Optical Polarisation Bistability in χ^2 Media at Large Desynchronisation Regime

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In this work we present results of calculations concerning bistability of optical polarisation in birefringent media. Second-order optical nonlinearity is assumed to be dominant and represented by one parameter. In our model the second-harmonic generation regime is assumed. Light transmission is analysed in terms of standard Fabry–Perot model of feedback. We derived formulas for the Stokes parameters describing the polarisation state. Numerical results showed that three parameters may exhibit bistable behaviour. This creates new possibilities for optical switching.

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1. Introduction

Due to its potential applications optical polarisation bistability has been the subject of many papers. Its physical background is described in several textbooks (see [1–3] for a review). Nevertheless, there still remain some open questions which can enlighten this phenomenon. Firstly, in contrast to dominant tendency, we focus our attention on media with quadratic nonlinearity. Secondly, we explicitly take into account the phase mismatch, which plays important role in our considerations.

To explain second-order bistability we have to take into account the cascade process [4, 5] connected with the second-harmonic generation (SHG). The generated wave of frequency 2ω has a weak intensity and does not change essentially the amplitude of the incoming wave of frequency ω. However, its influence on the phase of primary wave is crucial. These phase changes depend on the intensity of incoming wave and result in the behaviour of polarisation state giving rise to the polarisation bistability of this wave.

2. Physical model

In birefringent crystals the condition of phase matching may not be fully satisfied (e.g. $n^{(\mathrm{o})}(\omega) < n^{(\mathrm{e})}(2\omega)$ for negative birefringence; here "o" stands for ordinary, "e" — for extraordinary wave). In order to maximise the SHG process we assume that the fundamental wave propagates along the Oz axis which is perpendicular to the main optical axis (in our model — the Oy axis) and to the surface of the sample. The wave vector $\boldsymbol{k} = (0, 0, k_\omega)$ maintains its directions and is perpendicular to the electric vector $\boldsymbol{E}_\omega = (E_\omega^x, E_\omega^y, 0)$.

The smallest phase mismatch in SHG is achieved in two kinds of processes

$$E_\omega^{(\mathrm{o})} + E_\omega^{(\mathrm{o})} \to E_{2\omega}^{(\mathrm{e})} \quad \text{and} \quad E_\omega^{(\mathrm{o})} + E_{2\omega}^{(\mathrm{e})} \to E_\omega^{(\mathrm{o})}. \tag{1}$$

The ordinary wave is polarised in the Ox direction, the extraordinary one — in the Oy direction. Thus $E_\omega^x = E_\omega^{(\mathrm{o})}$, $E_{2\omega}^x = 0$ and $E_\omega^y = E_\omega^{(\mathrm{o})}$, $E_{2\omega}^y = E_{2\omega}^{(\mathrm{e})}$. The component $E_\omega^{(\mathrm{o})}$ is not involved in the nonlinear interaction considered and its evolution follows the linear optics rules.

The quadratic nonlinearity is represented by the tensor χ^2. In the processes described above only one component $(\chi_{yxx}^{(2)}(2\omega))$ of it is effectively involved; it will be denoted further by γ. The presence of nonlinearity couples the primary wave (ω, indexed by "1") with its second harmonics (2ω, index "2"). As a result, an energy transfer between them takes place. It may exhibit spatial periodicity named the cascaded process. Such processes may lead to nonlinear phase shifts of waves considered. These phase changes create a basis for the optical bistability.

Following the standard procedure (see, for example, [6]) we get the following equations describing the SHG process:

$$\frac{\mathrm{d}E_1}{\mathrm{d}z} = -\mathrm{i}\kappa_1 E_2 E_1^* \exp\left(\mathrm{i}\Delta k z\right), \tag{2a}$$

$$\frac{\mathrm{d}E_2}{\mathrm{d}z} = -\mathrm{i}\kappa_2 E_1^2 \exp(-\mathrm{i}\Delta k z), \tag{2b}$$

where

$$\Delta k = k_2 - 2k_1 = \frac{2\omega}{c}(n_2^L - n_1^L) \tag{3}$$

is the wave-vector mismatch (n — refraction index). The parameters κ_j are proportional to γ:

$$\kappa_j = \frac{\mu_0 c\omega}{2n_j}\gamma. \tag{4}$$

Since $n_2 > n_1$, the mismatch $\Delta k > 0$.

In the following we will assume that Δk is sufficiently large. This implies the absence of depletion of the amplitude $|E_1|$ of the fundamental wave. However, its phase φ_1 depends on z. This variation obeys the rule

$$\delta\varphi_1 = \frac{\kappa_1\kappa_2}{\Delta k} z \left[1 - \frac{\sin(\Delta k z)}{\Delta k z}\right] |E_1|^2, \tag{5}$$

where $\delta\varphi_1 = \varphi_1(z) - \varphi_1(0)$. As $\Delta k > 0$ hence $\delta\varphi_1$ is also positive for each value of z.

