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Transmission and Reflection of Spin-Polarized Electrons Propagating through a Model Domain Wall

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An analytically tractable simple model of a magnetic domain wall in a ferromagnetic metal is considered and, assuming the ballistic regime of electronic transport, transmission and reflection coefficients of such a wall are calculated within the stationary scattering theory. It is rederived that for realistic values of electron energies and domain wall widths the transmission coefficient is very close to one and thus an ideal domain wall itself (i.e. not taking into account other aspects such as disorder) does not essentially represent a hindrance to the transport.

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1. Introduction

Electronic transport through magnetic multilayers, including the transport through a domain Bloch wall in a ferromagnetic metal, has recently attracted attention of both experimentalists and theorists [1, 2]. While some experiments show a positive contribution of the wall to the material resistivity (nota bene giant magnetoresistance), others indicate a negative one [3, 4]. As has become clearer, quite a few distinct phenomena influence the resulting domain wall contribution. It is therefore essential to gain a truthful picture of the relative strengths of these contributions. This article aims to elucidate the effect of the changing magnetization in the wall on the electronic transport, leaving all other possible factors unattended. We designed a model domain wall the properties of which are to some extent analytically accessible. The results obtained are not completely new [5, 6], but are nontheless demonstrated on the simplest possible basis. This simplicity constitutes the main contribution of the presented work.

2. Model domain wall

In our proposed one-dimensional model of a domain wall two identical jellium-like ferromagnets with opposite magnetization directions are connected by a wall of finite width. We consider only the simplest case of a 180° domain wall. The magnetization inside the wall (an intermediate region, I) is supposed to rotate uniformly between the left (L) and right (R) boundary values, see Fig. 1. Although this is not fully realistic, it may still be taken as an approximation that makes the model analytically manageable. (For a more precise description of a wall anatomy see e.g. [7].) The Hamiltonian describing the electronic movement then reads (in x-representation)

$$H = p^2 + \gamma \boldsymbol{\sigma} \cdot \boldsymbol{M}(x), \quad p = -\mathrm{i}\partial_x, \ \boldsymbol{M}(x) = \begin{cases} (1,0,0), \ x \in \mathrm{L}, \\ (\cos qx, \sin qx, 0), \ x \in \mathrm{I}, \\ (-1,0,0), \ x \in \mathrm{R}, \end{cases} \tag{1}$$

where the first term represents the kinetic energy (we set $\hbar = 2m = 1$), γ characterizes the strength of the exchange splitting due to the magnetization $\boldsymbol{M}$, q determines the rate at which the magnetization rotates in the wall, x is the distance along the wall ($x = 0$ at the L/I interface and $x = \pi/q$ at the I/R interface) and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices. The value of γ is considered negative, so that the electron energy is lowest when its spin is parallel to the local magnetization.

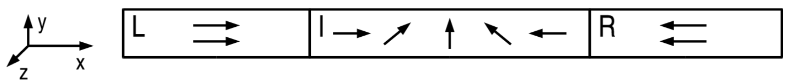

Fig. 1. Schematic structure of a domain wall.

Assuming the system is in the ballistic regime we calculate the reflection and transmission coefficients of an electron propagating through the wall. Thanks to the simplicity of our model, the eigenfunctions of the Hamiltonian can be explicitly found. The solutions in the homogeneous ferromagnets L and R are

$$\psi_{\mathrm{L}} = 1 \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mathrm{e}^{\mathrm{i}kx} + r_1 \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mathrm{e}^{-\mathrm{i}kx} + r_2 \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathrm{e}^{-\mathrm{i}\bar{k}x}, \tag{2}$$

$$\psi_{\mathrm{R}} = t_1 \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mathrm{e}^{\mathrm{i}\bar{k}x} + t_2 \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathrm{e}^{\mathrm{i}kx}, \tag{3}$$

where the values of k, $\bar{k}$ are determined from the energy relation $E = k^2 - |\gamma| = \bar{k}^2 + |\gamma|$, and r_1, r_2, t_1, t_2 are coefficients to be found, determining the individual transmission and reflection coefficients $R_1 = |r_1|^2$, $R_2 = |r_2|^2 \bar{k}/k$, $T_1 = |t_1|^2 \bar{k}/k$, $T_2 = |t_2|^2$ for real $\bar{k}$, otherwise $R_2 = T_1 = 0$.

In the form of the solutions (2), (3) we have incorporated our decision to deal only with fully spin-polarized incoming electrons (the first term of ψ_{L} with coefficient 1 with polarization along $\boldsymbol{M}$) and included only outgoing waves in ψ_{R} (k is always meant positive and real, $E > -|\gamma|$, so that there is a propagating incoming wave in L, while $\bar{k}$, if real, is also taken positive, and if purely imaginary, the solution with positive imaginary part is chosen).

