Vol. **108** (2005) *ACTA PHYSICA POLONICA A* No. 5

Proceedings of the XXXIV International School of Semiconducting Compounds, Jaszowiec 2005

Josephson-Like Effect in Mesoscopic Loop Structures

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The quantum interference is considered in time-dependent magnetic field that is quantum beats in mesoscopic loop structure. The similarities between this effect and Josephson, scalar Aharonov–Bohm and Aharonov–Casher effects, as well as their differences are treated and a possible application of the effect to the construction of the device, complementary to SQUIDs, is analysed.

PACS numbers: 78.47.+p, 03.67.–a, 73.23.Ad, 75.47.Jn, 85.35.Ds

1. Introduction

In recent years a new paradigm of electronics based on the spin degree of freedom of the electron has begun to emerge, so even the name for this new branch of the science and technology was coined, "spintronics". Plainly speaking, spintronics is an attempt to substitute the electron charge by its spin in order to use it for the variety of practical applications [1].

One of the main ideas which underpins different possible applications of "spin transport", including information storage and computation, is that the spins of electrons in semiconductors may have very long quantum coherence times [2], or in other words, electrons can travel a long way without flipping their spins. But this also gives the possibility to observe quantum effects which involve the interference of electron waves. The destruction of quantum coherence is controlled by the phase relaxation time or phase relaxation length. Since for the electron spin this length may be very long, it is naturally to expect that the spin interference can reveal itself in the conductance oscillations similar to the ones which are due to Aharonov–Bohm effect. Most of the researchers who dealt with the Aharonov–Bohm effect in the solid state [3] considered mainly the Hamiltonian $\hat{H} = (\boldsymbol{p} - (e/c)\boldsymbol{A})^2/2m^* + U(y,z)$, where $U(y,z)$ is the energy corresponding to the transverse motion, and

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almost nobody takes into account the spin-part $\mu_{\mathrm{B}}\hat{\boldsymbol{\sigma}}\boldsymbol{B}$ of the Hamiltonian (μ_{B} is the Bohr magneton, $\hat{\boldsymbol{\sigma}}$ is the electron spin operator, $\boldsymbol{B}$ — magnetic field). However, if the quantum interference is concerned, the quantity of main importance is the coherence length. If one considers the total Hamiltonian which includes Pauli term, one can write down the electron wave function in a factorized form as a tensor product: $\Psi(\boldsymbol{r}, s) = \varphi(\boldsymbol{r}) \otimes \chi(s)$ and consider the coherence of each part separately. As a result, it is possible to introduce two phase relaxation lengths, the first one for the "orbital part" of the electron wave function, $L_{\varphi}^{(\mathrm{e})}$, and the second one, $L_{\varphi}^{(\mathrm{s})}$ for the spin part of the wave function. It turns out [4–6] that $L_{\varphi}^{(\mathrm{s})} \gg L_{\varphi}^{(\mathrm{e})}$ which is in total agreement with the experiment [2]. The physics which is behind that is the following. An electron during its transfer along some path in the solid (semiconductor, for definiteness) interacts all the time with the environment. As a rule rigid scatters such as impurities and other defects of crystalline structure do not contribute to the phase relaxation; only dynamical scatters like phonons do. On the other hand, the electron scattering by phonons is mainly inelastic, while impurity scattering is mainly elastic, so we can say that only inelastic scattering contributes to the phase relaxation. But what does it mean inelastic scattering in case of spin? It means spin flips caused by spin–orbit interaction accompanied by phonon interaction, since there must be an agent which adds or subtracts the Zeeman energy to the electron spin. This kind of interaction is very weak and that is why the spin flips are rare events and the phase relaxation length for the spin part of the electron wave function is very long. If the structure length L is chosen to be $L_{\varphi}^{(\mathrm{s})} > L > L_{\varphi}^{(\mathrm{e})}$, it is possible to "wash out" the quantum interference related to phase coherence of the "orbital part" of the wave function retaining at the same time the phase coherence of the spin part one and hence, to reveal the corresponding conductance oscillations of the microstructure. Such model was considered in the papers [4–6], where the theory of the quantum interference in a loop structure due to Larmor precession of electron spin in semiconductor microstructure was presented. The aim of this paper is to attract one to another quantum interference effect which can occur in a loop structure similar to that considered in [4, 5] in a time-dependent magnetic field.