3. Polarisation state of the primary wave

The most convenient tool of analysing optical wave polarisation are the Stokes parameters S_m which may be defined as follows:

$$S_1 = \frac{I_x - I_y}{I}, \qquad S_2 = \frac{2\sqrt{I_x I_y}}{I}\cos\theta, \qquad S_3 = \frac{2\sqrt{I_x I_y}}{I}\sin\theta, \tag{6}$$

where $I_x = |E_x|^2$ and $I_y = |E_y|^2$ denote two components of the electric field of the wave considered, $I = I_x + I_y$. The parameter θ stands for the difference of phases between ordinary and extraordinary component of the field $\boldsymbol{E}$ on input or output from the Fabry–Perot (F–P) cavity

$$\frac{E_y}{E} = \frac{|E_y|}{|E_x|}\mathrm{e}^{\mathrm{i}\theta}. \tag{7}$$

The three Stokes parameters can be introduced for any value of z. On the input to the F–P cavity ($z = 0$) we will index them by "in", on the output ($z = L$) — by "out". The difference

$$\delta\theta = \theta^{\text{out}} - \theta^{\text{in}}$$

may be determined from the formula

$$\exp(\delta\theta) = \sqrt{\frac{T_x}{T_y}}\frac{1}{1 - R\exp(2\mathrm{i}k_y L)}\left[\exp\left(\mathrm{i}(k_y - k_x)L - \mathrm{i}n^{(2)}\frac{1}{1-R}I_x^{\text{out}}\right)\right.$$

$$\left. - R\exp\left(\mathrm{i}(k_x + k_y)L + \mathrm{i}n^{(2)}\frac{R}{1-R}I_x^{\text{out}}\right)\right], \tag{8}$$

where R is the reflection coefficient of the cavity, and

$$T_x = \frac{I_x^{\text{out}}}{I_x^{\text{in}}} = \left[1 + \frac{4R}{1-R}\sin^2\left(k_x L + \frac{1+R}{2(1-R)}n^{(2)}I_x^{\text{out}}\right)\right]^{-1}, \tag{9a}$$

$$T_y = \frac{I_y^{\text{out}}}{I_y^{\text{in}}} = \left[1 + \frac{4R}{1-R}\sin^2\left(k_y L + \frac{\theta^{\text{in}}}{2}\right)\right]^{-1}. \tag{9b}$$

The quadratic part $n^{(2)}$ of the refraction index is defined by the formula: $n = n^L + n^{(2)}I$ and is connected with the phase shift (5) in a way analogous to cubic nonlinear media

$$\delta\varphi = n^{(2)}I. \tag{10}$$

The Stokes parameters are used to determination of the polarisation ellipse. We have calculated three characteristics of the ellipse: the ratio b/a of its axes, the angle ϕ of the inclination of the axis a and the sign of the polarisation parameter p. It describes the rotation of the electric vector and is equal to the ratio S_3/S_0, S_0 being the intensity of the wave.

4. Results of numerical calculations

We have performed calculations of polarisation ellipse parameters for different values of reflection coefficient R and for different polarisation states on the input. The results are collected in Figs. 1–3. These figures show the dependence of b/a, ϕ, and p on the intensity I^{in} of the incident wave for three types of its polarisation: linear, circular, and elliptic ones. In majority of cases we observed a bistable behaviour of the polarisation state. It appears only for sufficiently large values of reflection coefficient R (in all figures $R = 0.9$). For $R < 0.5$ the bistability effect is absent. The desynchronisation parameter (3) was chosen in a way suitable to Ag_3AsS_3 for which $\Delta n = 0.073$. Besides, λ was put to be 1.06×10^{-6} m.

Polarisation bistability allows a much wider set of possible states. In contrast to standard intensity bistability when we have only two states (0 or 1), the polarisation bistability offers eight different states. They may be ordered as follows:

$\delta = 0$: (**0**; 0-1-0) or (**0**; 1-0-1),

$\delta = \pi/2$: (**0**; 1-0-1) or (**1**; 0-1-0),

$\delta = \pi/4$: (**0**; 1-1-1) or (**1**; 0-0-1),

$\delta = -\pi/4$: (**0**; 1-1-0) or (**1**; 0-0-0).

Here δ describes the polarisation ellipse of the incident wave. The first (bold) digit in the parenthesis denotes intensity state of the incoming wave (0 — lower part of the bistability curve, 1 — upper part). Next three digits describe, respectively, similar values of b/a, ϕ, and p. Other values of δ will create other types of polarisation states.

