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Deformation of Two Coupled Scalar Fields with Cosmological Solution

J. Sadeghi* and A.R. Amani

Sciences Faculty, Department of Physics, Mazandaran University
P.O. Box, 47415-416, Babolsar, Iran

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We study the first-order formalism for the two coupled scalar fields with the superpotential $W(\phi,\chi)$. As we know, the cosmological solution crucially depends on the coupled scalar fields. Here, we deform the corresponding superpotential and obtain the solution for some cosmological parameters. Finally, we compare the deformed and non-deformed solutions with the different figures.

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1. Introduction

We know that the cosmic acceleration in modern cosmology are discussed in Refs. [1–3]. The amount of dark energy is almost 70% of total energy of the universe. This energy causes the present cosmic acceleration. The dark energy is a hypothetical form of energy which permeates the whole space and tends to increase rate of expansion of the universe. The existence of dark energy is the most popular way to explain recent observations that the universe appears to be expanding at an accelerating rate. The standard Friedman–Robertson–Walker (FRW) model with the real scalar fields can be a candidate to describe the dark energy. Also the presence of acceleration in cosmology leads us to consider scalar fields. In order to investigate the rate of expansion we obtain first-order differential equation which solves the corresponding equation of motion. Evidently, the presence of first-order equation eases the process of solving specific models and also can be used to fully understand the related cosmic evolution. The key motivation for the present work has appeared after the work [4] which deals with FRW cosmology driven by real scalar field. As we know, if the scale factor $a(t)$ is known, the potential $V(t)$ and

*corresponding author; e-mail: pouriya@ipm.ir

the field $\phi(t)$ are also known [4]. Therefore, the evolution equation for two scalar fields in FRW model helps us to obtain the $H(t)$ and also $\rho(t)$ and $p(t)$.

Here, we also discuss models of two scalar fields. These models have also been used to describe complex phenomena such as entrapment of topological defects and braneworld scenario in five dimensions [5–11]. We note that the field deformation method introduced in [12] for one field, also works for systems of two scalar fields. The equations arising in this extended procedure are much more complicated than the case of single scalar field. In that case two deformation functions are required. Therefore, by deforming the two-field system we impose the orbit constraint. Finally, we obtain deformed parameters as scale factor, energy density, pressure and ω.

The paper is organized as follows. In Sect. 2 we introduce the first-order formalism for the single scalar field. The function $W(\phi)$ with Hubble's parameter leads us to scenarios of current interest in cosmology. In Sect. 3 we also apply the first-order formalism for two coupled scalar fields and define the function $W(\phi,\chi)$. These fields tell us how Hubble's parameter evolves in time by function of $W(\phi,\chi)$.

In Sect. 4 we consider the special example of superpotential which not guarantees Hubble's parameter to be additive. In that case the scale factor, Hubble's parameter, energy density, pressure, acceleration parameter, and equation of state are obtained.

In Sects. 5 and 6 we deform these two systems also to obtain the cosmological solution.

2. First-order formalism for the one scalar field

To make this idea effective, we consider the spatially flat Friedman–Robertson–Walker universe which has the following space-time metric:

$$ds^2 = dt^2 - a(t)^2\left[dr^2 + r^2 d\Omega^2\right], \tag{1}$$

where $a(t)$ is the scale factor. We consider the action of single field in four dimensions; the model that we investigate is described by the action

$$S = \int d^4x\sqrt{-g}\left[\frac{-R}{16\pi G} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right], \tag{2}$$

where $V(\phi)$ is potential and R is radius of curvature, for simplicity $4\pi G = 1$. By using the following Einstein equation

$$R^\mu_\nu - \frac{1}{2}g^\mu_\nu R = 2T^\mu_\nu \tag{3}$$

and energy-momentum tensor

$$T^\mu_\nu = \mathrm{diag}(\rho, -p, -p, -p) \tag{4}$$

one can obtain

$$H^2 = \frac{2}{3}\rho, \tag{5}$$

and

$$\frac{\ddot{a}}{a} = -\frac{1}{3}(\rho + 3p). \tag{6}$$

With the help of the following energy-momentum equation:

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} L, \tag{7}$$

and

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) \tag{8}$$

the density, pressure, and equation of motion can be obtained

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{9}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{10}$$

and

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0, \tag{11}$$

where $V_\phi = \frac{\mathrm{d}V}{\mathrm{d}\phi}$. From Eqs. (5), (6) and (9) one can obtain H as,

$$\dot{H} = -\dot{\phi}^2, \tag{12}$$

$$H^2 = \frac{1}{3}\dot{\phi}^2 + \frac{2}{3}V. \tag{13}$$

Deceleration parameter is defined by

$$q = \ddot{a}a/\dot{a}^2$$

and also we have

$$q = 1 + \frac{\dot{H}}{H^2}. \tag{14}$$

The equation of state is

$$\omega = \frac{p}{\rho}. \tag{15}$$

All equations depend on time evolution, such as $\phi = \phi(t)$, $a = a(t)$, $H = H(t)$, and $q = q(t)$.