2. Quantum interference in time-dependent magnetic field (quantum beats)

Whenever one deals with a physical phenomenon in which the motion of some object can be represented as the superposition of two harmonic oscillations with two angular frequencies ω_1, ω_2 which are very close to each other ($\omega_1 \approx \omega_2$), one can expect to observe the beats. That is, the resulting *almost* harmonic oscillation occurs at the frequency $\bar{\omega} = (1/2)(\omega_1 + \omega_2)$ and with the slowly varying amplitude, $A(t) \sim \cos(\omega_{\mathrm{mod}} t)$, where $\omega_{\mathrm{mod}} = (1/2)|\omega_1 - \omega_2|$.

Quantum beats are usually associated with coherence-sensitive experimental methods, such as four-wave mixing, photon echoes, coherent Raman scattering, and resonance fluorescence. When two nondegenerate states are coherently driven by an optical field, the induced polarizations in the medium interfere and give rise to a beating in the difference between the energies of these states. In semiconductors, the continuous density of states results in a rapid decay of the coherence of the induced polarization. The aim of this work is to treat the influence of time-dependent magnetic field on the spin coherent transport in a loop structure, to point out the possibility to get quantum beats somewhat different from those reported earlier [7–9] and to stress its amazing similarity to Josephson effect in superconductors.

Let us consider the loop structure similar to that one of Ref. [4, 6]. The main difference however now is that an external magnetic field is time dependent and its amplitude is supposed to be somewhat different in the channels 1 and 2 (Fig. 1).

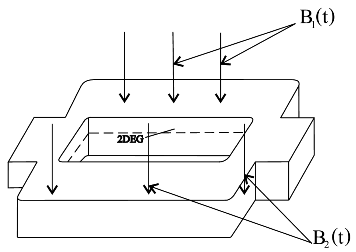

Fig. 1. A sketch of a two-channel semiconductor mesoscopic structure in external time-dependent magnetic fields of different amplitudes accross the channels.

Suppose the Hamiltonian of an electron is (see also [10]): $H(t) = H_0(t) \otimes I_0 + H_1(t) \otimes I_1$, where

$$H_0 = (1/2)m^*(\boldsymbol{p} - (e/c)\boldsymbol{A}(t))^2 + U(\boldsymbol{r}), \quad H_1 = -\mu_\mathrm{B}\hat{\boldsymbol{\sigma}} \cdot \boldsymbol{B}(\mathrm{t}). \tag{1}$$

Here I_0, I_1 are the unit operators acting in the state spaces of H_0 and H_1, respectively, m^* is the electron effective mass, $\boldsymbol{A}$ is the vector potential corresponding to the magnetic field $\boldsymbol{B}$, μ_B and $\hat{\boldsymbol{\sigma}}$ are Bohr magneton and the spin operator, respectively. We also assume that $U(\boldsymbol{r})$ describes conduction bands bending due to space charge and discontinuities of any band. Since H_0 does not depend on spin, the wave function is the tensor product: $\Psi(\boldsymbol{r}, t, s) = \varphi(\boldsymbol{r}, t) \otimes \chi(s, t)$. Ever since for convenience we shall refer to $\varphi(\boldsymbol{r}, t)$ as the "orbital part" of the total wave function, keeping in mind that it corresponds to H_0 describing the charge-field interaction, and we shall refer to $\chi(s, t)$ as the spin-part of the wave function related to H_1, the spin part of the Hamiltonian (1).

Assume the characteristic time scale of magnetic field changing is much longer than all characteristic electron scattering times, spin relaxation time including, then the main results of Refs. [4, 6] are applicable to this particular case of time-dependent magnetic field.

