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Anisotropy of the Conductivity in the Asymmetric Quantum Wells

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Gorbatsevich et al. and Kibis suggested that a number of interesting galvano-magnetic effects could be observed in quantum structures where the symmetry with respect to the space coordinates inversion and time-reversal are broken simultaneously. In the paper of Kibis for example, the infinite triangular quantum well in an external magnetic field was considered and the anisotropy of electron momentum transfer due to interaction with *phonons* was predicted. The role of magnetic field was to provide the time-invariance breaking. In this work we considered the effect of anisotropy of electron momentum transfer due to interaction with *polarized light* using more realistic model of *finite* triangular quantum well. This anisotropy leads to the anisotropy of the real part of photoconductivity and as it follows from our calculations, the effect though not very great, could be measurable for the attainable values of magnetic field $B \approx 5$ T and the widths of quantum well.

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1. Introduction

In Ref. [1] it was suggested that a number of new interesting effects could be observed in quantum structures where the symmetry with respect to the space coordinates inversion and time-reversal are broken simultaneously. As an example of such structure, in Ref. [1] the asymmetric double-quantum well (QW) in an external magnetic field is considered. In this model, the space asymmetry is introduced by the δ-barriers of different heights, while external magnetic field provides time-invariance breaking. Further on, in Ref. [2] the anisotropy of mometum transfer which occurs in the *infinite* triangular quantum well in an external magnetic field was considered. The models discussed in Refs. [1, 2], despite their remarkable insights are to be amended however, since the model of δ-barriers, as wells as the infinite triangular quantum well do not seem to be very realistic. Therefore, the aim of the paper is to consider the anisotropy of momentum transfer and photoconductivity which can occur under light absorption in *finite* triangular QW in an external magnetic field.

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2. Finite triangular quantum well in an external magnetic field

To treat the two-dimensional electron gas in an external magnetic field, we start with the conventional approach based on effective mass equation of the form [3]:

$$\left[E_{\mathrm{c}}+\frac{(\mathrm{i}\hbar\nabla+e\boldsymbol{A})^2}{2m^*}+U(z)\right]\psi(\boldsymbol{r})=E\psi(\boldsymbol{r}),\tag{1}$$

where E_{c} is the bottom of the semiconductor conduction band, $\boldsymbol{A}$ — vector potential, e and m^* are the electron charge and effective mass, respectively. We choose the gauge $\boldsymbol{A}=(Bz,0,0)$ and assume the potential $U(z)$ to be

$$U(z)=\begin{cases} U_0, & z<0,\\ eEz, & 0<z\le d, \quad |eEd|=U_0\end{cases}\tag{2}$$

and suppose that the electron wave function is

$$\phi_k(x)=C\varphi(k_x,z)\exp(\mathrm{i}k_x+ik_y),\tag{3}$$

where C is the normalizing constant. Then, by means of Eqs. (1)–(3), one can arrive at the next ordinary differential equation for the $\varphi(k_x,z)$-function

$$-\frac{\hbar^2}{2m^*}\frac{\mathrm{d}^2\varphi(k_x,z)}{\mathrm{d}z^2}+\left[\frac{\hbar^2}{2m^*}+\frac{\hbar eBk_xz}{m^*}+\frac{(eBz)^2}{2m^*}+eEz-\epsilon\right]\varphi(k_x,z)=0,\tag{4}$$

where $\epsilon=E_{\mathrm{c}}+\epsilon(k_x)-\frac{\hbar^2k_y^2}{2m^*}$.

