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## Spin Resonance Absorption in a 2D Electron Gas

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We analyze the power absorption at electron spin resonance excited by the Bychkov–Rashba spin–orbit field in a 2D electron gas. We show that, as long as the absorbed power is dissipated by the usual spin lattice relaxation mechanisms, the resonance line shape is expected to be of pure absorption type, i.e., it can be described by the imaginary part of the dynamic magnetic susceptibility. Therefore, the Dysonian line shape observed in ESR of 2D electron gas cannot result from this type of the resonance signal.

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### 1. Introduction

The experimentally observed line shape and amplitude of spin resonance of a 2D electron gas in a Si quantum well depends on the experimental geometry and on the applied microwave power in a very complex way [1–5]. In particular, the analysis of the ESR line shape shows that, in addition to the classical absorption signal described by the imaginary part of the dynamic magnetic susceptibility,  $\chi''(\omega)$ , a strong dispersion component occurs in the absorption signal, that corresponds to the real part,  $\chi'(\omega)$ . The dependence on the sample position [2] and its orientation indicates that the signal is induced not only by the magnetic but also by the electric component of microwave field. Moreover, the temperature dependence of the signal amplitude shows that the ESR signal amplitude scales with the carrier mobility, indicating that spin resonance is excited rather by the microwave current than directly by the electric field [2, 5–8], as expected in electric dipole spin resonance [9–13].

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All these experimental findings and theoretical considerations indicate that in the presence of a spin–orbit interaction one has to consider a possible excitation of ESR by both magnetic and electric dipole transitions and by the electric current. In this paper we analyze the power absorption when ESR is excited simultaneously by all components of the effective magnetic field. We assume, however, that the spin relaxation rate is constant, as in the classical Bloch equations. In other words, we assume that the classical spin relaxation channel is the only possible way of dissipating the precession energy.

These considerations lead us to the conclusion that the absorbed power is proportional to  $\chi''(\omega)$  only. This result indicates that the dispersive character of the ESR originates from a peculiar energy dissipation. In particular, one should consider an antenna effect, when the precessing magnetic moment induces an additional eddy currents leading to a modification of the Joule heating [5]. Because the emitted power, proportional to the precession amplitude, varies at the resonance condition, the Joule heat reflects the ESR signal. In such a case, however, the dissipation power, in contradiction to the assumption of this paper, is not constant, but varies at the resonance condition. Here we show that the mechanism of the spin excitation by both components of the microwave field cannot explain the occurrence of the dispersive component observed in the absorption ESR signal. The modeling of the ESR signal in a high mobility 2D electron gas requires thus not only the discussion of the complex excitation mechanisms [6], as discussed in this paper, but also a detailed analysis of the dissipation mechanism which is beyond the scope of this paper.

## 2. Current induced Bychkov–Rashba field

Lack of mirror symmetry results in the occurrence of the Bychkov–Rashba (BR) spin splitting [13]:

$$\hbar\omega_{\text{BR}} = \alpha_{\text{BR}}[\mathbf{k} \times \mathbf{n}] = g\mu_{\text{B}}\mu_0\mathbf{h}_{\text{BR}}(\mathbf{k}), \quad (1)$$

which is proportional to the electron  $k$ -vector. Here the unit vector  $\mathbf{n}$  is oriented perpendicularly to the sample layer, indicating the direction of the built-in electric field, which causes the spin–orbit coupling. The splitting is equivalent to an effective magnetic field,  $\mathbf{h}_{\text{BR}}$ , seen by spin of individual electrons. In Eq. (1)  $g$  is  $g$ -factor,  $\mu_{\text{B}}$  is the Bohr magneton, and  $\mu_0$  — the vacuum permeability.