Indeed, introducing the factorized form of the wave function as the tensor product of orbital and spin parts, one not only can, but rather have to introduce simultaneously two phase relaxation lengths, the first one for the "orbital part" of the electron wave function, $L_\varphi^{(e)}$, and the second one, $L_\varphi^{(s)}$ for the spin part. As we already mentioned in the Introduction, it turns out [4, 6] that $L_\varphi^{(s)} \gg L_\varphi^{(e)}$, which is in total agreement with the experiment [2]. As a rule rigid scatterers such as impurities and other defects of crystalline structure, do not contribute to the phase relaxation, but inelastic interactions may and in general do destroy the phase coherence of the wave function. In the papers [4, 6] we have considered different mechanisms of spin decoherence and shown that indeed $L_\varphi^{(s)} \gg L_\varphi^{(e)}$.

Let us consider now the non-relativistic motion of an electron with the spin $|s| = 1/2$ in a loop structure of Fig. 1, where the magnetic fields in the two arms of the structure are: $B_1 = B_{01}\cos\omega t$ and $B_2 = B_{02}\cos\omega t$. In accordance with general quantum mechanical approach (see, for instance, Aharonov–Bohm phase description given by Feynman [11]), time-dependent phase of precessing spin (actually, the Larmor rotating angle around the field B) can be introduced as follows:

$$\theta(t) = (\mu_{\mathrm{B}} g/\hbar) \int_0^t B(t')\mathrm{d}t'. \tag{2}$$

Here we again suppose the majority of electrons to be at the Fermi surface and to have the velocity equal to the Fermi velocity v_{F}.

Then using the technique described in Refs. [4, 6] one can calculate the current through the structure, which is equal to

$$I = (2e/h)\, K(A + D\cos(\Delta\theta(t))), \tag{3}$$

where K, A, D are the coefficients dependent on the peculiarities of the structure. The phase shift $\Delta\theta(t)$ acquired by the spin wave function of an electron moving through the structure, is

$$\Delta\theta(t) = (\mu_{\mathrm{B}} g \Delta B/\hbar\omega)\sin(\omega t), \tag{4}$$

where $\Delta B = B_{01} - B_{02}$.

Figure 2 represents the current through the loop structure plotted according to the formulae (3, 4) for different semiconductors and some chosen values of ΔB and ω; we also assumed $A \sim D$. It is clearly seen that in all cases the curves representing current versus time consist of a spire-like pieces separated by rather more slow undulations. Obviously, spire-like pieces correspond to relatively great values of the prefactor $(\mu_{\mathrm{B}} g \Delta B/\hbar\omega)$ in (4) and the values of $\sin\omega t$ which are very close to unity, while relatively slow undulations correspond to those values of $\sin\omega t$ which are very nearly to zero. Therefore, if one would measure the current through the structure in question by means of the device with relatively rough time resolution, this fine spire-like structure would be smeared out and one could observe only that current undulates up and down from its average value almost

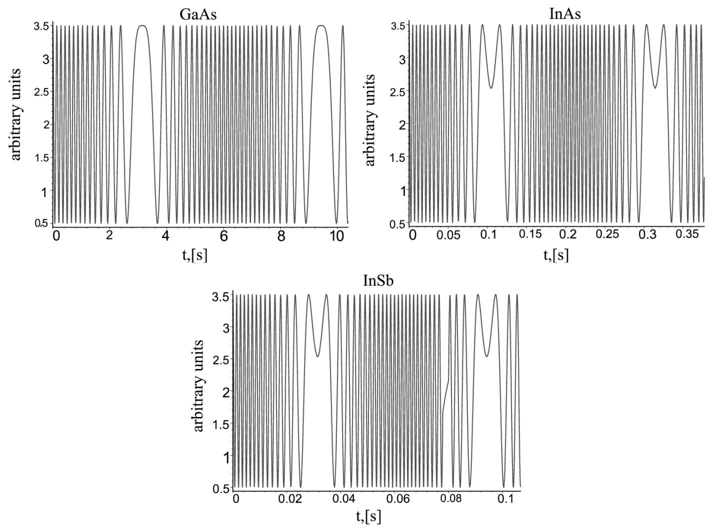

Fig. 2. Quantum beats in mesoscopic loop structure due to spin coherent transport and Larmor precession. Three pictures represent current through the structure versus time for three different semiconductors. The parameters chosen for the calculations are: InSb — $|g| = 50.7$, $\omega = 50$ Hz; InAs — $|g| = 15.3$, $\omega = 15$ Hz; GaAs — $|g| = 0.50$, $\omega = 0.5$ Hz; $\Delta B = 10^{-5}$ Gs throughout.

