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The Effects of Relativistic Warm Plasma Waveguide and High Frequency Electrical Field on Beam–Plasma Interaction

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The paper is concerned with investigation of the influence of an intense high frequency electric field and relativistic warm plasma waveguide on electrostatic oscillations of nonuniform bounded plasma. The stabilizing effect of a strong high frequency (pump) electrical field on beam–plasma interaction in a cylindrical relativistic warm plasma waveguide is discussed. A new mathematical technique "separation method" was applied to the two-fluid plasma model to separate the equations, which describe the system, into two parts, time and space parts. Plasma electrons are considered to have a relativistic, thermal velocity. It is shown that a high frequency electric field has no essential influence on dispersion characteristics of unstable surface waves excited in a relativistic warm plasma waveguide by a low-density electron beam. The region of instability is only slightly narrowing and the growth rate decreases by a small parameter and this result was reduced comparing to cold, nonrelativistic plasma. Also, it is found that the plasma electrons did not affect the solution of the space part of the problem.

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1. Introduction

The investigation of beam–plasma instabilities evokes interest not only from the point of view of understanding this particular type of instability but also as a method of modeling the instabilities caused by resonant interactions in more complex cases.

In particular, in that kind of modeling the transition from the laminar to the turbulent state results from instabilities and the control of instabilities.

It is well known that (e.g., [1, 2]) propagation of charged particle beams through plasma leads to development of great variety of instabilities. Sometimes such instabilities could be used as a basis for the construction of effective generators of electromagnetic radiation, for heating of magnetically confined plasmas, and for particle acceleration. But often beam instabilities are undesirable. This is true particularly for the experiments on magnetic fusion research where instabilities may cause an appreciable enhancement in the transport of particles and heat across the confining magnetic field. Therefore stabilization of instabilities excited by a beam of charged particles in plasmas is an important part of the more general problem of controlling instabilities in magnetic traps.

Treatments of the beam–plasma instability of a bounded electron beam in an unbounded plasma usually study the behavior of the growth rate as a function of the parameters of the problem for one or two oscillation modes which have the largest growth rate. This approach has permitted considerable insight into the important role of transverse transport of the oscillation energy with the group velocity out of the beam region in determining the behavior of the beam–plasma interaction [3–8].

The control of instabilities, in particular their suppression, can be achieved by means of intense high-frequency (HF) fields (e.g., [9, 10]). Based on existing experimental and theoretical results it may be deduced that the effect of an intense electromagnetic radiation on plasma may produce qualitative change in its main properties (e.g., [10–14] and references therein). In particular, the plasma dispersion characteristics can change to a considerable extent, the absorption of an external (pump) electromagnetic field energy by plasma particles increases comparing to collision absorption, the possibility exists for parametric excitation of plasma waves and also for stabilization of numerous plasma instabilities.

The stabilization effect of a uniform HF electric field on a two-stream (Buneman) instability in uniform (or nonuniform) unbounded (or bounded) plasma has been investigated by Aliev and Silin [15] and Demchenko et al. [16]. They obtained the dispersion equation for characteristic frequencies of electrostatic oscillations excited by the relative motion of electrons and ions in an HF electric field. The presence of a pump wave strongly modifies the dispersion equation of the Buneman instability.

Consequently, the growth rate of instability is reduced in comparison with the growth rate at vanishing external field amplitude.

The method described is used for the solution of the effect of an HF electric field and the instabilities of a low-density electron beam passing through a plasma waveguide in the presence of an HF electric field are investigated in [16].

Moreover, the instabilities resulting from the interaction of charged particle beams with plasma were greatly stimulated by the circumstance that, apart from their negative effects such as anomalous diffusion due to low-frequency oscillations or limitations of the highest attainable currents in plasma accelerators and other devices, there exists a great number of useful applications of these instabilities.

Resonant instabilities appear as a consequence of a wave–particle interaction occurring when the phase velocity of particles becomes the same as drift velocity.

These instabilities are in general overstable (e.g. trapping [17, 18], two-stream [19, 20] and bump on tail instabilities [21]). Non-resonant instabilities (also called bunching instabilities [22, 23]) do not depend so much on the behavior of the particle distribution in the neighborhood of a particular speed (e.g., pinch [24, 25] and mirror [26, 27] instabilities).

Instabilities are characterized by the properties of the plasma. Thus one speaks of instabilities of a homogeneous or inhomogeneous plasma, collissional or collisionless plasma etc.