In thermal equilibrium, the mean field,  $\mathbf{H}_{\text{BR}}$ , averaged over the ensemble of electrons, vanishes. But it appears when an electric current is applied [5, 14, 6–8]:

$$\mu_0\mathbf{H}_{\text{BR}} = \beta_{\text{BR}}(\mathbf{v}_{\text{d}} \times \mathbf{n}) = \frac{\beta_{\text{BR}}}{n_{\text{s}}e}(\mathbf{j} \times \mathbf{n}). \quad (2)$$

The current induced BR field is proportional to the drift velocity,  $\mathbf{v}_{\text{d}}$ , and the material parameter  $\beta_{\text{BR}} = \alpha_{\text{BR}}m^*/g\mu_{\text{B}}$ . Here  $m^*$  is the effective mass and  $n_{\text{s}}$  is the sheet electron concentration.  $\mathbf{H}_{\text{BR}}(t)$  is oriented in-plane and perpendicular to the local current density  $\mathbf{j}$ .

A dc current causes a static effective field which allows for the resonance tuning [5]. An alternating current causes also an ac  $\mathbf{H}_{\text{BR}}$ . The transverse component (to the external magnetic field,  $\mathbf{H}_0$ ) of this field induces ESR.

In the microwave cavity the current is induced by the ac electric field,  $\mathbf{E}_1(t)$ . Within the Drude model the current density,  $\mathbf{j}$ , is described by the complex conductivity

$$\mathbf{j} = \sigma(\omega)\mathbf{E}_1 = \frac{n_s e^2}{m^*} \frac{\tau}{1 - i\omega\tau} \mathbf{E}_1. \quad (3)$$

For low mobility layers,  $\omega\tau \ll 1$ ,  $\mathbf{j}$  stands for the drift current, which occurs in phase with the microwave electric field,  $\mathbf{E}_1$ , whereas in the high mobility limit, a displacement current occurs. In the latter limit the “current induced” ESR tends [5, 6, 13] to the well-known electric dipole spin resonance [9–12]. The phase shift between electric field and current,  $\varphi_{jE}$ , is connected to the parameter  $\omega\tau$ , by  $\tan\varphi_{Ej} = \omega\tau$ . Throughout this paper we use the convention of complex vector components oscillating with a frequency  $\omega$ , and thus  $\mathbf{j}(t) = \text{Re}(\mathbf{j}e^{-i\omega t}) = \text{Re}(\sigma\mathbf{E}_1e^{-i\omega t})$ , where the complex quantities,  $\mathbf{j}$ ,  $\sigma$ , and  $\mathbf{E}_1$ , contain the phase shift information.

### 3. Precession motion

We analyze the case when spin resonance is excited by the sum of microwave magnetic fields,  $\mathbf{H}_1(t)$  and of the spin-orbit field,  $\mathbf{H}_{\text{BR}}(t)$ . Their spatial distribution requires the self-consistent solution of Maxwell’s equations for the loaded microwave cavity. It turns out, however, that the line shape describing the power absorption happens to be independent of the local variation of the microwave fields:  $\mathbf{H}_1(t)$  and  $\mathbf{E}_1(t)$ .

These vectors rule the motion of the local spins contributing thus to the integral ESR signal amplitude but the local contributions are characterized by a common line shape. As a consequence, the spatial distribution of microwave field affects the magnitude of the integrated signal but it does not influence the line shape.

For an arbitrary orientation and position of sample in the microwave cavity these vectors are oriented in different directions. For simplicity we begin with the simplest experimental geometry. We analyze a long, stripe-like sample oriented along the  $\mathbf{E}_1(t)$  field ( $z$ -axis), with the sample layer perpendicular to  $\mathbf{H}_1(t)$  ( $x$ -axis). We assume that the magnitude of both fields does not change along the  $z$ -axis. This is approximately the situation in the classical ESR cavity of TE<sub>102</sub> type. The external dc field,  $\mathbf{H}_0$ , is assumed to be applied along the  $z$ -axis. For such geometry there is no cyclotron motion and  $\mathbf{j}(t)$  is parallel to  $\mathbf{E}_1(t)$ . Both vectors are thus parallel to the  $z$ -axis.