periodically. From Fig. 2 one can easily estimate this periodicity: if one has the device which can probe the current with time resolution ~ 0.01 s, then the period of undulations for InSb would be about 0.06 s, for InAs — 0.05 s and for GaAs — about 6 s (notice that ΔB in all cases is supposed to be the same, while ω is chosen to be different in order to "compensate" the difference in Lande factor g for these three materials).

It is especially easy to grasp what is going on, if one suppose that ω, the frequency of magnetic field oscillations, is so small that the condition $\omega t \ll 1$ is fulfilled. Then, from (3), (4) we have

$$I \propto [A + D\cos((\mu_{\mathrm{B}} g \Delta B/\hbar)t)] \,. \tag{5}$$

It is clear that since the factor $\omega_0 = \mu_{\mathrm{B}} g \Delta B/\hbar$ in the last expression, generally speaking, is large enough (for example, if $\Delta B \sim 1$ T, $\omega_0 \sim 4.4 \times 10^{10}$ Hz), on average the current through the structure is very nearly to some constant. However, if ΔB becomes very small, say, 10^{-5}–10^{-6} Gs, the period $T = 2\pi/\omega_0$ of oscillations in (3) becomes equal to 0.14–1.4 s and this yields the possibility to observe current modulation.

Thus, the current through the structure should oscillate without any apparent change in the structure and this kind of oscillations can also be called *quantum beats*. These beats are very similar to the Josephson effect. Indeed, in this case the phase difference $\Delta\theta(t)$ is driven by the magnetic field (see (4), (5) which is analogous to the Josephson effect in superconducting tunnel junctions, where the phase of Cooper pairs in two superconductors separated by the insulator film (so-called weak link) is related according to $\Delta\theta(t) = (2eV/\hbar)t$, where V is the voltage applied to the junction and the superconducting Josephson current of

$I_{\rm S} = I_{\rm Smax} \sin \Delta\theta(t)$.

Let us analize an amazing similarity of these two effects more thoroughly. Indeed, while the phase difference of Cooper pairs in Josephson junction driven by an applied voltage is equal $\Delta\theta(t) = 2eVt/\hbar$, the phase difference of the electrons spin wave function, acquired during electron transport through the channels of the loop structure driven by magnetic field, is equal to $\Delta\theta(t) = \mu_{\rm B} g \Delta B t/\hbar$. The effect discussed here differs, however, from Josephson one in some important respect. In order to grasp it, let us look at the effect discussed in the paper, from another point of view. Namely, from the point of view of its possible applications to measure extremely small magnetic fields deviations from the spatial uniformity.

It is well known that superconducting quantum interference devices (SQUIDs) operating at 4 K have been unchallenged as ultrahigh, sensitivity magnetic field detectors [12]. They have enabled, for instance, biomagnetic imaging, such as mapping of the heart activity, mapping of the magnetic fields produced by the brain and so on. Since the current through the single SQUID (that is, two Josephson junctions in parallel, making a superconducting loop) is proportional to $\cos|2e\Phi/\hbar|$, where Φ is the magnetic flux threading the superconducting loop, one has, in order to enhance sensitivity of the device, to increase the flux and hence, the area of the loop. To this end one has to put a set of 10, 20 or even more Josephson junctions close together and equally spaced. Note that $2e\Phi/\hbar$ is nothing else but Aharonov–Bohm phase acquired by Cooper pairs during their transport along the superconducting loop. Now it is clear in what respect the effect discussed in the paper differs from Aharonov–Bohm one. The phase difference acquired by the electron spins *does not* depend on the flux threading the loop, but does depend on the magnetic field difference in the channels 1 and 2. It means that this effect perhaps could be used for constructing the devices complementary to SQUIDs in that respect that they could enable us to measure the slightest magnetic field deviations from the spatial uniformity in extremely small scale. How small the measuring area could be, one can estimate in a following way. In order to satisfy the condition $L_\varphi^{(\rm s)} > L > L_\varphi^{(\rm e)}$, it is sufficient to get $L \sim 1.5 \times 10^{-2}$ cm, while the distance between the channels 1 and 2 could be about 4.0×10^{-6} cm with the width of the channel of about 1.0×10^{-6} cm each. Then the measuring area can be about 9.0×10^{-8} cm^2; compare this value with the millimeter-scale spatial resolution of the SQUIDs or even with a somewhat smaller spatial resolution of a recently reported subfemtotesla atomic magnetometer [13]. The sensitivity of the device, based on the effect discussed here, can be estimated as to be 10^{-11} THz$^{-1/2}$, while its operating temperature could be about 40 K or even more. Indeed, since in the formula (1) $A \sim D$, the "contrast" of the "interference pattern" is determined only by the ratio $\sqrt{(E_{\rm F} - k_{\rm B}T)/E_{\rm F}}$ which at a temperature of about 40 K is of the order of 90%.