Considering the infinite triangular QW and using the analytical solutions of corresponding Schrödinger equation, the author of Ref. [2] came to the conclusion that the electron energy spectrum is anisotropic with respect to transverse motion, that is $\epsilon(v_x)\neq\epsilon(-v_x)$. According to [2], the physical reason for that is this: as an electron travels at the velocity v_x, the Lorentz force which is due to in-plane magnetic field, acts on it in the direction $\langle -x\rangle$, with the result that the maximum of the electron wave function is shifted in the direction $\langle -z\rangle$. If the electron velocity changes to $-v_x$, the Lorentz force reverses direction and the maximum of the electron wave function shifts to the direction $\langle z\rangle$. Hence in asymmetric potential $U(z)\neq U(-z)$ the electron energy $\epsilon(v_x)\neq\epsilon(-v_x)$. This claim is correct in principle and in case of *finite* triangular QW the physical explanation of such asymmetry is similar. From (4) it immediately follows that the energy spectrum of electrons in QW depends on the direction of electron movement along x-axis. Indeed, since the electrons moving along x-axis in opposite directions are shifted by the Lorentz force in opposite directions with respect to z-axis, they behave as if they were in QWs of different effective depth, and hence, $\epsilon(+k_x)\neq\epsilon(-k_x)$.

Having in mind of what was said above concerning $\epsilon(+k_x)\neq\epsilon(-k_x)$, to make the next step it is convenient to consider two equations of the form

$$\frac{\mathrm{d}^2\varphi(\zeta)}{\mathrm{d}\zeta^2}=(\zeta-\tilde\epsilon_1)\varphi(\zeta),\quad \frac{\mathrm{d}^2\varphi(\zeta)}{\mathrm{d}\zeta^2}=(\zeta-\tilde\epsilon_2)\varphi(\zeta),\tag{5}$$

where ζ, $\tilde\epsilon_1$ and $\tilde\epsilon_2$ are dimensionless quantities which are equal: $\zeta=z/z_{01}$ for the first of Eqs. (5) and $\zeta=z/z_{02}$ for the second one; $z_{01}=$

$\left[\hbar^2/2m^*(eE+eB\hbar k_x/m^*)\right]^{1/3}$, $z_{02} = \left[\hbar^2/2m^*(eE-eB\hbar k_x/m^*)\right]^{1/3}$; $\tilde{\epsilon}_1 = \epsilon/\epsilon_{01}$, $\tilde{\epsilon}_2 = \epsilon/\epsilon_{02}$ while

$$\epsilon_{01} = \left\{[(eE+eB\hbar k_x m^{*-1})\hbar]^2/2m^*\right\}^{1/3},$$
$$\epsilon_{02} = \left\{[(eE-eB\hbar k_x m^{*-1})\hbar]^2/2m^*\right\}^{1/3}.$$

The numerical solutions to Eqs. (5) for $+k_x$, $-k_x$ and the boundary conditions $\psi(0)=0, \psi(d)=0$ are shown in Table. In the table, $\epsilon_1, \epsilon_2, \epsilon_3, \ldots$ stand for the energy eigenvalues of finite triangular QW.

Here we also compare the results of numerical calculations with the energy eigenvalues obtained by means of formulae for infinite triangular QW. It is seen that in general, there is an essential difference in energy eigenvalues for these two cases.In Fig. 1 an electron energy in the ground state of finite triangular QW is

TABLE

The numerical solutions to Eqs. (5) and the boundary conditions.

	$U_0 \longrightarrow \infty$ $E = 2.12\times10^7$ V/m ϵ_n [eV]	$d = 15$ nm $U_0 = 0.318$ eV ϵ_n [eV]	$U_0 \longrightarrow \infty$ $E = 1.59\times10^7$ V/m ϵ_n [eV]	$d = 20$ nm $U_0 = 0.318$ eV ϵ_n [eV]
		$B = 0.5$ T		
$n=0$	0.14155120	0.09429694	0.11910669	0.07787549
$n=1$	0.23969517	0.16500947	0.20021054	0.13608414
$n=2$	0.31966484	0.22293948	0.26629558	0.18390778
		$B = 3$ T		
$n=0$	0.14364455	0.09429699	0.12140644	0.07787529
$n=1$	0.24337781	0.16500925	0.20425628	0.13608410
$n=2$	0.32464247	0.22293979	0.27176399	0.18390776

