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s-Wave Order Parameter in the Presence of a Momentum-Dependent Impurity Scattering Potential

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Momentum dependence of the impurity potential leads to the anisotropy of a local density of states in isotropic s-wave superconductors. We complete this result by discussing the influence of a momentum-dependent impurity potential on the s-wave order parameter. The scattering process is considered in the T-matrix approximation, and the model impurity potential consists of two separate terms representing isotropic (on-site) and anisotropic (momentum-dependent) parts of the potential. We show that the effect of the isotropic part on the order parameter is local whereas the influence of the momentum-dependent term is nonlocal.

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1. Introduction

Lowered symmetry of the impurity potential realized by a momentum dependence of the scattering probability has impact on the local properties of superconductors in the immediate vicinity of the impurity [1–3]. Although nonmagnetic impurities do not change thermodynamic properties of an s-wave superconducting state [4], their influence can be distinguished in the local density of states or its Fourier transform [3, 5], as well as in the local change of the order parameter [6–9]. The nonmagnetic impurity effect on the s-wave order parameter has been discussed for point-like [6–9] and spherical [6] model scattering potentials. We complete these studies by including momentum dependence of the impurity potential and consider the evolution of a two-dimensional s-wave order parameter around such a scattering center. We perform our calculations in the particle–hole space using $\hat{\tau}_i$ $(i = 1, 2, 3)$ notation for the Pauli matrices, and $\hat{\tau}_0$ for the identity matrix.

2. Momentum-dependent impurity potential

A momentum-dependent impurity potential is assumed in a separable form [10]:

$$\hat{v}(\boldsymbol{k},\boldsymbol{k}') = [v_\mathrm{i} + v_\mathrm{a} f(\boldsymbol{k}) f(\boldsymbol{k}')]\hat{\tau}_3, \tag{1}$$

where v_i , v_a are potential amplitudes in the isotropic and anisotropic channels, respectively. We use a convenient α parameter which divides the scattering strength between these two channels $v_\mathrm{i} = \alpha v_0$, $v_\mathrm{a} = (1-\alpha)\, v_0$, where $0 \le \alpha \le 1$ and v_0 is a total potential amplitude. $\alpha = 1$ corresponds to isotropic and $\alpha = 0$ to a fully anisotropic scattering. We study the effect of the impurity potential of a tetragonal symmetry defined by the anisotropy function $f(\boldsymbol{k}) = \mathrm{sgn}(\cos 2\varphi)$, where φ is a polar angle of the wave vector $\boldsymbol{k}$.

3. T-matrix approximation

We consider a strong scattering regime and apply the T-matrix approach in a way appropriate for a single impurity

$$\hat{T}(\boldsymbol{k},\boldsymbol{k}',\omega) = \hat{v}(\boldsymbol{k},\boldsymbol{k}') + \sum_{\boldsymbol{k}''} \hat{v}(\boldsymbol{k},\boldsymbol{k}'')\hat{G}_0(\boldsymbol{k}'',\omega)\hat{T}(\boldsymbol{k}'',\boldsymbol{k}',\omega), \tag{2}$$

where $\hat{G}_0(\boldsymbol{k},\omega) = (\omega\hat{\tau}_0 - \xi_k\hat{\tau}_3 - \Delta_0\hat{\tau}_1)^{-1}$ is the retarded Green function of a uniform s-wave superconductor with the order parameter Δ_0 and $\xi_k = k^2/2m - \varepsilon_\mathrm{F}$ is the quasiparticle energy in the normal state with the Fermi energy ε_F. We take $\hbar = 1$. Because of a separable form of the impurity potential (1) the T-matrix reads

$$\hat{T}(\boldsymbol{k},\boldsymbol{k}',\omega) = \hat{T}_0(\omega) + \hat{T}_1(\omega) f(\boldsymbol{k}) f(\boldsymbol{k}') + \hat{T}_2(\omega) f(\boldsymbol{k}) + \hat{T}_3(\omega) f(\mathbf{k}'). \tag{3}$$

