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Cross-Correlations in Warsaw Stock Exchange

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We study the inter-stock correlations for the largest companies listed on Warsaw Stock Exchange and included in the WIG20 index. Our results from the correlation matrix analysis indicate that the Polish stock market can be well described by a one-factor model. We also show that the stock–stock correlations tend to increase with the timescale of returns and they approach a saturation level for the timescales of at least 200 min, i.e. an order of magnitude longer than in the case of some developed markets. We also show that the strength of correlations among the stocks crucially depends on their capitalization. These results combined with our earlier findings together suggest that now the Polish stock market situates itself somewhere between an emerging market phase and a mature market phase.

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1. Introduction

Since the pioneering work of Markowitz in 1950s [1], the financial cross-correlations are constantly a subject of extensive studies both at the theoretical and practical levels due to their fundamental relation to risk management and portfolio investing. In the field of econophysics, an interest in this type of correlations arose after it had been shown that they can be described [2–4] in the framework of random matrix theory (RMT) [5], expressing both a kind of universality and significant deviations from it. Stock market cross-correlations are typically quantified in terms of a correlation matrix, created for a set of N time series representing returns of different stocks. From this point of view, evolution of a stock market can be decomposed into N independent modes associated with eigenvalues of the correlation matrix. It occurs that a vast majority of these eigenvalues are concordant with the eigenvalue distribution of the relevant random matrix ensemble (the Wishart ensemble) [2, 6], which according to a common belief suggests that

these RMT modes do not carry any market-specific information beyond being a pure noise. Validity of this belief, however, has recently been challenged in some works [7]. As regards the remaining minority of the eigenvalues which deviate from the RMT predictions, there is a general agreement that they express the actual non-random linear dependences between the price fluctuations of different assets. Their particular number depends on a market and the number of analyzed stocks, but in each case there is an eigenvalue that strongly dominates, developing an “energy gap” that separates it from the subsequent eigenvalues. This peculiar eigenvalue is associated with a strongly delocalized eigenvector and is related to a “market mode”, i.e. a collective evolution of large group of stocks that usually closely mimics evolution of the market’s global index. From this perspective, magnitude of the largest eigenvalue reflects how collective is the evolution of an analyzed market. If, apart from the largest eigenvalue, there are also other eigenvalues which do not agree with the RMT spectrum, they correspond to smaller groups of interrelated stocks that can be usually identified with market sectors [4]. It has been found that the number of the non-random eigenvalues is highest for the largest, mature markets like New York, London, Frankfurt etc., while the less capitalized markets, e.g. the emerging ones, develop the spectrum which consists of a strongly repelled eigenvalue and the bulk with only minor disagreements in respect of the RMT prediction [8]. In fact, on small markets sectors and individual companies are too weak to be considered an optimal reference for the investors. Instead, the investors trade according to the behavior of the whole market or even they blindly follow the moves of the world’s largest markets.

Here we analyze high-frequency data from the Warsaw Stock Exchange (WSE) and inspect the correlation matrix eigenspectra calculated for a few selected groups of stocks. We address the question whether the correlation properties of the Warsaw stock market still situate it among the emerging markets or, conversely, it has already matured enough to be considered a developed market. An inspiration for rising this question is the fact that the WSE evolution shares some properties (like the broad multifractal spectra and the returns distributions which can be fitted by the q -Gaussians) with the well-established markets, as our earlier studies showed [9–11].

2. Methodology

Our tick-by-tick data covered the period from 17 November 2000 to 30 June 2005 and consisted of 39 stocks that were, at least temporarily, included during this period in WIG20 index. WIG20 is a capitalization-weighted index comprising the 20 largest companies traded on WSE. Its composition changes from time to time in order to reflect the current capitalization ranking of the WSE stocks. During the forementioned time interval only 10 stocks were constantly included in WIG20, while each of the remaining 29 stocks was contributing to WIG20 for a shorter period due to falling in the capitalization ranking or being delisted from WSE.

