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Condensation of Nonabelian Moore–Read Quasiholes in Correlated Composite Fermion Liquids

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Two- and three-body correlation functions (number of pairs or triplets vs. relative angular momentum) of electrons or Laughlin quasielectrons (i.e., composite fermions in their first excited Landau level) are studied numerically in several fractional quantum Hall liquids. It is shown directly that the $\nu_{\mathrm{e}} = 4/11$ liquid (corresponding to a $\nu = 1/3$ filling of composite fermions in their first excited Landau level) is a paired state of quasielectrons, hence interpreted as a condensate of "second-generation" quasiholes of Moore–Read $\nu = 1/2$ state of composite fermions.

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1. Introduction

Observed by Pan et al. [1] fractional quantum Hall (FQH) effect in a spin-polarized two-dimensional electron gas at filling factors $\nu_{\mathrm{e}} = 4/11$, $3/8$, $5/13$, etc. does not belong to the well-known Laughlin [2] or Jain [3] series of fractions at $\nu_{\mathrm{e}} = n(2pn \pm 1)^{-1}$ (with integral n and p). Jain states can be understood by assuming that each electron captures an even number $2p$ of magnetic flux quanta $\phi_0 = hc/e$, forming a so-called composite fermion (CF). The CFs exist in the reduced effective magnetic field $B^* = B - 2p\phi_0\varrho$ (ϱ being electron concentration) and can occupy their effective CF Landau levels (LLs), called CF-LL$_n$, indexed by $n \geq 0$. Laughlin liquid is a filled spin-polarized CF-LL$_0$, and its quasielectron (QE), quasihole (QH), and reversed-spin quasielectron (QE$_{\mathrm{R}}$) excitations correspond to a particle in CF-LL$_1$, a vacancy in CF-LL$_0$, and spin-flip particle in CF-LL$_0$, respectively. When CFs fill more LLs, FQH effect in Jain state is observed.

Spin-polarized incompressible liquids at $\nu_{\mathrm{e}} = 4/11$, $3/8$, and $5/11$ correspond to the situation when CFs full occupy their lowest CF-LL$_0$, but the second one is filled only partially. The total CF filling factor is $\nu_{\rm CF} = \nu_{\rm e}(1-2p\nu_{\rm e})^{-1} \equiv 1+\nu = 1+1/3,\ 1+1/2$, and $1+2/3$. These fractions can be obtained in hierarchical model [4], where once again we reapply CF procedure (flux attachment) to the CFs from partially occupied CF-LL_1. Unfortunately, this procedure, leading to "second generation" of CFs [5], has no justification in this case because of completely different short-range interactions between the CFs and between the electrons [6]. For QEs, repulsion at short range is strongly reduced, and it cannot produce a Laughlin incompressible quantum liquid of the QEs at the $\nu = 1/3$ filling of CF-LL_1 [7]. In this work we present results of numerical studies of two- and three--body correlations in these FQH liquids, whose origin of incompressibility remains controversial. We find evidence that in the state $\nu_{\rm e} = 4/11$ the CFs form pairs. Hence, this state can be interpreted as a condensate of quasiholes of the "second generation" Moore–Read state (of CFs).

2. Numerical calculations

In our model we take advantage of Haldane geometry [4], where N particles are confined to a spherical surface of radius R with Dirac monopole of strength $2Q = 4\pi R^2 B/\phi_0$ placed in the centre and producing radial magnetic field with strength B at the surface. Each particle on the n-th LL has angular momentum $l = Q + n$ and degeneracy of each shell is $g = 2l + 1$.

Large systems with a well-defined filling factor $\nu = N/g$ are represented on a sphere by a sequence of finite-size states with $2l = N/\nu - \gamma$, where $\gamma(\nu)$ is an integral "shift" function. For the Laughlin $\nu = 1/3$ state (e.g., of the electrons in LL_0), $\gamma = 3$; for the $\nu = 1/3$ state of the QEs (i.e. $\nu_{\rm e} = 4/11$ of the underlying electrons) it was identified as $\gamma = 7$ [6].