Also, instabilities are named according to the discoverer (Buneman, Čerenkov, etc.), according to the type of device in which the instabilities were found (tokamak, pinch, etc.) or according to the energy source.

Parametric interaction of external HF electric field with an electrostatic surface wave in an isotropic nonuniform plasma has been investigated in Ref. [17] using a special method based on the separation of variables.

Investigation of beam–plasma interaction presents a great interest for the development of effective methods via plasma instability, amplification and generation of electromagnetic waves, acceleration of charged particles in plasma, high frequency heating of plasma, and so on ([12, 28] and [29]).

It has been shown [30] that dispersion equations describing parametric excitation of surface waves at the boundary of isotropic plasma–vacuum to within the eigenfrequency renormalization coincide with the equations that determine the parametric excitation of volumetric waves in uniform unbounded plasma. Preceding from this conclusion the method for investigation of parametric interaction of external HF electrical field with electrostatic oscillations in isotropic bounded nonuniform plasma has been proposed [17]. The method makes it possible to separate the problem into two parts. The "dynamical" part describes the parametric build up of oscillations and corresponding equations within the renormalization of eigenfrequencies coinciding with equations for the parametrically unstable waves ([7, 30] and [31]). Therefore it is of special interest to apply the separation method to the solution of different problems involving parametric excitation of electrostatic waves in bounded nonuniform plasma.

Contrary to recent works [16], we take into consideration the influence of relativistic warm plasma electrons on the beam–plasma interaction in a cylindrical plasma waveguide pumped by HF electric field.

2. The influence of HF electric field on the instability of a low-density electron beam passing through a relativistic warm plasma waveguide

We assume that a uniform cold electron beam propagates along radially nonuniform cylindrical relativistic warm plasma waveguide. The radius of the beam R is supposed to coincide with the radius of the plasma cylinder (i.e. $R = r$).

Let us now assume that an electron beam of low density ($\varepsilon_{\mathrm{b}} = n_{0\mathrm{b}}/n_0 \ll 1$) is passing through quasineutral plasma with velocity $\boldsymbol{u}_{\mathrm{b}_0}$. We shall also suppose that both plasma components are at rest ($\boldsymbol{u}_{\mathrm{e}_0} = \boldsymbol{u}_{\mathrm{i}_0} = 0$). We take the vector of the external HF field $\boldsymbol{E}_{\mathrm{p}} = \boldsymbol{E}_0 \sin(\omega t)$ to be oriented along the axis of the plasma cylinder.

The "separation method" has been described [16, 32] in application to the problem of parametric excitation of surface waves in a cold isotropic plasma. Here we shall follow this paper. Representing the perturbations of velocity, density, and electrical potential in the form $\partial \boldsymbol{V}_\alpha,\ \partial n_\alpha,\ \Phi \sim \exp\left(\mathrm{i}(m\Psi + kz)\right)$ the linearized set of hydrodynamic equations together with the Poisson equation can be reduced to the form

$$\frac{\partial^2 \nu_{\alpha_1}}{\partial t^2} - \mathrm{i}ku_0(1-\gamma)\frac{\partial \nu_{\alpha_1}}{\partial t} + \eta_\gamma \nu_{\alpha_1} = -\frac{m_{\mathrm{e}}p^2}{e^2}\mathrm{e}^{\mathrm{i}A_\alpha} \sum_{\beta=\mathrm{b,e,i}} \nu_{\beta_1}\frac{e_\beta^2}{m_\beta}\mathrm{e}^{\mathrm{i}A_\beta}. \tag{1}$$

Here, γ $(relativistic\ effect) = (1 - u_0^2/c^2)^{-1/2}$, $\eta_\gamma = \gamma k^2 V_{\mathrm{th}}^2$, $\alpha = \mathrm{e}, \mathrm{i}$, or b and p is a separation constant, and ν_{α_1} is the temporal part of the density

$$A_\alpha = (ku_{\alpha_0})t - a_\alpha \sin(\omega_0 t) \quad \text{and} \quad a_\alpha \equiv \frac{e_\alpha k E_0}{m_\alpha \omega_0^2} \approx a_{\mathrm{e}}.$$

