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# Spontaneous Currents in a Ferromagnet– Normal Metal–Superconductor Trilayer

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We discuss the ground state properties of the system composed of a normal metal sandwiched between ferromagnet and superconductor within a tight binding Hubbard model. We solved the spin-polarized Hartree–Fock–Gorkov equations together with the Maxwell equation (Ampere’s law) and found a proximity induced Fulde–Ferrell–Larkin–Ovchinnikov state in this system. Here we show that the inclusion of the normal metal layer in between those subsystems does not necessarily lead to the suppression of the Fulde–Ferrell–Larkin–Ovchinnikov phase. Moreover, we found that depending on the thickness of the normal metal slab the system can be switched periodically between the state with the spontaneous current flowing to that one with no current. All these effects can be explained in terms of the Andreev bound states formed in such structures.

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## 1. Introduction

The proximity effect between a ferromagnet and a superconductor has attracted much attention recently due to its potential applications in such areas of technology as magnetoelectronics [1] or quantum computing [2]. The proximity effect in ferromagnet–superconductor (F–S) structures is also important from the point of view of the scientific interest as it allows the study of the interplay between ferromagnetism and superconductivity [3].

Due to time reversal symmetry breaking the proximity effect in F–S structures leads to new phenomena, not observed in usual normal metal–superconductor (N–S) proximity systems. Those are: oscillations of the superconducting transition temperature [4], density of states [5] and superconducting pairing amplitude in F–S multilayers [6], Josephson  $\pi$ -junction behavior in S–F–S heterostructures [7], a giant mutual proximity effect in S–F systems [8], spin valve [9] or spontaneous currents in F–S bilayers [10–12]. For review of the literature on those and related effects see [13].

We have recently examined F–S proximity system and found that a spontaneously generated current flows on both sides of the interface [10–12]. Such a current, which flows in the ground state of the system, is a hallmark of the Fulde–Ferrell–Larkin–Ovchinnikov (FFLO)-like state, originally predicted for a bulk superconductor in a magnetic exchange field acting on spins only [14]. It is the purpose of the present paper to see what the effect on the FFLO state will have a normal metal sandwiched between ferromagnet and superconductor (F–N–S structure). Will the spontaneous current still be generated? If so, how will it be modified? In the rest of the paper we show that the presence of the normal metal has nontrivial consequences on the generation of the spontaneous currents and on the FFLO state in general.

## 2. The model

Our model system is schematically shown in Fig. 1. It consists of a ferromagnet (layers  $n = -d_F, \dots, -d_N$ ), a normal (paramagnetic) region (layers  $n = -d_N, \dots, 0$ ) and a semi-infinite superconductor (layers  $n = 1, \dots$ ).

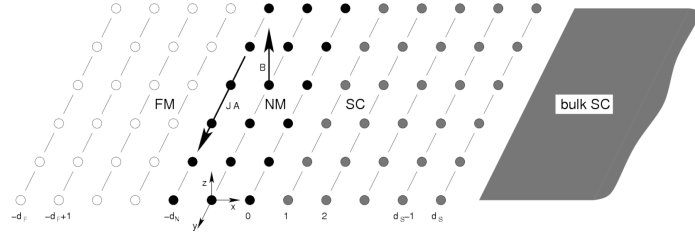


Fig. 1. Schematic view of our model system. The rows of empty circles ( $n = -d_F, \dots, -d_N$ ) describe ferromagnetic layers, filled ( $n = -d_N, \dots, 0$ ) — normal region, while shaded ( $n = 1, \dots$ ) — superconducting one. Directions of the magnetic field  $\mathbf{B}$ , vector potential  $\mathbf{A}$ , and current  $\mathbf{J}$  are also indicated.