In order to have a solution for the above equation we introduce superpotential function ($W = W(\phi)$) [13] which is defined by the following expression:

$$H = W(\phi), \tag{16}$$

so the first-order equation is

$$\dot{\phi} = -W_\phi. \tag{17}$$

Therefore the potential, energy density, pressure and equation state can be written in terms of superpotential as follows:

$$V = \frac{3}{2}W^2 - \frac{1}{2}W_\phi^2, \tag{18}$$

$$\rho = \frac{3}{2}W^2, \tag{19}$$

$$p = W_\phi^2 - \frac{3}{2}W^2, \tag{20}$$

$$q = 1 - \frac{W_\phi^2}{W^2}, \tag{21}$$

$$\omega = \frac{2}{3}\frac{W_\phi^2}{W^2} - 1. \tag{22}$$

Here we note that for the universe with equation of state, $\omega = 1/3, -1$, and 0 we have radiation, fluid, and dust, respectively.

3. First-order formalism for the two scalar fields

We are going to consider two coupled real fields. In particular, the Einstein–Hilbert action with two real fields is

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{-R}{4} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\phi,\chi)\right]. \tag{23}$$

Therefore, the evolution equation for two scalar fields in FRW model will have the following form:

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0, \tag{24}$$

$$\ddot{\chi} + 3H\dot{\chi} + V_\chi = 0. \tag{25}$$

In the spatially flat FRW universe, the effective density and the effective pressure of the scalar fields can be described by

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + V, \tag{26}$$

$$p = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 - V. \tag{27}$$

Now we are going to write the following first-order equation:

$$H = W(\phi,\chi), \qquad \dot{\phi} = -W_\phi, \qquad \dot{\chi} = -W_\chi, \tag{28}$$

and the FRW equation and potential are

$$\dot{H} = -\dot{\phi}^2 - \dot{\chi}^2, \tag{29}$$

$$H^2 = \frac{1}{3}\dot{\phi}^2 + \frac{1}{3}\dot{\chi}^2 + \frac{2}{3}V, \tag{30}$$

$$V = \frac{3}{2}W^2 - \frac{1}{2}W_\phi^2 - \frac{1}{2}W_\chi^2, \tag{31}$$

and also the corresponding energy density, pressure and declaration parameters are

$$\rho = \frac{3}{2}W^2, \tag{32}$$

$$p = W_\phi^2 + W_\chi^2 - \frac{3}{2}W^2, \tag{33}$$

$$q = 1 - \frac{W_\phi^2 + W_\chi^2}{W^2}. \tag{34}$$

4. Example of two scalar fields without deformation

As we know, the superpotential in two scalar fields system can appear in two ways. The first case is $W(\phi,\chi) = W_1(\phi) + W_2(\chi)$ and the potential can be written in the form of $V(\phi,\chi) = V_1(\phi)+V_2(\chi)+3W_1(\phi)W_2(\chi)$. Therefore, Hubble's parameter is additive $H(\phi(t),\chi(t)) = H_1(\phi(t)) + H_2(\chi(t))$, and the solution for the cosmological aspect will be simple.

In the other case which we have $W(\phi,\chi) \neq W_1(\phi)+W_2(\chi)$ and the potential cannot be written as above, so the Hubble parameter is not additive. Then the solution is more complicated than in the first case.

The example we demonstrate here is the second case and Hubble's parameter is not additive.