Assuming that the ions are at rest ($u_{\mathrm{i}_0} \equiv 0$) and that the frequency of HF field is much greater than the eigenfrequency of the excited surface waves ($\omega_0 \gg \omega_{\mathrm{SW}} \sim \omega_{\mathrm{P_e}}$), we may use the method of averaging [33] on Eq. (1). Using the Jacobi–Anger formula [34], $\exp(\pm \mathrm{i}a \sin(\omega_0 t)) = \sum_{m=-\infty}^{\infty} J_m(a) \exp(\pm \mathrm{i}m\omega_0 t)$ where $J_m(a)$ are the Bessel functions, for values $(\langle \nu_{\alpha_1} \rangle, \langle w_i \rangle) = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} (\nu_{\alpha_1}, w_{\mathrm{i}})\mathrm{d}t$, then from (1) we have

$$\begin{aligned}
&\frac{\mathrm{d}^2\langle \nu_{\mathrm{b}_1}\rangle}{\mathrm{d}t^2} + \varepsilon_{\mathrm{b}} p^2 \left\{\langle \nu_{\mathrm{b}_1}\rangle + \exp(\mathrm{i}(ku_{\mathrm{b}})t)\left[\langle \nu_{\mathrm{e}_1}\rangle + J_0(a)\langle w_{\mathrm{i}}\rangle\right]\right\} = 0,\\
&\frac{\mathrm{d}^2\langle \nu_{\mathrm{e}_1}\rangle}{\mathrm{d}t^2} - \mathrm{i}\Delta_\gamma \frac{\mathrm{d}\langle \nu_{\mathrm{e}_1}\rangle}{\mathrm{d}t} + \eta_\gamma \langle \nu_{\mathrm{e}}\rangle\\
&\qquad + p^2 \left(\langle \nu_{\mathrm{e}_1}\rangle + \exp\left(-\mathrm{i}(ku_{\mathrm{b}})t\right)\langle \nu_{\mathrm{b}_1}\rangle + J_0(a)\langle w_{\mathrm{i}}\rangle\right) = 0,\\
&\frac{\mathrm{d}^2\langle w_{\mathrm{i}}\rangle}{\mathrm{d}t^2} + \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}} p^2 \left[\langle w_{\mathrm{i}}\rangle + \left(\exp(-\mathrm{i}(ku_{\mathrm{b}})t)\langle \nu_{\mathrm{e}_1}\rangle + \langle \nu_{\mathrm{b}_1}\rangle\right) J_0(a)\right] = 0.
\end{aligned} \tag{2}$$

Here, $\varepsilon_{\mathrm{b}} = n_{0\mathrm{b}}/n_0$, $w_{\mathrm{i}} = \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\frac{\alpha_{\mathrm{i}}}{\alpha_e}\nu_{\mathrm{i}_1}$ and $\Delta_\gamma = ku_0(1-\gamma)$. The system of Eqs. (2) coincides (if $V_{\mathrm{th}} = 0$, $\gamma = 1$, $n_{0\mathrm{b}} = 0$, $\omega_{\mathrm{P_e}}^2 \to p^2$, $\omega_{\mathrm{P_i}}^2 \to (m_{\mathrm{e}}/m_{\mathrm{i}})p^2$) with the system describing the HF stabilization of the two-stream instability in uniform (or nonuniform) unbounded (or bounded) plasma ([15] or [16]).

If there is no externally injected beam ($\varepsilon_{\mathrm{b}} = (n_{0\mathrm{b}}/n_0) = 0$) then the system of Eqs. (2) coincides with the system describing the HF stabilization of the Buneman instability in a nonuniform bounded relativistic cold plasma [35, 36].

3. Solution of the time-dependent equations

According to Eqs. (2), plasma oscillations are then described by the dispersion equation

$$(\omega-\omega_{\mathrm{LF}}^2)[(\omega-ku_{\mathrm{b}})^2(\omega^2-\omega_{\mathrm{HF}}^2-\eta_\gamma+\varDelta_\gamma)$$
$$-\varepsilon_{\mathrm{b}}p^2\omega_{\mathrm{P_e}}^2(\omega^2-\eta_\gamma+\varDelta_\gamma)]=0. \tag{3}$$

Here

$$\omega_{\mathrm{LF}}^2(p)=\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}p^2[1-J_0^2(a)],$$
$$\omega_{\mathrm{HF}}^2(p)=p^2\left[1+\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}J_0^2(a)\right]. \tag{4}$$

In the case of vanishing electric field amplitude ($\boldsymbol{E}_0=\mathbf{0}$), and $V_{\mathrm{th}}=0$, $\gamma=1$, then Eq. (3) agrees with the dispersion equation which describes the unstable oscillations that are excited in a uniform unbounded (bounded) plasma by a low--density electron beam [1, 5]. We shall analyze Eq. (3) (as in [1, 5] in two cases).