The N-body Hamiltonian is diagonalized numerically in the configuration$-$interaction basis, in order to get the lowest eigenenergies and corresponding eigenvectors at different combinations of N and $2l$. It contains the cyclotron energy and the interactions. The two-body interaction within a LL is defined by a pseudopotential [8], i.e., interaction energy V_2 as a function of the relative pair angular momentum $\mathcal{R} = 2l - L$. For identical fermions, $\mathcal{R}$ is an odd integer, and larger $\mathcal{R}$ corresponds to a larger average squared distance between the particles.

3. Haldane amplitudes and clusterization

The energy of N particles interacting through $V_2(\mathcal{R})$ can be written as

$$E = \sum_{\mathcal{R}} \mathcal{N}_2(\mathcal{R}) V_2(\mathcal{R}), \tag{1}$$

where $\mathcal{N}_2(\mathcal{R})$ is the number of pairs with a given relative angular momentum $\mathcal{R}$. It is related to the so-called Haldane (pair) amplitude $\mathcal{G}(\mathcal{R}) = \binom{N}{2}^{-1} \mathcal{N}_2(\mathcal{R})$, normalized to $\sum_{\mathcal{R}} \mathcal{G}(\mathcal{R}) = 1$. Sometimes, the qualitative properties of the system (e.g., a general form of the correlations) are determined completely by a few details about pseudopotential. For example, every pseudopotential decreasing faster than linearly with $\mathcal{R}$ induces Laughlin two-body correlations [7]. Using a simple model of pseudopotential such as:

$$U_\alpha(1) = \alpha, \quad U_\alpha(\mathcal{R} > 1) = 1/\mathcal{R}^2 \tag{2}$$

and changing parameter α we can study a wide class of interactions, such as electrons in the lowest and excited LL, and CFs in their excited LL.

For such pseudopotential we calculated $\mathcal{G}(1)$, $\mathcal{G}(3)$, and $\mathcal{G}(5)$ for the ground states of two chosen configurations $(N, 2l)$, representative of the $\nu = 1/2$ and $1/3$ states in CF-LL$_1$. The results are ploted in Fig. 1. At $\alpha < -0.25$ the Haldane pair amplitude $\mathcal{G}(1)$ has its maximum value and the particles form one big $\nu = 1$ quantum Hall droplet. For greater α, in both cases, the number of pairs with $\mathcal{R} = 1$ decreases and takes the minimum value for $\alpha > 0.3$, which means no clusters and Laughlin correlations. The number of pairs $\mathcal{N}_2(\mathcal{R})$ changes values quasi-discontinuously as a function of α and shows, visible for both configurations ($N = 12,\ 2l = 21$ and $2l = 29$), series of plateaus. This suggests that, depending on interactions, the N particles group into various clustered configurations.

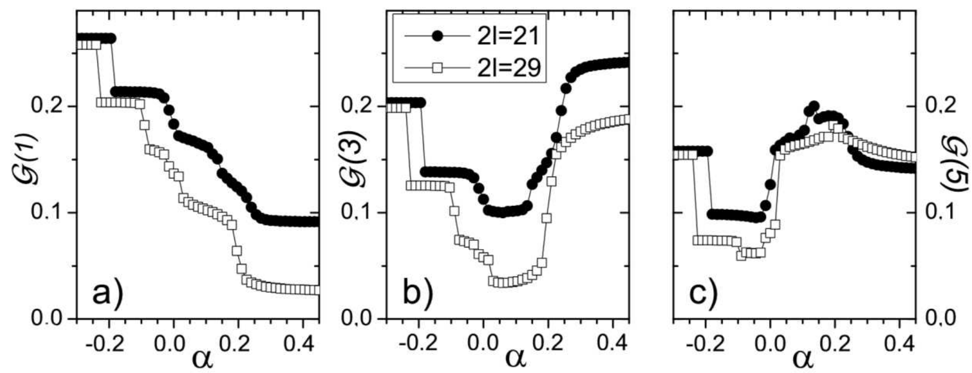

Fig. 1. Haldane pair amplitudes $\mathcal{G}(\mathcal{R})$, calculated for shells with $2l = 21$ and 29, as a function of parameter α of pseudopotential defined by Eq. (2), for the relative pair angular momenta $\mathcal{R} = 1$ (a), $\mathcal{R} = 3$ (b), and $\mathcal{R} = 5$ (c).