Therefore we consider the superpotential of two scalar fields as [14, 15]:

$$W(\phi,\chi) = \phi - \frac{1}{3}\phi^3 - r\phi\chi^2, \tag{35}$$

where ϕ and χ are two fields, and r is dimensionless and real parameter.
The first-order Eq. (29) leads us to

$$\dot{\phi} + 1 - \phi^2 - r\chi^2 = 0, \quad \dot{\chi} - 2r\phi\chi = 0. \tag{36}$$

In order to solve this system, we consider an elliptical orbit as

$$\chi = \sqrt{\frac{1-2r}{r}(1-\phi^2)}, \tag{37}$$

and the solutions for two fields are

$$\phi = \tanh(2rt), \quad \chi = \sqrt{\frac{1-2r}{r}}\operatorname{sech}(2rt). \tag{38}$$

From Eq. (32) one can obtain the potential as

$$V(t) = \frac{2}{3}[(-1+6r-9r^2)\operatorname{sech}^6(2rt) + 3(1-3r)\operatorname{sech}^4(2rt) \\ +3(2r^2+r-1)\operatorname{sech}^2(2rt) + 1]. \tag{39}$$

In Fig. 1 we plot two fields ϕ , χ an V with respect to time evolution. These plots of fields are similar kink and lump and V has minima in negative part and also have symmetry with respect to the axis.

Also the Hubble parameter and scale factor are, respectively:

$$H(t) = \frac{2}{3}\frac{\sinh(2rt)}{\cosh^3(2rt)}[\sinh^2(2rt) + 3r], \tag{40}$$

$$a(t) = \cosh^{\frac{1}{3r}}(2rt)\exp\left(\frac{1-3r}{6r}\operatorname{sech}^2(2rt)\right). \tag{41}$$

The graph of Hubble's parameter and scale factor with respect to time evolution have are in Fig. 2. This tells us that the universe always expands eternally for $r = 0.3$.

The energy density and pressure are given as,

$$\rho(t) = \frac{1}{6}\tanh^2(2rt)[2 + (6r-2)\operatorname{sech}^2(2rt)]^2, \tag{42}$$

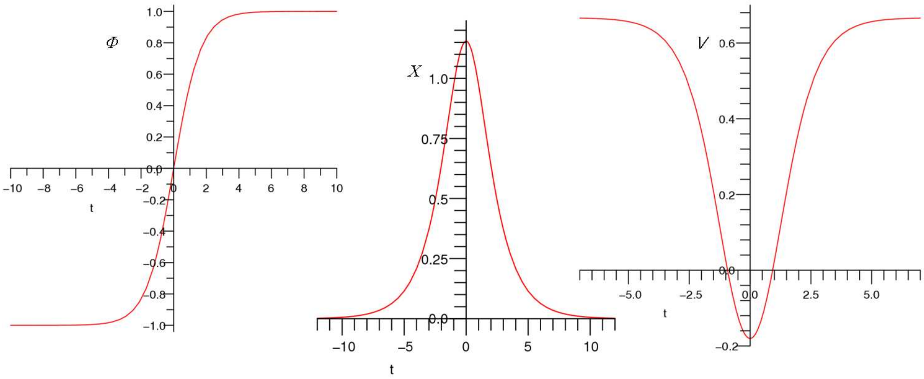

Fig. 1. The field $\phi(t)$ (left), the field $\chi(t)$ (middle) and the potential $V(t)$ (right) are plotted for $r = 0.3$.

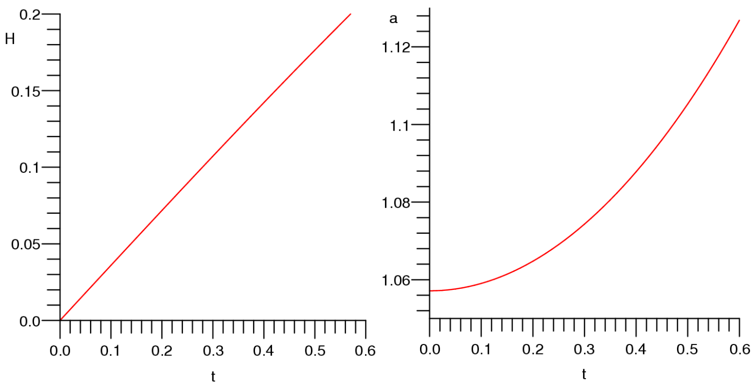

Fig. 2. The $H(t)$ (left) and scale factor a (right) are plotted for $r = 0.3$.

$$p(t) = \frac{2}{3}[(9r^2 - 6r + 1)\text{sech}^6(2rt) - 3(9r^2 - 6r + 1)\text{sech}^4(2rt) + 3(4r^2 - 4r + 1)\text{sech}^2(2rt) - 1]. \tag{43}$$

We can see in Fig. 3 for $r = 0.3$ the pressure decreases asymptotically, and going to negative. The energy density starts positively.