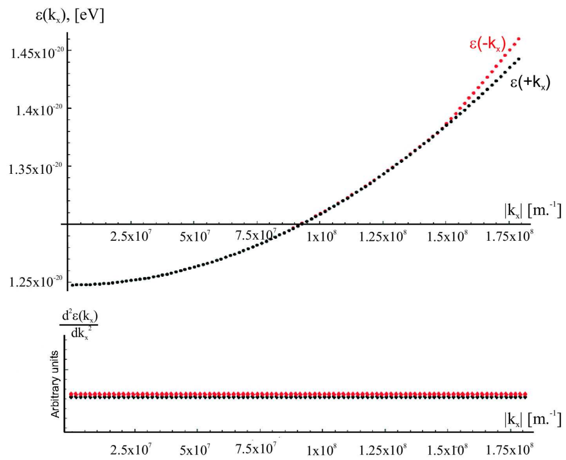

Fig. 1. Electron energy in the ground state of finite triangular QW and its second derivative versus $|k_x|$.

plotted versus $|k_x|$; it is clearly seen that indeed, starting from some definite values of $|k_x|$, $\epsilon(+k_x) \neq \epsilon(-k_x)$. In the same picture the second derivatives of energy $\epsilon(\pm k_x)$ with respect to k_x are also shown; let us notice that they are represented by straight lines up to the $k_x = k_{\rm F}$, where $k_{\rm F}$ is the Fermi wave vector.

3. Anisotropy of the photoconductivity

The anisotropy of energy $\epsilon(+k_x) \neq \epsilon(-k_x)$ suggests that the anisotropy of momentum transfer could occur under light absorption within this QW in an external magnetic field. Indeed, let us suppose that the light linearly polarized along x-axis with the wave vector $\boldsymbol{k}_{\rm ph} = (0, 0, k_{z,{\rm ph}})$ is incident on the semiconductor structure which makes a finite triangular QW (Fig. 2). That the second derivatives

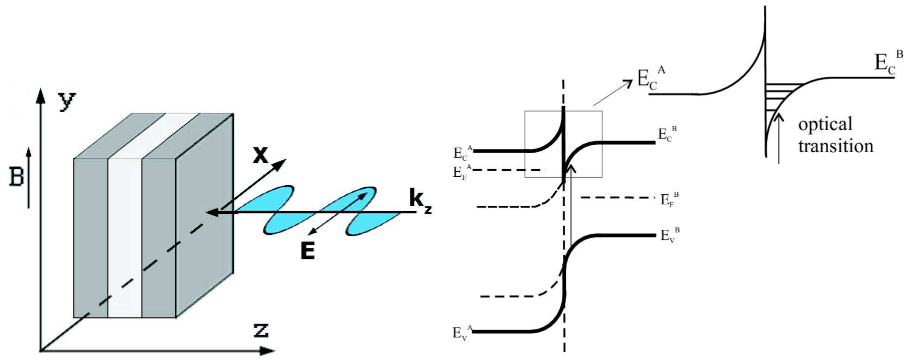

Fig. 2. Schematic representation of the structure containing triangular QW and the optical transitions.

of $\epsilon(\pm k_x)$ with respect to electron wave vector k_x are the straight lines within the wide range of changes of k_x, enables to characterize an electron movement along x-axis, as we shall see later, by means of "renormalized" effective masses which are *different* for $\langle -x \rangle$ and $\langle +x \rangle$. Then despite the fact that absorbed photons deliver to the electrons with $+k_x$ and $-k_x$ the *same momentum*, the electron momenta corresponding to $\langle -x \rangle$ and $\langle +x \rangle$ are *different*, because the "renormalized" effective masses corresponding to $+k_x$ and $-k_x$ are different.