No.	Company Name	Ticker	In WIG20
1	KGHM	KGH	20001117-20050630
2	PROKOM	PKM	20001117-20050630
3	ORBIS	ORB	20001117-20050630
4	PEKAO	PEO	20001117-20050630
5	SOFTBANK	SFT	20001117-20050630
6	TPSA	TPS	20001117-20050630
7	AGORA	AGO	20001117-20050630
8	BRE	BRE	20001117-20050630
9	COMPLAND	CPL	20001117-20050630
10	PKNORLEN	PKN	20001117-20050630
11	BUDIMEX	BDX	20020221-20041217
12	HANDLOWY	BHW	20001117-20010316
13	INGBSK	BSK	20010319-20020621
14	COMARCH	CMR	20010319-20040319
15	DEBICA	DBC	20001117-20010316 20021223-20050617
16	GRUPAONET	GRO	20001117-20020920
17	MOSTALEXP	MSX	20001117-20010921
18	PGF	PGF	20020923-20040917
19	POLIMEXMS	PXM	20041220-20050318
20	ŻYWIEC	ZWC	20001117-20010316
21	STALEXP	STX	20040621-20050617
22	ŚWIECIE	MPP	20010319-20050630
23	NETIA	NET	20001117-20020920 20030606-20050630
24	KETY	KTY	20010924-20050630
25	MILLENNIUM	MIL	20001117-20010921 20020624-20040319
26	ELEKTRIM	ELE	20001117-20020920
27	CERSANIT	CST	20040322-20040618 20050321-20050630
28	BANKBPH	BPH	20020103-20050630
29	AMICA	AMC	20020923-20030321
30	OPTIMUS	OPT	20020923-20021220
31	TVN	TVN	20050620-20050630
32	BZWBK	BZW	20010924-20050630
33	PBK	PBK	20001117-20020102
34	NETIA2	NET2	20030324-20030605
35	MOL	MOL	20050620-20050630
36	KABLEHOLD	ELK	20001117-20020220
37	BACA	BCA	20040322-20041217
38	PKOBP	PKO	20041220-20050630
39	GTC	GTC	20040920-20050630

Fig. 1. Stocks included in WIG20 during the period 17.11.2000–30.06.2005, divided into four groups N_1 , N_2 , N_3 , N_4 . See text for details.

Although it is associated with a relatively small number of companies, WIG20 is considered the most important and influential index on WSE. This

is because the companies included in WIG20 are viewed as belonging to a core of Polish economy. Thus it is not unreasonable to expect that their statistical and correlation properties differ, at least to some extent, from the properties of other companies that are not a part of this index. Also a particular stock may change its behavior after being included in or removed from WIG20. In order to investigate this issue we divided our set of signals into 4 partially overlapping groups (see Fig. 1):

(1) Group 1 of $N_1 = 29$ signals associated with the stocks listed on WSE over the entire period.

(2) Group 2 of $N_2 = 20$ signals constituting WIG20 with changing stock content according to the actual WIG20 basket composition; signals (at the level of normalized returns) representing the replaced stocks were cut and joined with the ones representing the replacing stocks.

(3) Group 3 of $N_3 = 10$ signals for the stocks that were permanently included in WIG20; these are also the companies with the largest capitalization.

(4) Group 4 of $N_4 = 10$ signals representing the stocks with the least capitalization among the ones that were temporarily included in WIG20. We performed our calculations for each of the above groups independently.

For each individual company α , $\alpha = 1, \dots, N_k$, from the raw tick-by-tick data we extracted a time series of price evolution $p_\alpha(t_i)$, $i = 1, \dots, T$ sampled with 1 min frequency and calculated the corresponding returns according to the usual definition: $G_\alpha(t_i) = \ln p_\alpha(t_i + \tau) - \ln p_\alpha(t_i)$, where τ is the time lag. After normalizing the time series of returns to have unit variance and zero mean, the resulting length of each signal was equal to $T = 415,000$.

From N_k time series we construct an $N_k \times T$ data matrix \mathbf{M} and the correlation matrix \mathbf{C} that are related by

$$\mathbf{C} = (1/T)\mathbf{M}\mathbf{M}^T. \quad (1)$$

By diagonalizing the correlation matrix

$$\mathbf{C}\mathbf{v}^j = \lambda_j\mathbf{v}^j, \quad (2)$$

one obtains a set of its eigenvalues λ_j , $j = 1, \dots, N_k$ and eigenvectors $\mathbf{v}^j = \{v_\alpha^j\}$.

An ensemble of random matrices which can be used as a null hypothesis in our context is the ensemble of the Wishart matrices. It offers an analytic expression for a distribution of eigenvalues, known as the Marchenko–Pastur formula [12, 6]. Here we are interested in its upper λ_{\max} and lower λ_{\min} bounds only

$$\lambda_{\min}^{\max} = \sigma^2(1 + 1/Q \pm 2\sqrt{1/Q}), \quad (3)$$

with $Q = T/N_k \geq 1$ and time series variance $\sigma^2 = 1$.