The model Hamiltonian is given by single orbital Hubbard model

$$H = \sum_{ij\sigma} [t_{ij} + (\varepsilon_{i\sigma} - \mu)\delta_{ij}] c_{i\sigma}^\dagger c_{j\sigma} + \frac{1}{2} \sum_{i\sigma} U_i \hat{n}_{i\sigma} \hat{n}_{i-\sigma}, \quad (1)$$

where, in the presence of a vector potential, the nearest neighbor hopping integral is  $t_{ij} = -t \exp[-ie \int_{\mathbf{r}_i}^{\mathbf{r}_j} \mathbf{A}(\mathbf{r}) \cdot d\mathbf{r}]$ . The site energy levels  $\varepsilon_{i\sigma}$  are equal to  $\frac{1}{2} E_{ex} \sigma$

in the ferromagnet and 0 in the normal and superconducting region, and  $\mu$  is the chemical potential.  $U_i$  is the on-site electron–electron interaction, which is assumed to be negative on superconducting side and zero elsewhere,  $c_{i\sigma}^+$  ( $c_{i\sigma}$ ) are the usual electron creation (annihilation) operators and  $\hat{n}_{i\sigma} = c_{i\sigma}^+ c_{i\sigma}$  is the electron number operator.

We study the above model in the spin-polarized Hartree–Fock–Gorkov (SPHFG) approximation, assuming the Landau gauge for the magnetic field  $\mathbf{B} = (0, 0, B_z(x))$ , thus  $\mathbf{A} = (0, A_y(x), 0)$  (see Fig. 1). In the following we assume periodicity of our model in the direction parallel to the interface and therefore we work in  $\mathbf{k}$  space in the  $y$  direction but in real space in the  $x$  direction.

As usual, self-consistency is assured by the relations determining superconducting (SC) order parameter  $\Delta_n$ , current  $J_{y\uparrow(\downarrow)}(n)$  and the vector potential  $A_y(n)$  on each layer  $n$ :

$$\Delta_n = U_n \chi_n = -\frac{U_n}{\pi} \sum_{k_y} \int d\omega \text{Im} G_{nn}^{12}(\omega, k_y) f(\omega), \quad (2)$$

$$J_{y\uparrow(\downarrow)}(n) = -\frac{2et}{\pi} \sum_{k_y} \sin[k_y - eA_y(n)] \int d\omega \text{Im} G_{nn}^{11(33)}(\omega, k_y) f(\omega), \quad (3)$$

$$A_y(n+1) - 2A_y(n) + A_y(n-1) = -4\pi J_y(n), \quad (4)$$

where  $G_{nm}^{\alpha\beta}(\omega, k_y)$  is the  $4 \times 4$  Nambu–Green function and  $f(\omega)$  the Fermi distribution. Equation (4) is a lattice version of the Ampere law  $\frac{d^2 A_y(x)}{dx^2} = -4\pi J_y(x)$ . The above equations have to be solved self-consistently, using the method described in [10].

### 3. Results

Figure 2 shows the spatial dependence of the normalized superconducting pairing amplitude  $\tilde{\chi}_n = \chi_n/\chi_{\text{bulk}}$  (Fig. 2a) and spontaneous current  $J_n$  (Fig. 2b) for a number of normal metal  $d_N$  layers ( $d_F = 20 - d_N$ ). As in our previous work on F–S structures [6, 10–12], the superconducting pairing amplitude shows

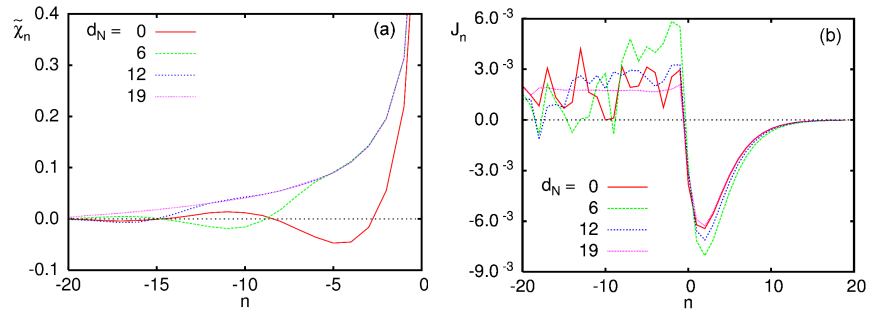


Fig. 2. Spatial dependence of the normalized superconducting pairing amplitude  $\tilde{\chi}_n = \chi_n/\chi_{\text{bulk}}$  (a) and spontaneous current  $J_n$  (b) for a number of  $d_F$  and  $d_N$  ( $d_F + d_N = 20$ ) layers. The model parameters are:  $E_{ex} = 0.517$ ,  $\Delta_S = 0.528$  and  $T = 10^{-2}$  in units of  $t$ .