Defining in our case the effective masses as

$$\tilde{m}_n^*(\pm k_x) = \frac{\hbar^2}{m_{\rm e}} \frac{\epsilon_n(\pm k_x)}{dk^2}$$

and calculating them numerically, we can see that $\tilde{m}_n^*(+k_x) \neq \tilde{m}_n^*(-k_x)$. We term this effective masses "renormalized" and denote them by tilde, because they depend on magnetic field B as well as an electric field E.

We now proceed to the analysis of anisotropy of photoconductivity, induced by the polarized light beam incident on the structure in question, as it is shown in Fig. 2. Here two scenarios are possible. The first one corresponds to optical inter-subband transitions between the states of two triangular QWs, one for the electrons in conduction band and another one for the holes in valence band. The second corresponds to optical transitions between valence band and the states of electrons in triangular QW in the conduction band (see Fig. 2). This situation occurs for instance, for the doped n-AlGaAs/GaAs heterojunction.

Using the standard approach (see, for instance [4]), one gets the general expression for the real part of the photoconductivity which we denote by σ:

$$\sigma = \frac{\pi e^2}{m_{\rm e}^2 \omega} \frac{2}{\Omega} \sum_{i,j} |\langle |j|\boldsymbol{e}\cdot\hat{\boldsymbol{p}}|i\rangle|^2 \left[f(E_i) - f(E_j)\right] \delta\left(E_j - E_i - \hbar\omega\right),$$

where, since the electromagnetic wave is polarized along z, $\boldsymbol{e} = (0,0,1)$ and $\boldsymbol{e}\cdot\hat{\boldsymbol{p}} = -{\rm i}\hbar\partial/\partial z$. The factor of 2 in front of the summation is for spin, $f(E_i), f(E_j)$ are the corresponding Fermi factors and Ω stands for the volume of the system. Then doing exactly in the same way as in Ref. [4], after some manipulations which include the summation of Dirac-comb, one gets the next expression for the inter--subband transitions in triangular QWs

$$\sigma^{\pm} = \frac{\pi e^2}{m_{\rm e}^2 d\omega} \sum_{n,m} |\boldsymbol{e}\cdot\boldsymbol{p}_{{\rm c}n,{\rm v}m}|^2 |\langle {\rm c}n|{\rm v}m\rangle|^2 \left(m_{\rm e}\tilde{m}^{*(\pm)}_{{\rm c}n,{\rm v}m}/\pi\hbar^2\right) \times \Theta\left[\hbar\omega - (E_{\rm g} + \epsilon_{{\rm c}n} - \epsilon_{{\rm v}m})\right]. \tag{6}$$

Here

$$\boldsymbol{e}\cdot\boldsymbol{p}_{{\rm c}n,{\rm v}m}\langle {\rm c}n|{\rm v}m\rangle \equiv \boldsymbol{e}\cdot\boldsymbol{p}_{{\rm c}n,{\rm v}m} \int \varphi^*_{{\rm c}n}(z)\varphi_{{\rm v}m}{\rm d}z \approx \langle {\rm c}n\boldsymbol{k}|\boldsymbol{e}\cdot\hat{\boldsymbol{p}}|{\rm v}m\boldsymbol{k}\rangle,$$

where "c" and "v" indicate the conduction and valence bands, n and m label the bound states within the QWs and $\boldsymbol{k}$ stands for the transverse wave vector, while the matrix element $\boldsymbol{e}\cdot\boldsymbol{p}_{{\rm c}n,{\rm v}m}$ depends on the nature of the Bloch functions and on the polarization $\boldsymbol{e}$; $\Theta(\ldots)$ is the step function and the $\pm$-superscripts of σ and $\tilde{m}^*_{{\rm c}n,{\rm v}m}$ correspond to $+k_x$ and $-k_x$, respectively. The main difference between formula (6) and the analogous standard formula from, for example Ref. [4], is that here instead of reduced effective mass $m^*_{\rm cv}$, we have $\tilde{m}^{*\pm}_{{\rm c}n,{\rm v}m}$, which are defined as follows:

$$\left(\tilde{m}^{*+}_{{\rm c}n,{\rm v}m}\right)^{-1} = \left(\tilde{m}^{*+}_{{\rm c}n}\right)^{-1} + \left(\tilde{m}^{*+}_{{\rm v}m}\right)^{-1}, \quad \left(\tilde{m}^{*-}_{{\rm c}n,{\rm v}m}\right)^{-1} = \left(\tilde{m}^{*-}_{{\rm c}n}\right)^{-1} + \left(\tilde{m}^{*-}_{{\rm v}m}\right)^{-1},$$

for the first scenario and

$$\left(\tilde{m}^{*+}_{{\rm c}n,{\rm v}m}\right)^{-1} = \left(\tilde{m}^{*+}_{{\rm c}n}\right)^{-1} + \left(m^*_{\rm h}\right)^{-1}, \quad \left(\tilde{m}^{*-}_{{\rm c}n,{\rm v}m}\right)^{-1} = \left(\tilde{m}^{*-}_{{\rm c}n}\right)^{-1} + \left(m^*_{\rm h}\right)^{-1},$$

for the second one. Here $\tilde{m}^{\pm}_{{\rm c}n}$ and $\tilde{m}^{\pm}_{{\rm v}m}$ are the renormalized effective masses of charge carriers in the QWs of conduction and valence band, respectively, while $m^*_{\rm h}$ is the standard hole effective mass.

The results of our calculations corresponding to the first scenario are presented in Fig. 3 as the $\Delta\sigma(B)/\sigma(0)$-curves plotted versus B for different d, that is the different effective QW-widths.

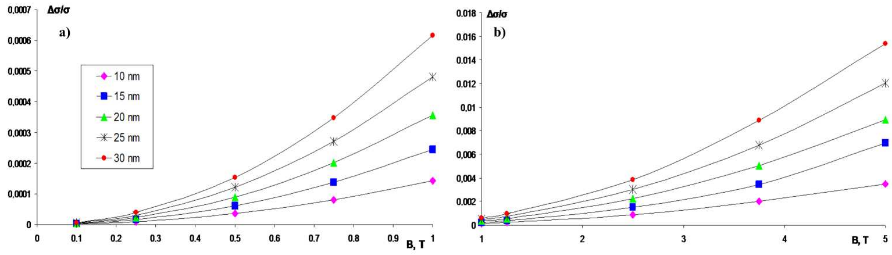

Fig. 3. $\Delta\sigma/\sigma(0)$ versus magnetic field for different QW widths.

Here $\Delta\sigma = \sigma^+ - \sigma^-$ and $\sigma(0)$ is the usual photoconductivity at zero magnetic field. The results for optical transitions between valence band and the states of electrons in triangular QW in the conduction band are very similar to these ones. It is clearly seen from Fig. 3 that the anisotropy of the photoconductivity should occur in the finite triangular QW in an external magnetic field and could be observable. Indeed, in the magnetic field of about 5 T the $\Delta\sigma/\sigma(0)$-ratio, that is the relative measure of the effect, is about 0.016 which is quite measurable value.

4. Conclusion

The results of our study can be summarized as follows. We considered the anisotropy of momentum transfer in finite triangular QW in an external magnetic field which occurs due to interaction with polarized light. We prove, by numerical solution of the Schrödinger equation, that the electron energy $\epsilon_n(+k_x) \neq \epsilon_n(-k_x)$ and that the effect mentioned above really exists. This one leads to the anisotropy of the real part of photoconductivity $\sigma(+k_x) \neq \sigma(-k_x)$ and the effect, though not very great, could be observable as it seems, for the attainable values of magnetic field $B \approx 5$ T and the widths of QW, because as it follows from our calculations, the $\Delta\sigma/\sigma(0)$-ratio, that is the relative measure of the effect, is about 0.016, which is quite measurable value.

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