3. Results

Figure 2 shows the eigenvalue spectra for each of the 4 considered groups of stocks; two time lags are used: $\tau = 10$ min (top) and $\tau = 360$ min, i.e. 1 trading day (bottom). In accordance with the remarks done in the introductory section, for

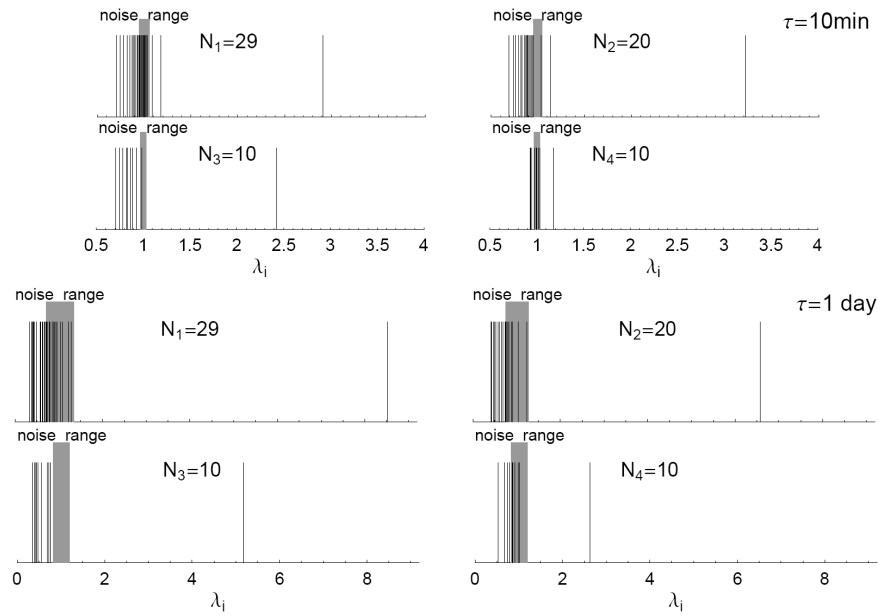


Fig. 2. Empirical eigenvalue spectrum of the correlation matrix C (vertical lines), calculated for 4 group of companies over the period 17.11.2000–30.06.2005. Two timescales are shown: $\tau = 10$ min (top) and $\tau = 360$ min = 1 day (bottom). According to RMT predictions, eigenvalues of a Wishart matrix have to lie only within the shaded region.

all the groups and for both time lags the largest eigenvalue λ_1 is repelled from the RMT range defined by Eq. (3). Clearly, for Groups 1, 2, 3 the corresponding shift is stronger than for Group 4. As regards the rest of the eigenvalues, they are close to the random matrix region; the observed discrepancies between their position and the RMT bounds can be attributed to the “squeezing” effect of large λ_1 exerted on the smaller eigenvalues [2, 7] which are shifted towards zero. The larger magnitude of λ_1 , the stronger is this effect. Agreement between the empirical eigenspectrum and the RMT prediction can be significantly improved by removing a mode associated with λ_1 from the analyzed signals (see e.g. Ref. [7, 13]). The largest eigenvalue is related to the temporal evolution of the market mode. As we see in Fig. 2, practically no other eigenvalue considerably exceeds the RMT upper bound λ_{\max} , which indicates that — besides the idiosyncratic fluctuations — the market mode is the principal and unique factor responsible for the behavior of individual stocks. A lack of other deviating eigenvalues is the evidence of weakness of a sectorization in the WSE market.

The magnitudes of the largest eigenvalues for different groups of stocks cannot be directly compared based on Fig. 2. This is because of a lack of a proper normalization: the correlation matrices for different groups have different size and different trace. Thus, in each case we divide λ_1 by the matrix trace. As a result

we obtain normalized values of λ_1 expressing a fraction of the maximum possible magnitude (equal to the matrix trace) that is absorbed by the largest eigenvalue. It thus describes the “rigidity” of a given group’s temporal evolution. The corresponding results are exhibited in Fig. 3. For both timescales, the largest normalized magnitude of λ_1 is observed for Group 3, comprising the largest companies permanently listed in WIG20. The difference between this and other groups is especially substantial for the larger timescale of 1 trading day (Fig. 3, bottom part). This indicates that the stocks for the largest companies are particularly strongly coupled with each other. Evidently smaller strength of collective movements can be seen for Group 2 (representing the actual content of WIG20), which is associated with the second-largest magnitude of λ_1 . The stocks within the remaining Groups 3 and 4 are relatively weakly correlated. In contrast to Groups 1 and 2, which at each moment comprise the stocks belonging to the WIG20 basket, the stocks from Groups 3 and 4 are not necessarily included in WIG20 during the whole studied interval of time: at each particular moment some of them belong to WIG20 and some of them do not. This also means that their capitalization is, on average, smaller than the one for the stocks from Groups 1 and 2.