We note that the acceleration parameter and ω in this model are given by

$$q(t) = \frac{\cosh^6(2rt) - 3(6r^2 - 5r + 1)\cosh^4(2rt)}{\sinh^2(2rt)[\sinh^2(2rt) + 3r]^2} + \frac{3(12r^2 - 7r + 1)\cosh^2(2rt) - 9r^2 + 6r - 1}{\sinh^2(2rt)[\sinh^2(2rt) + 3r]^2},$$

$$\omega = \frac{-\cosh^6(2rt) + 3(4r^2 - 4r + 1)\cosh^4(2rt)}{\sinh^2(2rt)[\sinh^2(2rt) + 3r]^2}. \tag{44}$$

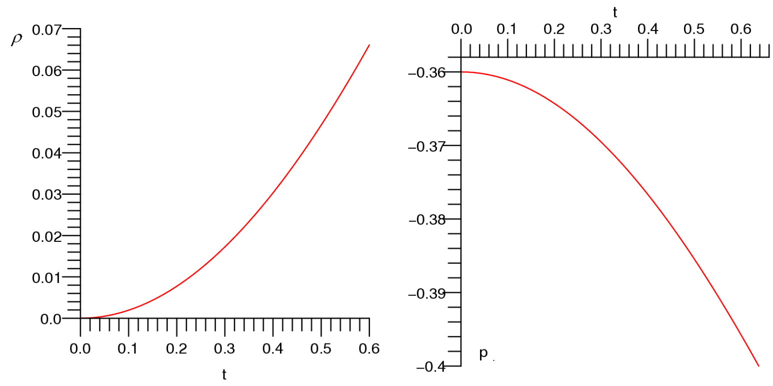

Fig. 3. The energy density of $\rho(t)$ (left) and pressure $p(t)$ (right) are plotted for $r = 0.3$.

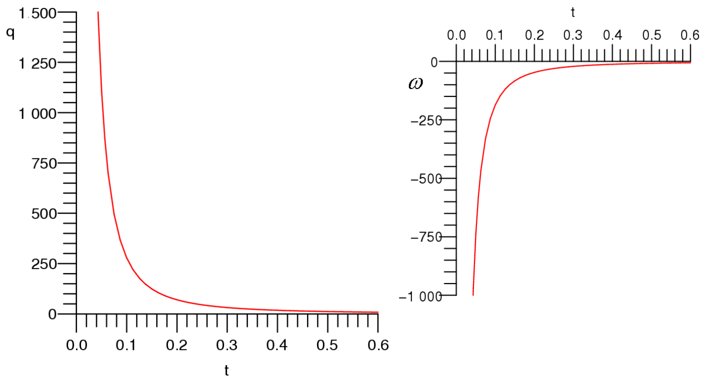

Fig. 4. The acceleration parameter q (left) and equation of state ω (right) are plotted for $r = 0.3$.

$$-\frac{3(9r^2-6r+1)\cosh^2(2rt)+9r^2-6r+1}{\sinh^2(2rt)[\sinh^2(2rt)+3r]^2}. \tag{45}$$

The graphs of acceleration parameter and equation of state are shown in Fig. 4. The acceleration parameter decreases from positive value and becomes constant in $t = 0$. Equation of state increases from negative value to zero asymptotically.

5. Deformation procedure for single scalar field

We are going to discuss deformation procedure and investigate one scalar field model [16]. Therefore, we define deformation function as $\phi(t) = f(\widetilde{\phi}(t))$, that $\phi(t)$ and $\widetilde{\phi}(t)$ are initial real scalar field and deformed scalar field respectively. Deformation function is a differentiable and invertible function. We can write inverse of deformation function as $\widetilde{\phi} = f^{-1}(\phi)$. Then, the deformed superpotential $\widetilde{W}(\widetilde{\phi})$ for cosmological models in terms of $W(\phi)$ and $f(\widetilde{\phi})$ will be

$$\widetilde{W}_{\widetilde{\phi}}(\widetilde{\phi}) = \frac{W_\phi(\phi)}{\frac{\mathrm{d}f(\widetilde{\phi})}{\mathrm{d}\widetilde{\phi}}}, \tag{46}$$

where deformed potential of Eq. (19) can be written by

$$\widetilde{V}(\widetilde{\phi}) = \frac{2}{3}\widetilde{W}(\widetilde{\phi})^2 - \frac{1}{2}\widetilde{W_{\widetilde{\phi}}(\widetilde{\phi})}^2. \tag{47}$$