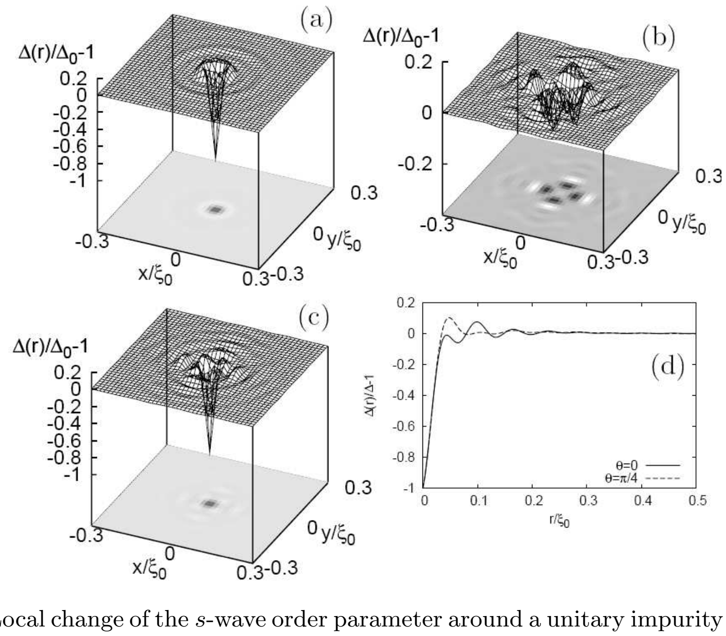

Fig. 1. Local change of the s-wave order parameter around a unitary impurity ($c = 0$) at the origin induced by (a) isotropic scattering channel, (b) anisotropic scattering channel, (c) momentum-dependent impurity potential, (d) cuts of (c) for [1,0] and [1,1] directions. The distance from the impurity is in the coherence length ξ_0 units.

For the s-wave superconductor we obtain a simple solution of Eq. (2):

$$\hat{T}_0(\omega) = \left(1 - v_\mathrm{i}\hat{\tau}_3\hat{G}_0\right)^{-1} v_\mathrm{i}\hat{\tau}_3, \qquad \hat{T}_1(\omega) = \left(1 - v_\mathrm{a}\hat{\tau}_3\hat{G}_0\right)^{-1} v_\mathrm{a}\hat{\tau}_3, \tag{4}$$

and $\hat{T}_2(\omega) = \hat{T}_3(\omega) = 0$. The T-matrix gives the impurity-induced change of the spatial Green function

$$\delta\hat{G}(\boldsymbol{r},\omega) = \sum_{\boldsymbol{k},\boldsymbol{k}'} \mathrm{e}^{\mathrm{i}(\boldsymbol{k}-\boldsymbol{k}')\boldsymbol{r}} \hat{G}_0(\boldsymbol{k},\omega)\hat{T}(\boldsymbol{k},\boldsymbol{k}')\hat{G}_0(\boldsymbol{k}',\omega), \tag{5}$$

whose off-diagonal element determines the evolution of the order parameter in the vicinity of the impurity

$$\Delta(\boldsymbol{r}) - \Delta_0 = -g\int_{-\omega_\mathrm{C}}^{\omega_\mathrm{C}} \frac{\mathrm{d}\omega}{2\pi}\delta G_{12}(\boldsymbol{r},\omega), \tag{6}$$

where $g > 0$ is an s-wave coupling constant and ω_C is a cut-off energy.

4. Results

We have evaluated the position-dependent order parameter in the vicinity of the impurity for the momentum-dependent scattering potential (1) of a tetragonal