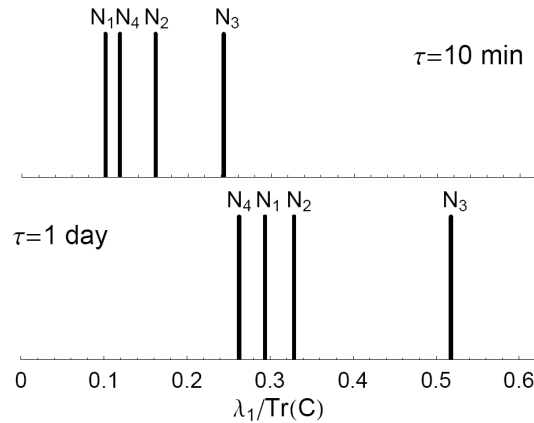


Fig. 3. Empirical eigenvalue spectrum of the correlation matrix \mathbf{C} (vertical lines), calculated for 4 groups of companies over the period 17.11.2000–30.06.2005. Two timescales are presented: $\tau = 10$ min (top) and $\tau = 360$ min = 1 day (bottom).

By comparing the results in both parts of Fig. 3, one can notice that for the longer timescale the eigenvalues assume larger magnitudes than do their counterparts for the shorter timescale. This can be a manifestation of the Epps effect [14, 15], i.e. increase in market cross-correlations with increasing timescale of the returns. In order to verify this supposition, we systematically inspected the functional dependence of λ_1 on time lag τ for a few distinct timescales $10 \text{ min} < \tau < 900 \text{ min}$. Indeed, the results collected in Fig. 4 confirm that the observed behavior of the largest eigenvalue is a consequence of the Epps effect. For all the

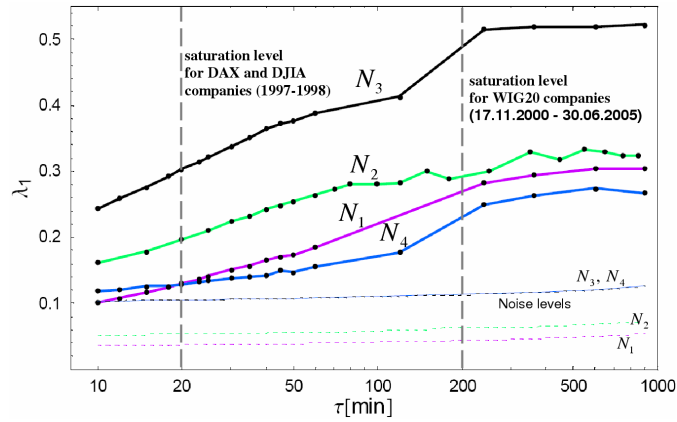


Fig. 4. Functional dependence of λ_1 on time lag τ for all analyzed groups of stocks.

groups, λ_1 increases from small values for the shortest time lags to the group-specific saturation levels for $\tau > 200$ min (compare with the American market with similar saturation occurring for $\tau > 20$ min). It is noteworthy that for Group 3 and Group 4 and for $\tau = 10$ min the largest eigenvalue is comparable in size with the noise level (λ_{\max}) predicted by RMT. This outcome resembles the analogous one obtained for the American stock market [15]: the smaller is the capitalization of a group of stocks, the less internally correlated and more noisy is the group's evolution, which — in consequence with the Epps effect — leads to a complete lack of actual inter-stock couplings for sufficiently short timescales. This phenomenon is related to the fact that investors need some time to fully react to new information and events on a stock market. Stocks of smaller companies are traded less frequently than stocks of large companies and therefore the amount of time needed to develop couplings between such stocks is considerably larger.

4. Summary

We investigated the inter-stock correlations for the relatively large companies traded on Warsaw Stock Exchange and included in the WIG20 index. We divided the full set of stocks into 4 groups depending on a particular stock's capitalization and a time interval in which the stock was included in WIG20. Our results show that the Polish stock market can basically be expressed by a one-factor model with the fully developed couplings to occur at timescales longer than half a trading day. Since these properties are characteristic of small and emerging markets and since, on the other hand, the Polish market reveals some features that are common to well-developed markets (q -Gaussian structure of the returns probability density functions (p.d.f.s) [10], multifractality [11]), we arrive at the conclusion that at present WSE is in a transition phase from being an emerging market to becoming a fully-established one. Our analysis also proved that the

strength of correlations among stocks crucially depends on their capitalization — this effect is universal for all the markets investigated so far in literature.

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