Thus, the deformed $\widetilde{H}(t)$ and $\widetilde{a}(t)$ parameters will be

$$\widetilde{H}(t) = \widetilde{W}(\widetilde{\phi}(t)), \tag{48}$$

$$\widetilde{a}(t) = \exp\left(\int \widetilde{H}(t)\mathrm{d}t\right). \tag{49}$$

The Einstein–Hilbert action with deformed fields is

$$S = \int \mathrm{d}^4x\sqrt{-\widetilde{g}}\left[\frac{-R}{4} + \frac{1}{2}\partial_\mu\widetilde{\phi}\partial^\mu\widetilde{\phi} + \frac{1}{2}\partial_\mu\widetilde{\chi}\partial^\mu\widetilde{\chi} - \widetilde{V}(\widetilde{\phi},\widetilde{\chi})\right]. \tag{50}$$

The equations of motion are

$$\ddot{\widetilde{\phi}} + 3\widetilde{H}\dot{\widetilde{\phi}} + \widetilde{V}_{\widetilde{\phi}} = 0, \tag{51}$$

$$\ddot{\widetilde{\chi}} + 3\widetilde{H}\dot{\widetilde{\chi}} + \widetilde{V}_{\widetilde{\chi}} = 0. \tag{52}$$

Thus, the deformed cosmological parameters will be

$$\widetilde{\rho} = \frac{3}{2}\widetilde{H}^2, \tag{53}$$

$$\widetilde{p} = -\dot{\widetilde{H}} - \frac{3}{2}\widetilde{H}^2, \tag{54}$$

$$\widetilde{q} = 1 + \frac{\dot{\widetilde{H}}}{\widetilde{H}^2}, \tag{55}$$

$$\widetilde{\omega} = -1 - \frac{2}{3}\frac{\dot{\widetilde{H}}}{\widetilde{H}^2}, \tag{56}$$

where $\widetilde{H}$, $\widetilde{a}$, $\widetilde{\rho}$, $\widetilde{p}$, $\widetilde{q}$ and $\widetilde{\omega}$ are deformed parameters of Hubble's parameter, scale factor, energy density, pressure, deceleration parameter and equation of state, respectively.

6. Deformation for two coupled scalar fields

The deformation for two fields has been considered before in Ref. [17]. Now we generalize deformation procedure for two scalar fields, so we change two initial scalar fields $\phi(t)$ and $\chi(t)$ in the form of deformed scalar fields $\widetilde{\phi}(t)$ and $\widetilde{\chi}(t)$. We have to introduce two deformation functions which are function of fields

$$\phi(t) = f_1(\widetilde{\phi}), \tag{57}$$

$$\chi(t) = f_2(\widetilde{\chi}). \tag{58}$$

Also the deformation functions (58) and (59) are differentiable and invertible. We can write inverse of deformation functions as $\widetilde{\phi} = f_1^{-1}(\phi)$ and $\widetilde{\chi} = f_2^{-1}(\chi)$. We define the following condition for the deformation function.

$$\frac{\mathrm{d}f_1(\widetilde{\phi})}{\mathrm{d}\widetilde{\phi}} = \frac{\mathrm{d}f_2(\widetilde{\chi})}{\mathrm{d}\widetilde{\chi}}. \tag{59}$$

In order to obtain deformation function, first we have to guess f_1, and then obtain f_2 from Eqs. (59) and (60). Therefore, one can rearrange the superpotential as follows:

$$\widetilde{W} = \int \frac{W_\chi}{\frac{\mathrm{d}f_2(\widetilde{\chi})}{\mathrm{d}\widetilde{\chi}}} \mathrm{d}\widetilde{\chi}. \tag{60}$$

As we showed before, the standard FRW cosmology can be described by the first-order differential equations, so the deformation function of superpotential leads us to obtain all parameters in FRW model.

Now we deform the superpotential (36) and obtain the cosmological solution. We choose initial function ϕ in term of deformed function $\widetilde{\phi}$ as follows:

$$\phi = f_1(\widetilde{\phi}) = \tan(\widetilde{\phi}). \tag{61}$$

With the use of Eq. (39), one can obtain $\widetilde{\phi}$ as

$$\widetilde{\phi} = \arctan(\tanh(2rt)), \tag{62}$$

and also the orbit equation gives us

$$\chi = \sqrt{\frac{1-2r}{r}(1-\phi^2)}, \tag{63}$$

where

$$\widetilde{\chi} = \int \frac{\mathrm{d}\widetilde{\phi}}{\mathrm{d}\phi} \mathrm{d}\chi. \tag{64}$$

In this case we have

$$\chi = f_2(\widetilde{\chi}) = \sqrt{\frac{2(1-2r)}{r}} \tanh\left(\sqrt{\frac{2r}{1-2r}}\widetilde{\chi}\right). \tag{65}$$

By using Eq. (39) we obtain function $\widetilde{\chi}(t)$ as

$$\widetilde{\chi} = \sqrt{\frac{1-2r}{2r}} \operatorname{arctanh}\left(\frac{1}{\sqrt{2}} \operatorname{sech}(2rt)\right). \tag{66}$$

In Fig. 5 we plot both the two deformed fields $\widetilde{\phi}$ and $\widetilde{\chi}$ in dependence on time for some choice of parameter.