Solutions in the region I of rotating magnetization can be found with the help of the operator $O = p + q\sigma_z/2$, which commutes with the Hamiltonian, as can be shown by direct calculation. The eigenfunctions of H can then be taken as simultaneous eigenfunctions of O, which are found to be

$$\left(p + \frac{q}{2}\sigma_z\right)\psi_{\mathrm{I}} = \chi\psi_{\mathrm{I}} \quad \Rightarrow \quad \psi_{\mathrm{I}}(x) = \begin{pmatrix} c_1 \exp\left(\mathrm{i}\left(\chi - \frac{q}{2}\right)x\right) \\ c_2 \exp\left(\mathrm{i}\left(\chi + \frac{q}{2}\right)x\right) \end{pmatrix}. \tag{4}$$

Solving the energy eigenvalue problem $H\psi_{\mathrm{I}} = E\psi_{\mathrm{I}}$ then yields two solutions for any χ^2 (labeled by $\pm$)

$$E_{\pm}(\chi^2) = \chi^2 + \frac{q^2}{4} \pm w, \quad \begin{pmatrix} c_{1\pm} \\ c_{2\pm} \end{pmatrix} = \frac{1}{\sqrt{2w(w \pm \chi q)}} \begin{pmatrix} \gamma \\ \chi q \pm w \end{pmatrix}, \tag{5}$$

where $w = \sqrt{\chi^2 q^2 + \gamma^2}$. Thus for any chosen energy E we have four generally complex χ values leading to four solutions in the I region. Generally, ψ_{I} is a linear combination of these four solutions, $\psi_{\mathrm{I}} = \sum_{i=1}^{4} m_i \psi_{\mathrm{I}}^i$ with some unknown coefficients $m_{1,2,3,4}$.

Altogether, for given E, γ and q — which completely characterize our problem — we have eight unknown coefficients $r_{1,2}$, $t_{1,2}$ and $m_{1,2,3,4}$. On the other hand, continuity of ψ and its first derivative at the interfaces imposes eight matching conditions. We can then solve the set of equations and calculate the desired reflection and transmission coefficients.

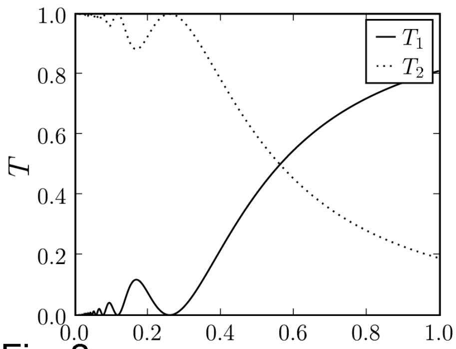

Fig. 2

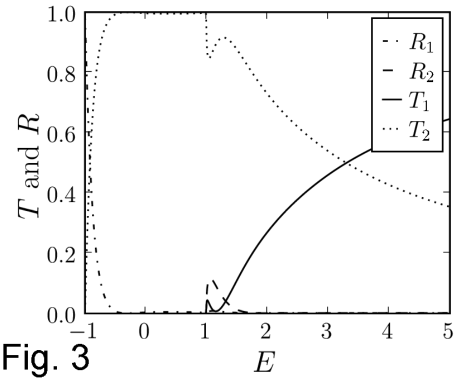

Fig. 3

Fig. 2. Transmission coefficients for fixed energy $E = 5$ as a function of q ($\gamma = -1$).

Fig. 3. Transmission and reflection coefficients for fixed $q = 0.7$ as a function of energy ($\gamma = -1$).

3. Results and discussion

The situation in common ferromagnets may be described [7] by the Fermi energy of about 5 eV, exchange splitting of about 1 eV, Fermi wave vector of about 10^{10} m$^{-1} \approx 2\pi/a$, a being the material lattice constant, and the wall width of about one hundred lattice constants. In the words of our parameters these values translate to $E = 5$, $\gamma = -1$, $k_\mathrm{F} = 2$ and $q = k_\mathrm{F}/100 = 0.02$. Using these values gives $R_{1,2} \approx T_1 \approx 0$, $T_2 \approx 1$, which confirms that the electron spin follows the change in local magnetization as it propagates through the material. This situation is practically unchanged as long as the wall width is much larger compared to the electron wavelength, i.e. $q \ll k$, $k \approx k_\mathrm{F}$. Figure 2 depicts this kind of behavior. Further, we see that from a certain value of q the wall width is too thin (its width equals π/q) for the electrons to adjust their spins and they more and more pass through the wall maintaining their original polarization. Figure 3 shows the dependence of the reflection and transmission coefficients for such higher (unrealistic) value of $q = 0.7$ (that corresponds to a wall about four lattice constants thick) on the energy of an incoming electron.

Acknowledgments

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