Now let us consider more realistic interactions. In Fig. 2a we show the number of pairs $\mathcal{N}_2$ for the ground states of $N = 12$ particles (QEs and electrons) as a function of double angular momentum $2l$. For electrons on their first LL we can recognize series of Jain states $\nu = n(2pn \pm 1)^{-1}$. For electrons on LL$_1$ and for QEs on CF-LL$_1$ we can identify the $\nu = 1/2$ series at $2l = 2N - 3$, the $\nu = 1/3$ series at $2l = 3N - 7$, and their particle–hole conjugates at $2l = 2N + 1$ and $3N/2 + 2$. As we can expect, for Laughlin/Jain states the number of pairs decreases as a function of $2l$, and it equals to zero for $\nu = 1/3$. Similarly, for $2l > 29$ in LL$_1$ the $\mathcal{N}_2$ drops almost linearly with increasing $2l$, aiming at zero

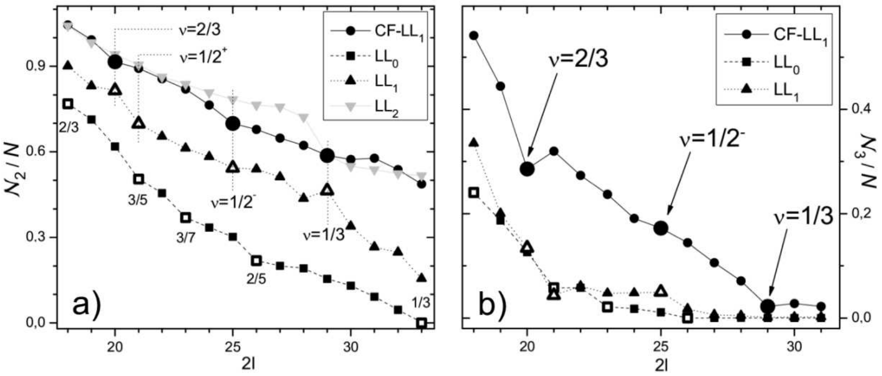

Fig. 2. Number of pairs $\mathcal{N}_2(\mathcal{R}=1)$ (a) and triplets $\mathcal{N}_3(\mathcal{T}=3)$ (b) as a function of double shell angular momentum $2l$ for interaction in different electron Landau levels (LL_n) and in the first excited CF Landau level (CF-LL_1).

for $2l \approx 35$. This suggests Laughlin correlations in LL_1 at sufficiently low filling. In comparison, in the CF-LL_1 there are considerably more pairs, almost the same number as in LL_2. We should emphasize that the $\nu = 1/2$ state on the electron LL_1 is the Moore–Read state, which is known to be paired. Indeed, in this case $\mathcal{N}_2 \approx N/2$. A similar value is obtained for the $\nu = 1/3$ state in CF-LL_1.

To consider the possible occurrence of clusters containing more than two particles it is convenient to use the three-body pseudopotential $V_3(\mathcal{T}, \beta_3)$, where $\mathcal{T} = 3l - L_3$ is the three-body relative angular momentum, with the allowed values $\mathcal{T} = 3$ or $\mathcal{T} \geq 5$ for identical fermions. Here, L_3 is the total triplet angular momentum and β_3 distinguishes between degenerate mulitiplets at the same $\mathcal{T}$. The many-body energy can be expressed as

$$E = \sum_{\mathcal{T},\beta_3} \mathcal{N}_3(\mathcal{T}, \beta_3) V_3(\mathcal{T}, \beta_3), \tag{3}$$

where $\mathcal{N}_3(\mathcal{T})$ is the number of triplets with a given $\mathcal{T}$. Larger $\mathcal{T}$ means larger expectation value of the area spanned by three particles. For $\mathcal{T} < 9$ there is only one possible triplet, so index β_3 can be omitted.

In order to show the short-range three-body correlations we plot in Fig. 2b the number of "compact" triplets with $\mathcal{T} = 3$ as a function of $2l$. Clearly, $\mathcal{N}_3$ quickly decreases with growing $2l$ for both LL_0 and LL_1, and at $2l = 21$ it actually reaches virtually zero. It means that for the sufficiently low ν, in both LLs there are no triplets, and thus that no clusters larger than pairs form. The number of QE triplets in CF-LL_1 is also a nearly linear function of $2l$, but it diminishes to zero at $2l = 3N - 7 = 29$, i.e. for $\nu = 1/3$. The vanishing of $\mathcal{N}_3$ together with having $\mathcal{N}_2 = N/2$ is the evidence of QE pairing at $\nu_e = 4/11$.