The 2D magnetization is defined by  $M_\alpha = M_\alpha^{3D}\delta = n_s g \mu_B \langle S_\alpha \rangle$ , where  $\delta$  is the effective thickness of the layer and the superscript stands for coordinate axes:  $\alpha = x, y$ , or  $z$ . The magnetization vector is assumed to be of the form

$\mathbf{M} = (m_x, m_y, M_0 - m_z)$ , where  $m_x$  and  $m_y$  are oscillating components of the precessing magnetization,  $M_0$  stands for thermal equilibrium value, and  $m_z$  describes the change of the projection of  $\mathbf{M}$  on the  $z$ -axis caused by the precession motion.

Within this notation the Bloch equation takes the form

$$\frac{d\mathbf{M}(t)}{dt} = -\gamma\mu_0\mathbf{M}(t) \times \mathbf{H}(t) - \left( \frac{m_x}{T_2}, \frac{m_y}{T_2}, -\frac{m_z}{T_1} \right), \quad (4)$$

where  $\gamma$  is the gyromagnetic factor, and  $\mathbf{H}(t) = (H_1(t), H_{\text{BR}}(t), H_0)$ .

Using the linear approximation, for small  $m_\alpha, H_1, H_{\text{BR}}$  (in comparison to  $H_0$ ), we obtain the following steady state solutions for the components of magnetization:

$$\begin{aligned} m_x &= \chi_{xx}(\omega) \left( H_1 + \frac{-i\omega + 1/T_2}{\omega_0} H_{\text{BR}} \right), \\ m_y &= \chi_{yy}(\omega) \left( H_{\text{BR}} - \frac{-i\omega + 1/T_2}{\omega_0} H_1 \right), \end{aligned} \quad (5)$$

where  $\chi_{xx}(\omega) = \chi_{yy}(\omega) = \chi'(\omega) + i\chi''(\omega)$  is the complex magnetic susceptibility and  $\omega_0 = \gamma\mu_0 H_0$ .

The power absorbed,  $P$ , within the unit area,  $A$ , by the oscillating magnetization is the mean value of  $dP/dA = \mu_0\mathbf{M}(t) \cdot d\mathbf{H}/dt$ , averaged over one period. For a magnetic vector oscillating according to Eq. (5) the power absorption is described by

$$\begin{aligned} \frac{dP}{dA} &= \frac{\omega}{2} \chi''(\omega) \mu_0 \left[ |H_1|^2 + |H_{\text{BR}}|^2 - \frac{2\omega}{\omega_0} \text{Im}(H_1 H_{\text{BR}}^*) \right] \\ &\quad - \frac{\omega}{T_2 \omega_0} \chi'(\omega) \mu_0 \text{Im}(H_1 H_{\text{BR}}^*). \end{aligned} \quad (6a)$$

For a narrow resonance line, when  $\omega_0 T_2 \gg 1$ , the resonance absorption occurs at  $\omega \cong \omega_0$  and the expression can be simplified to

$$\frac{dP}{dA} \cong \frac{\omega}{2} \chi''(\omega) \mu_0 \left[ |H_1|^2 + |H_{\text{BR}}|^2 - 2\text{Im}(H_1 H_{\text{BR}}^*) \right]. \quad (6b)$$

For the observed case of ESR in Si [1, 2] the observed line width is very narrow, and  $\omega_0 T_2$  is of the order of  $10^4$ . When  $H_{\text{BR}}$  is expressed by Eqs. (2) and (3), the density of the power absorption, for the discussed simplified geometry, takes the form

$$\frac{dP}{dA} \cong \frac{\omega}{2} \chi''(\omega) \mu_0 \left[ |H_1|^2 + \frac{e^2 \tau^2 \beta_{\text{BR}}^2 |E_1|^2}{\mu_0^2 m^* (1 + \omega^2 \tau^2)} + \frac{2e\tau\beta_{\text{BR}} |E_1| |H_1|}{\mu_0 m^* (1 + \omega^2 \tau^2)} \right]. \quad (7)$$