Similar results were recently obtained [7] for the case of two incident waves. In this paper we have shown that also one incident wave may behave in a bistable way provided that SHG process is taken into account.

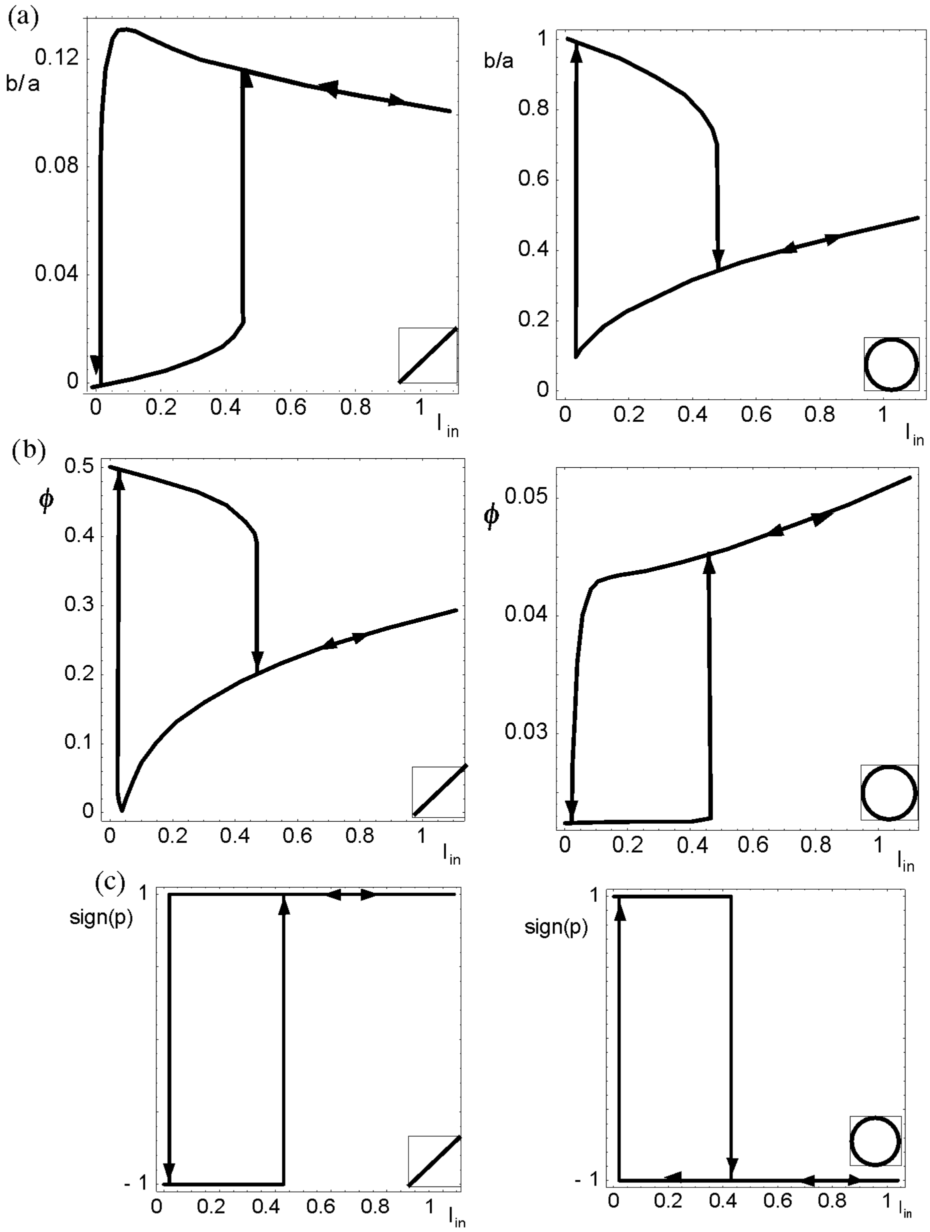

Fig. 1. Polarisation state parameters of the outgoing wave as functions of incident wave intensity $I_{\rm in}$ (in arbitrary units) for two different polarisations of the incident wave: linear (left part, $\delta = 0$) and circular (right part, $\delta = \pi/2$). (a) The ratio b/a, (b) the angle ϕ (in units of π), (c) the sign p of the rotation.