The deformed superpotential is given by an orbit equation as follows:

$$\widetilde{W} = (1-2r)\left(\arctan(\phi) - \frac{\phi}{1+\phi^2}\right). \tag{67}$$

Inserting equation $\phi = \tanh(2rt)$ in above equation we obtain deformed superpotential in terms of time evolution as

$$\widetilde{H}(t) = \widetilde{W}(t) = (1-2r)\left[\arctan(\tanh(2rt)) - \frac{\tanh(2rt)}{1+\tanh^2(2rt)}\right], \tag{68}$$

and the deformed potential is given by

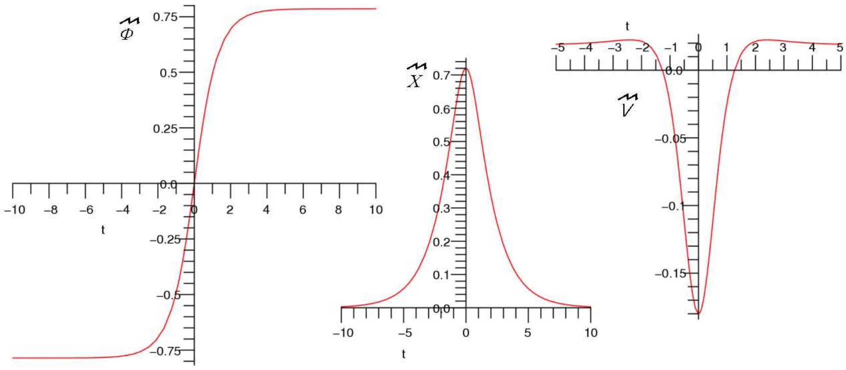

Fig. 5. The deformed field $\widetilde{\phi}(t)$ (left), the deformed field $\widetilde{\chi}(t)$ (middle), and the deformed potential $\widetilde{V}(t)$ (right) are plotted for $r = 0.3$.

$$\widetilde{V}(t) = \frac{3}{2}\widetilde{W}^2 - \frac{1}{2}\dot{\widetilde{\phi}}^2 - \dot{\widetilde{\chi}}^2. \tag{69}$$

Finally we have

$$\widetilde{V}(t) = \frac{3}{2}(1-2r)^2\left[\arctan(\tanh(2rt)) - \frac{\tanh(2rt)}{1+\tanh^2(2rt)}\right]^2$$
$$-\frac{2r\mathrm{sech}^2(2rt)}{[1+\tanh^2(2rt)]^2}[\mathrm{rsech}^2(2rt) - (1-2r)\tanh^2(2rt)^2]. \tag{70}$$

We can see the deformed potential in Fig. 5 and is similar to non-deformed potential.

Now we are going to discuss the Hubble parameter $\widetilde{H}(t)$; then we obtain all cosmological parameters. In this case, we firstly assume that $r = 0.3$, so the $\widetilde{H}(t)$ is

$$\widetilde{H}(t) = \frac{2}{5}\left[\arctan(\tanh(0.6t)) - \frac{\tanh(0.6t)}{1+\tanh^2(0.6t)}\right]. \tag{71}$$

Figure 6 shows $\widetilde{H}$ in terms of time evolution, and also it is similar to the non-deformed case. In order to calculate $\widetilde{a}(t)$, we cannot obtain the first term of integral, so for the simplicity we take

$$\arctan(\tanh(0.6t)) = 0.78\tanh(0.7t). \tag{72}$$

Because the two functions on the left and right hand side have the same graphs, so the deformed scale factor can be written by

$$\widetilde{a}(t) = \exp(\frac{i\pi}{30})\mathrm{sech}^{\frac{-2}{5}}(0.7t)(2\cosh(0.6t)-1)^{\frac{-1}{6}}, \tag{73}$$

and the real part is

$$\widetilde{a}(t) = \cos\left(\frac{\pi}{30}\right)\mathrm{sech}^{\frac{-2}{5}}(0.7t)[2\cosh(0.6t)-1]^{\frac{-1}{6}}. \tag{74}$$

In Fig. 6, $\widetilde{a}$ changes with respect to time evolution, and is similar to the non-deformed case which also changes exponentially.