For the case of arbitrary directions of the vectors  $\mathbf{H}_0$ ,  $\mathbf{H}_1$ , and  $\mathbf{H}_{\text{BR}}$  we get the following general expression for the power dissipation:

$$\begin{aligned} \frac{dP}{dA} &\cong \frac{\omega}{2} \chi''(\omega) \mu_0 \\ &\quad \times \left[ |H_{1x} + H_{\text{BR}x}|^2 + |H_{\text{BR}y}|^2 - 2\text{Im}((H_{1x} + H_{\text{BR}x})H_{\text{BR}y}^*) \right]. \end{aligned} \quad (8)$$

Here the  $z$ -axis has been chosen to be parallel to  $\mathbf{H}_0$ , and the  $x$ -axis to be parallel to the projection of  $\mathbf{H}_1$  on the plane perpendicular to  $\mathbf{H}_0$ . Expression (8) describes

the power absorbed by an arbitrarily oriented element of the sample area. In that sense, as long as we do not analyze the power absorbed by the total sample, the expression is general, and it holds for any arbitrary sample shape.

#### 4. Discussion and conclusions

Both cases, the simplified result (Eq. (6)) and the more general one (Eq. (8)), for the local power absorption contain the imaginary part of the dynamic susceptibility only. The magnitude of the total power absorption requires the integration of expressions (6), (7), and (8) over the sample area. The total amplitude of the ESR line is equal to the integral of the expressions in the square brackets. Therefore, for the evaluation of the signal amplitude, the details of the spatial distribution of  $\mathbf{H}_1$  and  $\mathbf{H}_{\text{BR}}$  and the knowledge about their phase shifts are required. But the line shape of the total power absorption is neither dependent on the experimental geometry nor on the real distribution of the microwave field. Moreover, it is also independent of the phase shift between  $\mathbf{E}_1(t)$  and  $\mathbf{j}(t)$ . Therefore this conclusion holds both for ESR induced by the electric current and by electric dipole transitions<sup>†</sup>. This is the main conclusion of this paper. It means that the Dysonian shape of ESR line cannot be directly related to the fact that the resonance is excited by  $\mathbf{H}_1 + \mathbf{H}_{\text{BR}}$ .

For an explanation of this discrepancy we have to consider different reasons: the spin relaxation rate depends on frequency in the vicinity of the resonance condition, e.g., due to the energy emission by the oscillating magnetic moment in the resonator and power dissipation by the change of the Joule heating at ESR condition as indicated in Ref. [5].

The magnitude of two discussed effective fields,  $\mathbf{H}_1$  and  $\mathbf{H}_{\text{BR}}$ , depends on the position and orientation of the sample in microwave cavity and on carrier mobility. According to Eqs. (2), (3),  $\mathbf{H}_{\text{BR}}$  scales with  $\tau$ , i.e., with the electron mobility. Consequently, the absorbed power scales with the square of the electron mobility. For the material parameters of Si/SiGe layers, the magnitude of  $H_1$  and  $H_{\text{BR}}$  are equal for  $\tau \approx 10^{-13}$  s when a comparison is made for the optimal position of the sample, i.e., for magnetic dipole excitation, the sample is placed in the maximum of  $H_1$  and for the current induced resonance, the sample is placed in the node  $H_1$  and the layer is parallel to  $\mathbf{E}_1(t)$ . The momentum relaxation rate for Si/SiGe layer reaches a value of  $\tau \approx 10^{-11}$  s. In such a case the current induced signal is by 4 orders of magnitude bigger than the magnetic dipole ESR signal. For other materials, e.g., for III-V compounds, where the spin-orbit coupling is much stronger, the domination of  $\mathbf{H}_{\text{BR}}$  is expected to be much more pronounced.

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<sup>†</sup>The problem of the relation between the ESR induced by current and by electric field is discussed in [15].

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