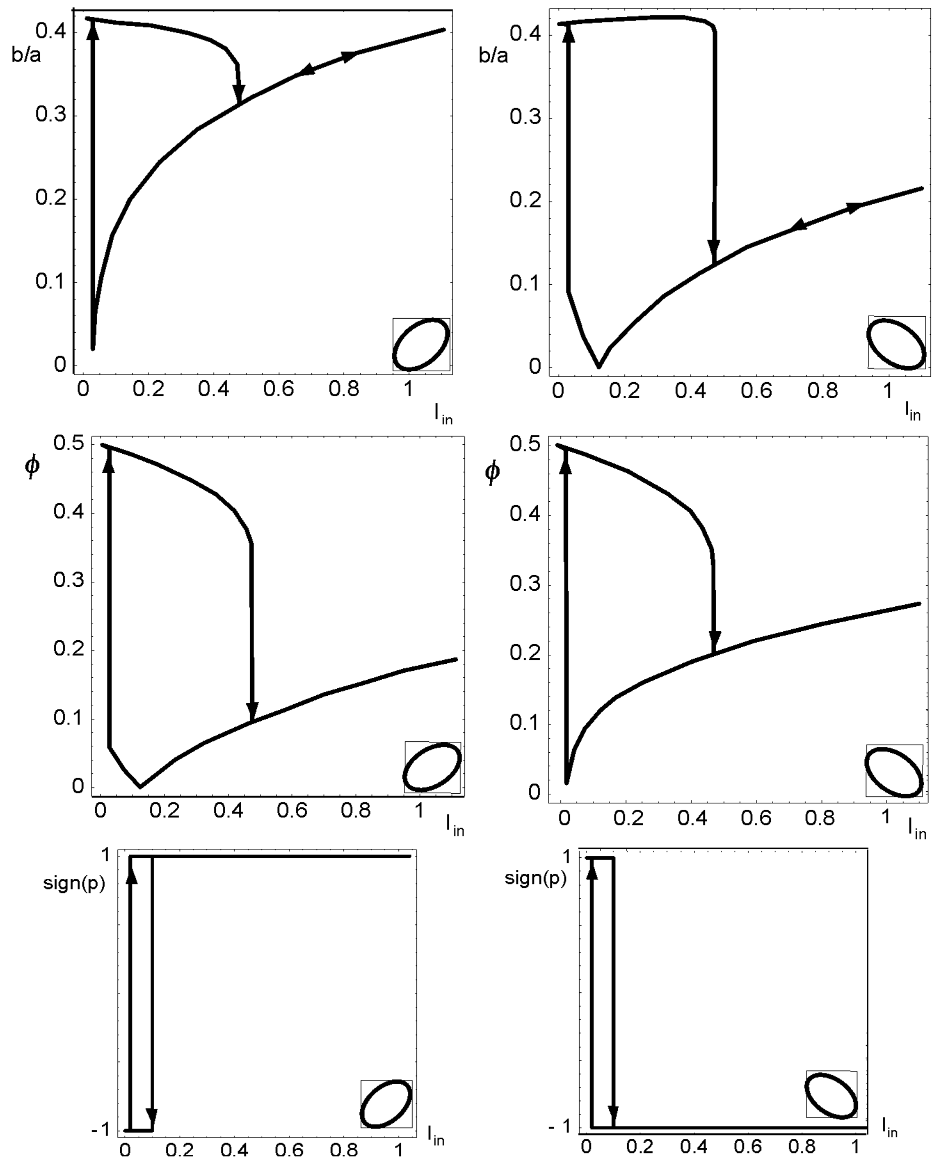

Fig. 2. Comparison of polarisation state parameters of the outgoing wave as function of the intensity of the incident wave with elliptic polarisation (two cases are shown in inserts in the right bottom corners. Left column corresponds to $\delta = \pi/4$, the right one to $\delta = -\pi/4$).

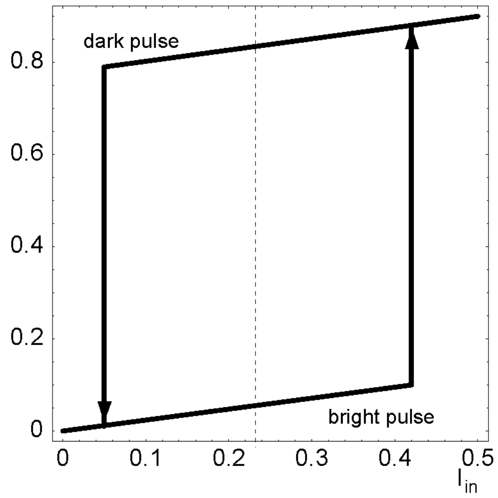

Fig. 3. The principle of switching between optical states of an optical logic gate. The logical state "1" is represented by the dark pulse, whereas the state "0" — by the bright pulse. The loop in the figure may be attributed to each polarisation parameter.

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