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Orthonormalization of Radiation Modes in Effective Resonator Model of Dielectric Multilayer Structure
ERRATUM

A. Rudziński*

Institute of Microelectronics and Optoelectronics
Warsaw University of Technology
Koszykowa 75, 00-665 Warszawa, Poland

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This paper contains an erratum to [A. Rudziński, *Acta Phys. Pol. A* **112**, 495 (2007)]

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In [1] it was shown that the set of non-orthonormal modes $\boldsymbol{\psi}_{k\epsilon}(\boldsymbol{r})$ of a dielectric multilayer structure, obtained with the help of the effective resonator model and given by:

$$\boldsymbol{\psi}_{k\epsilon}(\boldsymbol{r}) = \rho_\epsilon(\boldsymbol{k}) \sum_{j=-N_\mathrm{L}}^{N_\mathrm{R}} \chi_j(z) \times \Big(u_{jk\epsilon} \boldsymbol{e}_{k^{(j)}\epsilon} \exp\left(\mathrm{i}\boldsymbol{k}^{(j)}\boldsymbol{r}^{(j)}\right) + v_{jk_*\epsilon} \boldsymbol{e}_{k_*^{(j)}\epsilon} \exp\left(\mathrm{i}\boldsymbol{k}_*^{(j)}\boldsymbol{r}^{(j)}\right)\Big), \tag{11}$$

(equations have the same numbers as in [1]), can be transformed into the set of orthonormal modes $\boldsymbol{f}_{k\epsilon}(\boldsymbol{r})$ by linear combinations

$$\boldsymbol{f}_{k\epsilon}(\boldsymbol{r}) = \frac{1}{r_{k\epsilon}} \frac{\boldsymbol{\psi}_{k\epsilon}(\boldsymbol{r}) - s_{k\epsilon}\boldsymbol{\psi}_{k_*\epsilon}(\boldsymbol{r})}{1-|s_{k\epsilon}|^2}. \tag{36}$$

The above two expressions are correct, however, on their basis formula (37) in [1] (a more explicit form of (36)) has been too hastily calculated as:

$$\boldsymbol{f}_{k\epsilon}(\boldsymbol{r}) = \frac{1}{1-|s_{k\epsilon}|^2} \sqrt{\frac{1+|s_{k\epsilon}|^2}{F_{k\epsilon}}} \sum_{j=-N_\mathrm{L}}^{N_\mathrm{R}} \chi_j(z) \Big[(u_{jk\epsilon} - s_{k\epsilon} v_{jk\epsilon}) \boldsymbol{e}_{k^{(j)}\epsilon} \times \exp\left(\mathrm{i}\boldsymbol{k}^{(j)}\boldsymbol{r}^{(j)}\right) + (v_{jk_*\epsilon} - s_{k\epsilon} u_{jk_*\epsilon}) \boldsymbol{e}_{k_*^{(j)}\epsilon} \exp\left(\mathrm{i}\boldsymbol{k}_*^{(j)}\boldsymbol{r}^{(j)}\right)\Big]. \tag{37}$$

*e-mail: a.rudzinski@elka.pw.edu.pl

It has been overlooked by the author that the expression for $\boldsymbol{\psi}_{k_*\epsilon}(\boldsymbol{r})$ is obtained not by a simple replacement of every wave vector $\boldsymbol{k}^{(j)}$ in (11) by $\boldsymbol{k}_*^{(j)}$. The correct expression is

$$\boldsymbol{\psi}_{k_*\epsilon}(\boldsymbol{r}) = \rho_\epsilon(\boldsymbol{k}) \sum_{j=-N_\mathrm{L}}^{N_\mathrm{R}} \chi_j(z) \\ \times \left(u_{jk_*\epsilon} \boldsymbol{e}_{k_*^{(j)*}\epsilon} \exp\left(\mathrm{i}\boldsymbol{k}_*^{(j)*}\boldsymbol{r}^{(j)}\right) + v_{jk\epsilon} \boldsymbol{e}_{k_*^{(j)*}\epsilon} \exp\left(\mathrm{i}\boldsymbol{k}^{(j)*}\boldsymbol{r}^{(j)}\right)\right),$$