3.1. Nonresonant case ($ku_{\mathrm{b}}\not\approx\omega_{\mathrm{HF}}$)

We have from Eq. (3):

$$\omega=ku_{\mathrm{b}}\pm\sqrt{\varepsilon_{\mathrm{b}}}\frac{p[(ku_{\mathrm{b}})^3-\eta_\gamma+\varDelta_\gamma)]^{1/2}}{[(ku_{\mathrm{b}})^2-\omega_{\mathrm{HF}}^2-\eta_\gamma+\varDelta_\gamma)]^{1/2}} \tag{5}$$

under the conditions

$$p^2\left[1+\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}J_0^2(a)\right]>ku_{\mathrm{b}}(ku_{\mathrm{b}}+\varDelta_\gamma-\eta_\gamma)>0. \tag{6}$$

The roots of Eq. (5) are complex and one of them corresponds to instability with the growth rate

$$\gamma_{\mathrm{NR}}=\frac{\sqrt{\varepsilon_{\mathrm{b}}}p[(ku_{\mathrm{b}})^2-\eta_\gamma+\varDelta_\gamma)]^{1/2}}{[\omega_{\mathrm{HF}}^2(p)-(ku_{\mathrm{b}})^2-\eta_\gamma+\varDelta_\gamma)]^{1/2}}. \tag{7}$$

At $\gamma=1$ and $V_{\mathrm{th}}=0$ the result agrees with the result for cold plasma [14, 33].

3.2. Resonant case ($ku_{\mathrm{b}}\approx\omega_{\mathrm{HF}}$)

The frequency of unstable oscillations can be represented in the form $\omega=\omega_{\mathrm{HF}}(p)+\Delta\omega$, where

$$\mathrm{Re}\Delta\omega=-\frac{\varepsilon_{\mathrm{b}}^{1/3}p\left[1+\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}J_0^2(a)\right]^{1/6}}{2^{4/3}\left[1+\left(2(\eta_\gamma-\varDelta_\gamma)/\left(p^2\left(1+\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}J_0^2(a)\right)\right)\right)\right]^{1/3}}, \tag{8}$$

$$\gamma_{\mathrm{R}}=\mathrm{Im}\Delta\omega=\frac{\sqrt{3}}{2^{4/3}}\frac{\varepsilon_{\mathrm{b}}^{1/3}p\left[1+\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}J_0^2(a)\right]^{1/6}}{\left[1+\left(2(\eta_\gamma-\varDelta_\gamma)/\left(p^2\left(1+\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}J_0^2(a)\right)\right)\right)\right]^{1/3}}. \tag{9}$$

Here p is determined by the equation of the space part of the problem.

It follows from expressions (5)–(9) that the HF electric field has no essential influence on the dispersion characteristics of unstable surface waves excited in a plasma waveguides by a low-density electron beam. The region of instability only slightly narrows and the growth rate decreases by a small parameter.

The results obtained are in full agreement with the conclusion that an external HF field may have a stabilizing effect on the electron beam–plasma interaction in uniform (or nonuniform) plasma ([16, 32] and [36]). The relativistic warm plasma reduced the growth rate.

We conclude that the growth rate of the electron beam–plasma interaction decreases more in the relativistic warm plasma than in the previous works ([16, 32] and [36]).

4. Solution of the spatial part of the problem

The main feature of the expressions (5), (7) and (9) consists in an existence of a separation constant p, which enables us to consider the plasma boundaries.