clear oscillations in the ferromagnet. The period of the oscillations is related to the ferromagnetic coherence length  $\xi_F = 2t/E_{ex}$ , i.e.  $\chi_n \propto \frac{\sin(n/\xi_F)}{n/\xi_F}$ . No such effect is observed in the N region. One could expect that the inclusion of the normal metal suppresses the oscillations of  $\chi_n$  in the F region. Interestingly, the amplitude of the  $\chi_n$  oscillations on the ferromagnetic side remains almost unchanged. Only the phase of the oscillations can change. Moreover, as for F–S bilayer [10], the vanishing of the pairing amplitude at  $n = -d_F$  is related to the crossing of the Andreev bound states (ABS) through the Fermi energy of the system. Furthermore, in such a situation, spontaneous current is generated [10]. The current flowing produces a magnetic field, which splits this zero energy ABS, thus lowering the total energy of the system. The energy of this splitting is given in our model by  $\delta \approx 2et\bar{A}_y$ , where  $\bar{A}_y$  is the layer averaged vector potential.

As we already mentioned, the zero energy ABS is responsible for the generation of the spontaneous current. The typical distribution of such current, flowing parallel to the interface, is shown in Fig. 2b. The current flows in the negative  $y$  direction on the superconducting side and in the positive direction in the whole F–N region, giving a total current equal to zero, as should be in the true ground state. Interestingly, the current in the ferromagnet shows also oscillatory behavior, however, with a different period than the pairing amplitude does. The oscillations of the current are related to the oscillations of the density of states at the Fermi energy [10, 11]. On the other hand, in the N region, the distribution of the current is rather smooth, similarly to the pairing amplitude (let us compare the solid and thin dotted lines in Fig. 2).

The state with spontaneous current is a true ground state, as it has lower energy than the state with no current. This can be seen in Fig. 3, which shows the change in total energy of the system  $\Delta E_{\text{tot}}$  between the solution with spontaneous current and the one where the current is constrained to be zero is shown. Another important finding is that the system can be periodically switched between the states with and without spontaneous current by increasing the thickness of the normal metal region. For  $d_F + d_N = 20$  layers and  $E_{ex} = 0.517$  the period is equal to six (see Fig. 3).

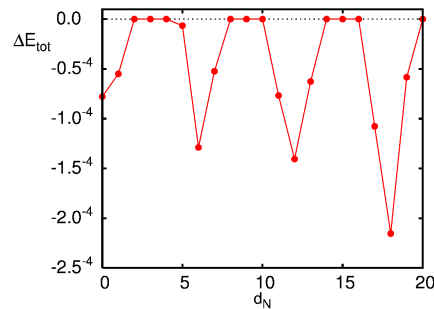


Fig. 3. The difference in total energy between solutions with and without current.

All the above results show that inclusion of the normal metal slab between the ferromagnet and superconductor has non-trivial effects on the physics of the proximity induced FFLO state. It cannot be argued that the normal metal simply acts as interface transparency, as in our case the N slab is in the clean (ballistic) regime, and thus does not suppress the proximity effect. However, it leads to the suppression of the oscillatory behavior of the pairing amplitude, spontaneous current and the density of states at the Fermi energy. Moreover, it strongly modifies the positions of the ABS, which leads to the periodical switching between the states with and without current. Interface transparency also leads to such periodical switching of the current [11], however, at the same time it also kills the proximity effect, suppressing the current and changing the period of the pairing amplitude oscillations. No such effects are caused by the inclusion of the normal metal slab. Perhaps the effect of the N slab would be more similar to the effect of reduced interface transparency if the normal metal was in the dirty regime.

#### 4. Conclusions

In conclusion, we have studied the ground state properties of F–N–S proximity system. We have observed oscillatory behavior of the superconducting pairing amplitude in ferromagnet, but not in the normal region. Similarly to F–S structures, we have found spontaneously generated currents flowing in the whole F and N regions and within a distance of a few  $\xi_S$  on superconducting side. Interestingly, the system can be switched periodically (changing  $d_N$ ) between the states with and without the spontaneous current. All this suggests possible realization of the proximity induced FFLO state in F–N–S heterostructure.

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