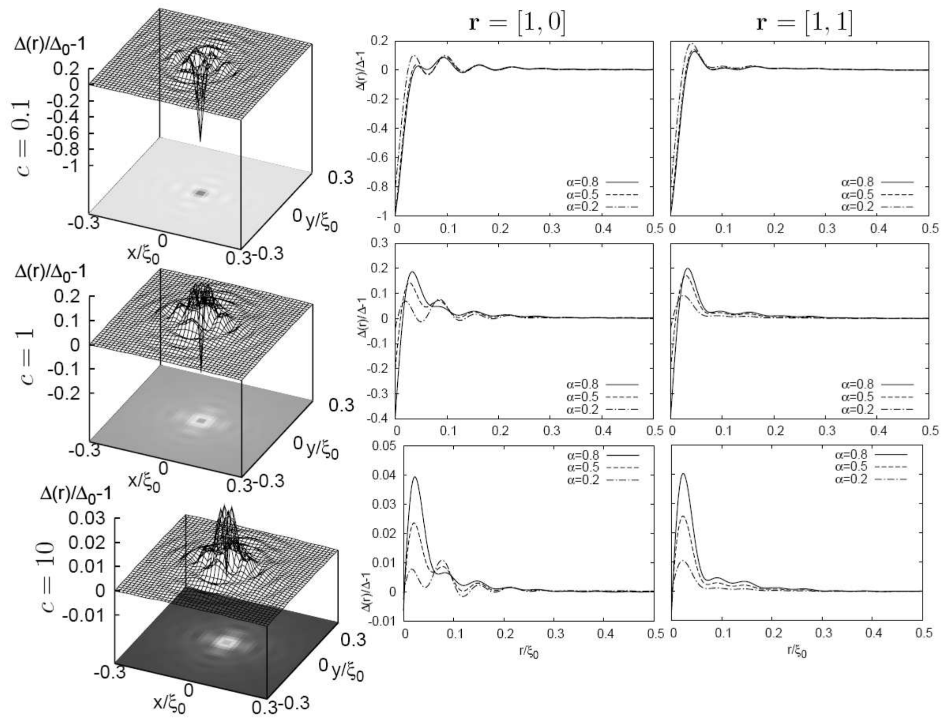

Fig. 2. Evolution of the s-wave order parameter around the impurity (first column), $\Delta(\boldsymbol{r})$ for $\boldsymbol{r} = [1,0]$ (second column) and for $\boldsymbol{r} = [1,1]$ (third column) for a finite scattering strength (from the top) $c = 0.1,\ 1.0,\ 10$ and the potential partition $\alpha = 0.8$, $\alpha = 0.5$, $\alpha = 0.2$.

symmetry. In Fig. 1 we show the evolution of the order parameter around a unitary impurity. The effect of a sole isotropic part of the impurity potential is presented in Fig. 1a while the influence of a momentum-dependent channel is shown in Fig. 1b. We note that the isotropic potential leads to a strong on--site order parameter suppression, whereas the changes induced by the anisotropic channel are nonlocal. The impact of a full anisotropic impurity scattering potential (1) on the s-wave order parameter in the unitary limit along with $\Delta(\boldsymbol{r})$ plots for [1,0] (polar angle $\Theta = 0$) and [1,1] ($\Theta = \pi/4$) directions are shown in Figs. 1c, d. We also present in Fig. 2 a local effect of a finite impurity potential on the s-wave order parameter. In the numerical computations the cut–off energy in the gap equation $\omega_{\mathrm{C}}/\Delta_0 = 18$, and the Fermi energy $\varepsilon_{\mathrm{F}}/\Delta_0 = 12\pi$ were applied. The results are plotted in gN_0 units, where $N_0 = m/2\pi$ is the normal state density of states per spin at the Fermi level, and a convenient notation of the scattering strength $c = (\pi N_0 v_0)^{-1}$ is used.

5. Conclusions

In conclusion, we note a nonlocal suppression of the order parameter by the momentum–dependent term of the scattering potential and a strong on–site effect of the isotropic part of the impurity potential. Comparison of the isotropic and momentum–dependent impurity potentials shows that the anisotropy of the scattering potential leads to a weak suppression of the order parameter. Also the symmetry of the position–dependent order parameter is determined by the symmetry of the momentum–dependent impurity potential.

References

[1] P. Pisarski, G. Harań, *Physica C* **390**, 270 (2003).
[2] A. Maciag, P. Pisarski, G. Harań, *Acta Phys. Pol. B* **34**, 479 (2003).
[3] P. Pisarski, G. Harań, *Phys. Status Solidi B* **242**, 426 (2005).
[4] P.W. Anderson, *J. Phys. Chem. Solids* **11**, 26 (1959).
[5] L. Capriotti, D.J. Scalapino, R.D. Sedgewick, *Phys. Rev. B* **68**, 014508 (2003).
[6] A.L. Fetter, *Phys. Rev.* **140**, A1921 (1965).
[7] T. Xiang, J.M. Wheatley, *Phys. Rev. B* **51**, 11721 (1995).
[8] G. Litak, *Physica B* **359-361**, 566 (2005).
[9] A. Ghosal, M. Randeria, N. Trivedi, *Phys. Rev. Lett.* **81**, 3940 (1998).
[10] G. Harań, A.D.S. Nagi, *Phys. Rev. B* **58**, 12441 (1998).