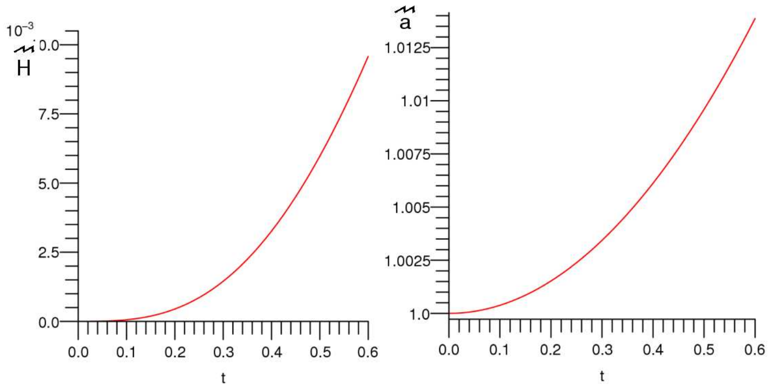

Fig. 6. The deformed Hubble parameter $\widetilde{H}(t)$ (left) and deformed scale factor $\widetilde{a}(t)$ (right) are plotted for $r = 0.3$.

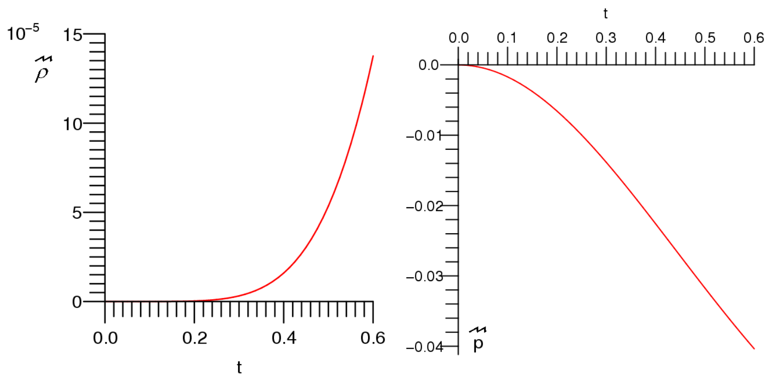

Fig. 7. The deformed energy density $\widetilde{\rho}(t)$ (left) and deformed pressure $\tilde{p}(t)$ (right) are plotted for $r = 0.3$.

The deformed density energy and pressure are given by,

$$\widetilde{\rho} = \frac{3}{2}\widetilde{H}^2 = \frac{6}{25}\left[\arctan(\tanh(0.6t)) - \frac{\tanh(0.6t)}{1+\tanh^2(0.6t)}\right]^2, \tag{75}$$

$$\widetilde{p} = -\dot{\widetilde{H}} - \widetilde{\rho} = -\frac{12}{25}\frac{\tanh^2(0.6t)\mathrm{sech}^2(0.6t)}{(1+\tanh^2(0.6t))^2} \\ -\frac{6}{25}\left[\arctan(\tanh(0.6t)) - \frac{\tanh(0.6t)}{1+\tanh^2(0.6t)}\right]^2. \tag{76}$$

In Fig. 7 we see the variation of $\widetilde{\rho}$ and $\widetilde{p}$ with respect to time evolution. In that case pressure is initially positive and decreases asymptotically and also goes to be negative. Also we say that the energy density starts positive and similary as in non-deformed case also changes by scale.

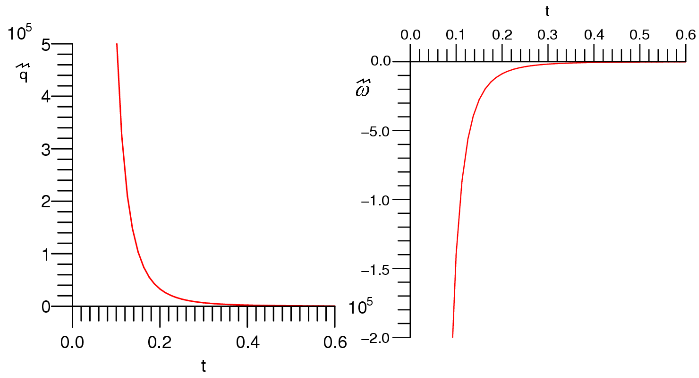

Fig. 8. The deformed acceleration parameter $\widetilde{q}(t)$ (left) and deformed equation of state $\widetilde{\omega}$ (right) are plotted for $r = 0.3$.