It is also instructive to compare the effect discussed in the paper, with other cyclic phenomena such as scalar Aharonov–Bohm (SAB) [14] and Aharonov–Casher (AC) [15] effects.

It is well known that in the Aharonov–Bohm (AB) effect a charge moving around magnetic flux filament in a region with vanishing electromagnetic fields, accumulates the phase shift. This is due to gauge invariant coupling between the current and electromagnetic vector potential and for that reason, the *locally* accumulated phase is not gauge invariant. Therefore, AB effect is sometimes termed as being nonlocal. On the other hand, in SAB effect, as it was argued by Peshkin [14] and in AC effect, as it was shown in [16], the magnetic moment of a neutral particle couples directly to the field strenghts, either $\boldsymbol{B}$ (as in case of SAB) or to $\boldsymbol{E}$, as in case of AS.

The effect discussed here is very similar to SAB and AC, because it is also brought about by an ordinary action of the Maxwell field and hence, has the properties of all other local interactions. The AB effect is nonlocal because the electron experiences no force and exchanges no momentum, energy or angular momentum with the electromagnetic field. In our case, just like in case of SAB and AC, the Hamiltonian and the equation of motion involve contemporaneous Maxwell field in the domain of the electron's position; thus, the effect is not entirely topological in character. The main difference between this effect and SAB and AC is that the former deals with charged particles (electrons) in semiconductor and could be observable only under special conditions, when the "phase memory" related to the orbital part of the wave function is "washed out", while the phase coherence of the spin part of the wave function remains intact. Thus, this spin coherence can reveal itself in corresponding spin current oscillations.

3. Conclusion

A simple theory of the quantum interference phenomenon, quantum beats which are due to Larmor precession of spins in mesoscopic loop structure, is presented. We have shown that if the amplitudes of magnetic fields in the channel 1 and 2 of the loop structure are a little bit different, say $\Delta B \sim 10^{-5}$–10^{-6} Gs, the quantum beats can occur in the structure which are the spin current modulation with a period of about 0.14–1.4 s.

The similarities, as well as differences between the quantum beats in a loop structure and Josephson effect are also discussed in the paper. It is worthy to stress that the effect considered in this work could be used for developing the device complementary to SQUIDs, devices which are also used for measuring small magnetic fields.

Small electric currents which occur in a human body are the sources of time--dependent magnetic fields whose amplitudes vary from few fT (such fields are generated by human brain) to 50000 fT (fields generated due to heart activity). Measuring small magnetic fields are extremely important in biomagnetic imaging and medical diagnostics where the use of the SQUIDs enables us to localize the sources of epilepsy with the accuracy of a few mm. The use of the hypotetical device proposed in the paper could perhaps reduce the measuring area to 1.5×10^{-2} cm $\times 5 \times 10^{-6}$ cm. This is because in order to enhance the sensitivity of the device like SQUID; one has to enhance the measuring area in order to make the magnetic flux threading the SQUID-loop greater, while sensitivity of the device, proposed in the paper, does not depend on the magnetic flux.

Acknowledgments

I am grateful to my supervisor Prof. I. Tralle for encouragement and numerous advices and discussions.

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