture free energy [27] simplifies as

$$\mathcal{F} = \frac{1}{2}\chi_N C^2 \int \mathrm{d}x\mathrm{d}y \times \left[ K(\nabla \Phi)^2 + \sum_{i,j} |\partial_i \hat{d}_j|^2 + \xi_D^{1-2}(\hat{d}_x^2 + \hat{d}_y^2) \right] , \tag{5}$$

where $\chi_N$ and $C$ are the spin susceptibility and the spin--wave velocity, and $\xi_D^{1-2} = \xi_D^{-2} - \xi_H^{-2}$. Here

$$K(t) = \frac{\rho_{\mathrm{s}}(t)}{\rho_{\mathrm{sp}}(t)} = \frac{1 + \frac{F_1}{3}}{1 + \frac{F_1^a}{3}} \frac{1 + \frac{F_1^a}{3}[1 - \rho_{\mathrm{s}}^0(t)]}{1 + \frac{F_1}{3}[1 - \rho_{\mathrm{s}}^0(t)]} \,, \tag{6}$$

where $t = T/T_{\mathrm{c}}$, $\rho_{\mathrm{s}}(t)$, $\rho_{\mathrm{sp}}(t)$ and $\rho_{\mathrm{s}}^0(t)$ are the superfluid density, the spin superfluid density and the bare superfluid density. In $PrOs_4Sb_{12}$, $\rho_{\mathrm{s}}^0(t)$ is well approximated by $\rho_{\mathrm{s}}^0(t) = 1 - 0.460572t - 0.53428t^{1.729}$ [29]. Experimentally $m^*/m = 50$ in $PrOs_4Sb_{12}$ and, if we ignore $F_1^a$,

$$K(t) = \left[1 - \left(1 - \frac{m}{m^*}\right)\rho_{\mathrm{s}}^0(t)\right]^{-1} . \tag{7}$$

Here we note that the temperature dependences of $K(t)$ in $UPt_3$ and $Sr_2RuO_4$ are very similar to that of $PrOs_4Sb_{12}$ [22, 23].

Now let us compare the free energy of an Abrikosov vortex and a bound pair of half quantum vortices by the $\hat{d}$-soliton. For this purpose, we consider the free energy of an Abrikosov vortex in a unit circular cell with radius $a$ which encloses a flux quantum. Then $a$ is given by

$$a = \left(\frac{\phi_0}{\pi H}\right)^{1/2} , \tag{8}$$

where $H$ is the field strength. Let us note also a similar relation due to de Gennes

$$H_{c2}(t) = \frac{\phi_0}{2\pi\xi^2(t)}\,, \tag{9}$$

where $\xi(t)$ is the coherence distance. This indicates, for $H < H_{c2}(t)$, $a > \sqrt{2}\xi(t)$. The free energy for an Abrikosov vortex is readily given by

$$\mathcal{F}_{\rm A} = \pi\chi_N C^2 K \ln(a/\xi)\,. \tag{10}$$

For a bound pair of HQVs separated by a distance $R$ and inside the unit cell,

$$\mathcal{F}_{\rm BP} = \frac{\pi}{2}\chi_N C^2\left[K\ln\left(\frac{a+\sqrt{a^2-R^2/4}}{2\xi}\right) + \ln(a/\xi)\right.$$
$$-\frac{R}{4a}\sin^{-1}\left(\frac{R}{\sqrt{a^2-R^2/4}}\right)$$
$$\left.+\frac{1}{4}\left(\frac{R}{\xi'_D}\right)^2\ln\left(\frac{2\xi'_D}{a}\frac{K+1}{\sqrt{2K+1}}\right)\right]. \tag{11}$$

Also, for $K(t) \gg 1$, this equation is optimized by choosing $R/2a = \sqrt{2K+1}/K+1$ and simplifies as [30]:

$$\mathcal{F}_{\rm BP} = \frac{\pi}{2}\chi_N C^2\left[K\ln\left(\frac{a}{2\xi}\frac{2K+1}{K+1}\right) + \ln\left(\frac{a}{\xi}\right)\right.$$
$$-\frac{1}{2}\frac{\sqrt{2K+1}}{K+1}\sin^{-1}\left(\frac{2\sqrt{2K+1}}{K+1}\right)$$
$$\left.+\frac{2K+1}{(K+1)^2}\left(\frac{a}{\xi'_D}\right)^2\ln\left(\frac{2\xi'_D}{a}\frac{K+1}{\sqrt{2K+1}}\right)\right]. \tag{12}$$

Now from $\mathcal{F}_{\rm A} = \mathcal{F}_{\rm BP}$, we find

$$H^*(t)/H_{c2}(t) = [2^{-(K-2)}(2K+1)^{(2K+1)}$$
$$\times(K+1)^{-2(K+1)}]^{-1/(K-1)}, \tag{13}$$

where $H^*(t)$ is the phase boundary below which HQVs are stable. For $PrOs_4Sb_{12}$, we show the stability region of HQVs in Fig. 3 [24], which is close to the phase boundary determined by Izawa et al. [31]. According to the present picture, the B phase discovered in Ref. [31] is the vortex state filled with HQVs. Also we have reached a similar conclusion for the B phase in $UPt_3$ [22, 23]. As to $Sr_2RuO_4$, there is no clear sign of such a phase transition. However, we suspect that the anomalous angle-dependent magnetospecific heat found by Deguchi et al. [32] below 300 mK is due to HQVs.

Finally, we comment that HQVs are accessible by scanning tunneling microscopy (STM) [33], micromagnetometry [34] and small angle neutron scattering (SANS) below, say, $T < 300$ mK. We show a local magnetic field associated with the pair of HQV in Fig. 4 [23].

## 4. Concluding remarks

We have shown that the triplet superconductors so far discovered can be put into 2 classes.

1) Type A which comprises $UPt_3$, $Sr_2RuO_4$, and $PrOs_4Sb_{12}$ etc., is characterized by $\hat{l}$ (the chiral vector) and $\hat{d}$ (the spin vector) as in $^3$He-A. We expect half quantum vortices to appear at low temperature (say below $T_c/5$) which can be accessible by STM, micromagnetometry and SANS. Physics of HQVs has to be explored.

2) Type $A_1$ which comprises most of non-centrosymmetric triplet superconductors. In these superconductors, an Abrikosov vortex does not split into a pair of HQVs.

3) All vortices in these triplet superconductors harbor the Majorana fermions. Clearly the role of these Majorana fermions has to be clarified.

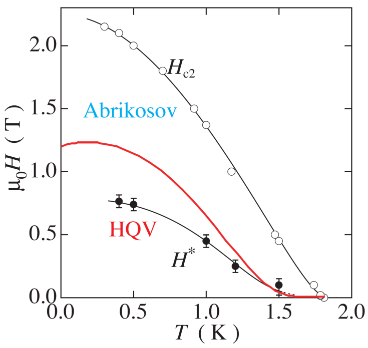


Fig. 3. For $PrOs_4Sb_{12}$, the stability region of HQV (inside the line without circles) is shown [24], which is close to the phase (whose boundary is given by black circles) determined by Izawa et al. [31].

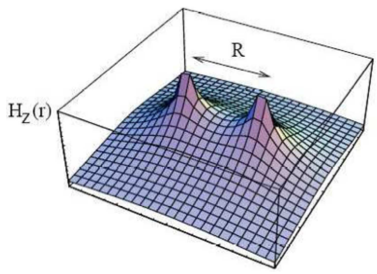


Fig. 4. A local magnetic field associated with the pair of HQV [23].

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