Also we obtain deformed deceleration parameter and equation of state as follows:

$$\widetilde{q} = 1 + \frac{\dot{\widetilde{H}}}{\widetilde{H}^2} = 1 + 3\frac{\tanh^2(0.6t)\mathrm{sech}^2(0.6t)}{[(1+\tanh^2(0.6t))\arctan(\tanh(0.6t)) - \tanh(0.6t)]^2}, \quad (77)$$

$$\widetilde{\omega} = -1 - 2\frac{\tanh^2(0.6t)\mathrm{sech}^2(0.6t)}{[(1+\tanh^2(0.6t))\arctan(\tanh(0.6t)) - \tanh(0.6t)]^2} \quad (78)$$

In Fig. 8, graphs of deformed acceleration parameters and equation of state are similar to their non-deformed forms.

7. Conclusion

In this paper we have considered models described by two coupled real scalar fields. This model leads us to introduction of the function $W(\phi, \chi)$. The relation of this function to Hubble's parameter leads us to discussion about cosmological solution. We have investigated this system for deformed and non-deformed cases. Therefore, finally we say that the deformation procedure for two coupled scalar field just changes the energy density by scale but that gives us the exact variation of pressure of the universe. It may be interesting to discuss this paper to Anti-de-Sitter and de-Sitter geometry [18].

References

[1] A. Riess, A.V. Fillippenko, P. Challis, A Clocchiatti, A Diercks, P. Garnovich, R.L. Gilliland, C.J. Hogan, S. Iha, R.P. Kirschner, B. Leibundgut, M.M. Phillips, D. Reiss, B.P. Schmidt, R.A. Schommer, R.C. Smith, J. Spyromilio, C. Stubbs, N.B. Suntzeff, J. Tonry, *Astron. J.* **116**, 1009 (1998).

[2] S.J. Perlmutter, G. Aldering, G. Goldhaber, R.A. Knop, P. Nugent, P.G. Castro, S. Deustua, S. Fabbro, A. Goobar, D.E. Groom, I.M. Hook, A.G. Kim, M.Y. Kim, J.C. Lee, N.J. Nunes, R. Pain, C.R. Pennypacker, R. Quimby, *Astrophys. J.* **517**, 565 (1999).

[3] N.A. Bachall, J.P. Ostriker, S.J. Perlmutter, P.J. Steinhardt, *Science* **284**, 1481 (1999).

[4] J. Frieman, C. Hill, A. Stebbins, I. Waga, *Phys. Rev. Lett.* **75**, 2077 (1995).

[5] J.R. Morris, *Phys. Rev. D* **51**, 697 (1995).

[6] J.R. Morris, *Int. J. Mod. Phys. A* **13**, 1115 (1998).

[7] D. Bazeia, F.A. Brito, *Phys. Rev. D* **61**, 105019 (2000).

[8] D. Bazeia, F.A. Brito, *Phys. Rev. D* **62**, 101701 (R) (2000).

[9] D. Bazeia, F.A. Brito, L. Losano, *J. High Energy Phys.* **0611**, 064 (2006).

[10] P. Stucliffe, *Phys. Rev. D* **68**, 085004 (2003).

[11] J. Sadeghi, A. Mohammadi, *Eur. Phys. J. C* **49**, 859 (2007).

[12] D. Bazeia, L. Losano, J.M.C. Malbouisson, *Phys. Rev. D* **66**, 101701 (R) (2002).

[13] D. Bazeia, C.B. Gomes, L. Losano, R. Menzes, *Phys. Lett. B* **633**, 415 (2006).

[14] D. Bazeia, J.R.S. Nascimento, R.F. Riberio, D. Toledo, *J. Phys. A* **30**, 8157 (1997).

[15] D. Bazeia, M.J. dos Santo, R.F. Riberio, *Phys. Lett. A* **208**, 84 (1995).

[16] D. Bazeia, L. Losano, A.B. Pavan, astro-ph/0611021.

[17] V.I. Afonso, D. Bazeia, M.A. Gonzalez Leon, L. Losano, J. Mateos Guilarte, *Phys. Rev. D* **76**, 025010 (2007).

[18] J. Sadeghi, A. Mohammadi, *Mod. Phys. Lett. A* **22**, 1349 (2007).