The elementary excitations of the Moore–Read state at $2l > 2N - 3$ are the quarter-charged QHs (of the Laughlin liquid of pairs [9]) and the pair-breaking neutral-fermion excitations. Being paired [10–12], the QE state at $2l = 3N - 7$

can only contain the QHs but no pair-breakers. The interaction of Moore–Read QHs in CF-LL_1 is not known, but evidently causes QH condensation at $\nu = 1/3$ (even though the QE Moore–Read state at $\nu = 1/2$ is unstable).

For $2l = 20$ (i.e., at $\nu = 2/3$ in CF-LL_1, i.e., at $\nu_e = 5/13$) the number of triplets $\mathcal{N}_3/N \approx 1/3$, suggesting that the system of N QEs is divided into $N/3$ clusters, each containing three particles. For $2l = 25$ (i.e., at $\nu = 1/2$ or $\nu_e = 3/8$), $\mathcal{N}_3/N \approx 1/6$ implies a more complicated cluster configuration.

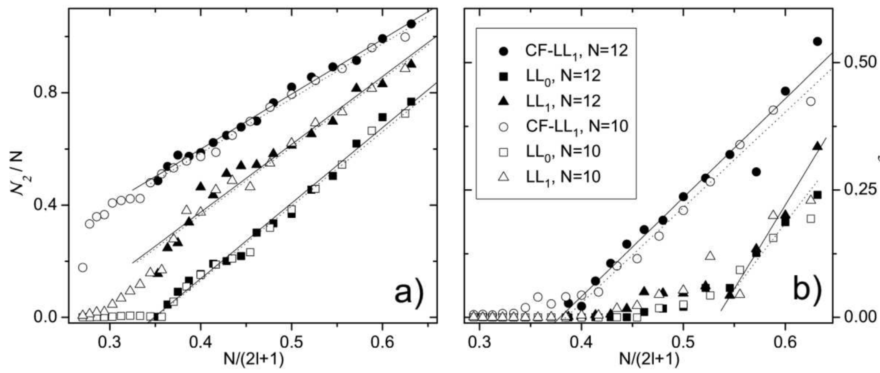

Fig. 3. Number of pairs $\mathcal{N}_2$ (a) and triplets $\mathcal{N}_3$ (b) as a function of the filling factor $N/(2l + 1) \approx \nu$ for systems containing $N = 10$ and $N = 12$ particles (electrons on LL_0 and LL_1 or quasielectrons on CF-LL_1).

Finally let us compare results obtained for different system sizes. Figure 3a shows the relative number of pairs $\mathcal{N}_2(1)/N$ as a function of $N/(2l+1) \approx \nu$. Data obtained for $N = 10$ reveal an almost identical, quasi-linear dependence as for $N = 12$, with a different slope for different interactions. The fewest pairs can be found for electrons on LL_0, and the most for CF-LL_1. In Fig. 3b we plot the relative number of triplets $\mathcal{N}_3(1)/N$ as a function of $N/(2l + 1)$. Also in this case the dependence seems to be linear for sufficiently large ν. The difference between data obtained for $N = 10$ and 12 is much better visible here, especially for electrons on the LL_0 and LL_1. However, we should remember that $N = 10$ is not divisible by 3, so it is not a good candidate to test three-body clusterization.

4. Conclusion

In numerical calculations for the systems of $N = 12$ particles we showed that the number of CF triplets with the minimum allowed $\mathcal{T} = 3$ decreases as a function of $2l$ from $\mathcal{N}_3 = N/3$ at $\nu = 2/3$ to zero at $\nu = 1/3$. At the same time, the number of CF pairs with $\mathcal{R} = 1$ decreases from $\mathcal{N}_2 = N$ to $N/2$. This is the evidence for CF pairing at $\nu = 1/3$, i.e., in the corresponding $\nu_e = 4/11$ FQH state, in analogy to the signature of pairing in the half-filled Moore–Read state of electrons. Hence, the $\nu_e = 4/11$ is interpreted as a condensate of the quasiholes of the Moore–Read state of the CFs.

Acknowledgments

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