because if the component $k_z^{(j)}$ of the wave vector in the j-th layer is imaginary, its sign does not depend on the direction of propagation (in this case it depends on the position of the source of the wave) and remains constant during the transformation $\boldsymbol{\psi}_{k\epsilon} \to \boldsymbol{\psi}_{k_*\epsilon}$. For this reason, the four complex conjugations appear (in polarization versors and exponents). Thus, expression (37) as given in [1] is accurate only in layers, in which $k_z^{(j)}$ is real, therefore it is correct only for some of radiation modes. In layers in which $k_z^{(j)}$ is imaginary, instead of the coefficient $(u_{jk\epsilon} - s_{k\epsilon} v_{jk\epsilon})$ one obtains $(u_{jk\epsilon} - s_{k\epsilon} u_{jk_*\epsilon})$ and $(v_{jk_*\epsilon} - s_{k\epsilon} u_{jk_*\epsilon})$ is replaced by $(v_{jk_*\epsilon} - s_{k\epsilon} v_{jk\epsilon})$. The corrected formula (37) can be written as:

$$\boldsymbol{f}_{k\epsilon}(\boldsymbol{r}) = \frac{1}{1-|s_{k\epsilon}|^2} \sqrt{\frac{1+|s_{k\epsilon}|^2}{F_{k\epsilon}}} \sum_{j=-N_\mathrm{L}}^{N_\mathrm{R}} \chi_j(z) \\ \times \left(U_{jk\epsilon} \boldsymbol{e}_{k^{(j)}\epsilon} \exp\left(\mathrm{i}\boldsymbol{k}^{(j)}\boldsymbol{r}^{(j)}\right) + V_{jk_*\epsilon} \boldsymbol{e}_{k_*^{(j)}\epsilon} \exp\left(\mathrm{i}\boldsymbol{k}_*^{(j)}\boldsymbol{r}^{(j)}\right)\right) \tag{37}$$

with coefficients $U_{jk\epsilon}$ and $V_{jk\epsilon}$ defined as following:

$$U_{jk\epsilon} = \begin{cases} u_{jk\epsilon} - s_{k\epsilon} v_{jk\epsilon}, & \text{if } k_z^{(j)} \in \mathbb{R}, \\ u_{jk\epsilon} - s_{k\epsilon} u_{jk\epsilon}^*, & \text{otherwise}, \end{cases}$$

$$V_{jk\epsilon} = \begin{cases} v_{jk\epsilon} - s_{k\epsilon}^* u_{jk\epsilon}, & \text{if } k_z^{(j)} \in \mathbb{R}, \\ v_{jk\epsilon} - s_{k\epsilon}^* v_{jk\epsilon}^*, & \text{otherwise}. \end{cases}$$

Apart from the error described above, in formula (35) complex conjugate and in formula (A.12) a numerical factor are missing. Correct forms are:

$$s_{k\epsilon} = \begin{cases} \frac{F_{k\epsilon} - \sqrt{F_{k\epsilon}^2 - |\tilde{F}_{k\epsilon}|^2}}{\tilde{F}_{k\epsilon}^*}, & \tilde{F}_{k\epsilon} \neq 0, \\ 0, & \tilde{F}_{k\epsilon} = 0. \end{cases} \tag{35}$$

$$\begin{pmatrix} u_{jk\epsilon} \\ v_{jk\epsilon}^* \end{pmatrix} = \left(\delta_{\epsilon\mathrm{TE}} + \frac{n_{(0)}}{n_{(j)}} \delta_{\epsilon\mathrm{TM}} \right) \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} 1 \\ \xi_\epsilon^*(\boldsymbol{k}) \end{pmatrix} \tag{A.12}$$

References

[1] A. Rudziński, *Acta Phys. Pol. A* **112**, 495 (2007).