To find an explicit expression for the constant p it is necessary to solve the following differential equation (for detail see [15]):

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[r\varepsilon(p,r)\frac{\mathrm{d}\Phi_2(r)}{\mathrm{d}r}\right]-\left(\frac{m^2}{r^2}+k^2\right)\varepsilon(p,r)\mathrm{d}\Phi_2(r)=0. \tag{10}$$

Here, $\varepsilon(p,r) = 1-\omega^2_{\mathrm{P_e}}(r)/p^2$. If the radial profile of the plasma density and boundary conditions are specified, the solution of Eq. (10) gives us the desired value of the separation constant p. The feature of Eq. (10) is that neither the amplitude of HF electric field nor electron beam parameters enter into it. Therefore Eq. (10) coincides with equation describing propagation of natural (free of external influence) electrostatic surface waves in a nonuniform plasma cylinder (see e.g. [16] and references therein). Let us suppose that plasma density is uniform and the interface between plasma and vacuum is sharp. In such case the relations determine the solution of Eq. (10)

$$\Phi_2(r<R)=C_1I_m(kr),\quad \Phi_2(r>R)=C_2K_m(kr). \tag{11}$$

Here, $I_m(kr)$ and $K_m(kr)$ are the modified Bessel functions and $C_{1,2}$ are the constants. Using the continuity condition of Φ_2 and $\varepsilon\mathrm{d}\Phi_2/\mathrm{d}r$ at the boundary, the following equation is found (see e.g. [16] and references therein)

$$\varepsilon_0(p)+\eta_m(kR)=0, \tag{12}$$

with

$$\varepsilon_0(p)=1-\frac{\omega^2_{\mathrm{P_e}}}{p^2},\quad \eta_m(kR)=-\frac{I_m(kR)K'_m(kR)}{K_m(kR)I'_m(kR)}, \tag{13}$$

where the stroke means differentiation with respect to argument. Equation (12) gives us the relation between the separation constant p, electron plasma frequency $\omega_{\mathrm{P_e}}$ and the axial wave number k:

$$p=\omega_{\mathrm{P_e}}(kR)^{1/2}[K_m(kR)I'_m(kR)]^{1/2}. \tag{14}$$

In the limiting case of small radius of plasma waveguide ($kR \ll 1$), from (14) we find

$$p_{m=0}=\omega_{\mathrm{P_e}}(kR)\left[\frac{1}{2}\ln\left(\frac{kR}{2}\right)\right]^{1/2},\quad p_{m\neq0}\approx\frac{\omega_{\mathrm{P_e}}}{\sqrt{2}}. \tag{15}$$

For $kr \gg m$ (“thick” waveguide) always $\eta_m(kR)\approx 1$ and we have $p=\omega_{\mathrm{P_e}}/\sqrt{2}$.

This equation is the same equation in cold plasma waveguide ([16, 32] and [36]) i.e., the relativistic warm plasma waveguide has no effect on the space part of the problem.

5. Conclusions

The paper deals with parametric excitation of the potential surface waves in bounded nonuniform relativistic warm plasma by monochromatic HF electrical field. It is shown that the problem can be reduced to the solution of the "temporal" (parametric) and "stationary" (spatial) parts. The "temporal" part determining frequencies and growth rates of unstable oscillations coincides with accuracy to redefinition of natural frequencies with equations describing parametric resonance in homogeneous plasma. Natural frequencies of oscillations and spatial distribution of the amplitude of the self-consistent electric field are determined from the solution of a boundary-value problem ("space" part) taking into account specific spatial distribution of the plasma density. The method described is used to solve the effect of HF field on the excitation of surface waves by an electron beam under the development of instability of low-density beam passing through a cylindrical relativistic warm plasma waveguide.

The method was used for the solution of the stabilization effect of a strong HF electric field on beam–plasma interaction in a cylindrical relativistic warm plasma waveguide. We solved the "temporal" (time-dependent) equations and obtained the corresponding dispersion Eq. (3) in a cylindrical geometry, which was analyzed for two cases: nonresonance instability ($ku_{\mathrm{b}} \not\approx \omega_{\mathrm{HF}}$) and resonance one ($ku_{\mathrm{b}} \approx \omega_{\mathrm{HF}}$). In both cases the frequency growth rates of the oscillations are obtained (relations (7) and (9)). The separation constant p is obtained from relation (14), the results are compared with the case when the external electric field is absent ($\boldsymbol{E}_0 = \mathbf{0}$) and nonrelativistic cold plasma $\gamma = 1$, $V_{\mathrm{th}} = 0$.

In conclusion it is shown that an HF electric field has no essential influence on dispersion characteristics of unstable surface waves excited in a relativistic warm plasma waveguide by a low-density electron beam. The region of instability only slightly narrows and the growth rate decreases by a small parameter and this result has been reduced compared to nonrelativistic cold plasma. Also, it is found that the plasma electrons have not affected the solution of the space part of the problem.

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